CHAPTER 7 QUANTUM THEORY AND ATOMIC STRUCTURE

The value for the speed of light will be 3.00×10^8 m/s except when more significant figures are necessary, in which cases, 2.9979×10^8 m/s will be used.

END–OF–CHAPTER PROBLEMS

7.1 All types of electromagnetic radiation travel as waves at the same speed. They differ in both their frequency, wavelength, and energy.

7.2 Plan: Recall that the shorter the wavelength, the higher the frequency and the greater the energy. Figure 7.3 describes the electromagnetic spectrum by wavelength and frequency.
Solution:
a) Wavelength increases from left (10^{-2} nm) to right (10^{12} nm) in Figure 7.3. The trend in increasing wavelength is: **x-ray < ultraviolet < visible < infrared < microwave < radio wave**.
b) Frequency is inversely proportional to wavelength according to the equation $c = \lambda\nu$, so frequency has the opposite trend: **radio wave < microwave < infrared < visible < ultraviolet < x-ray**.
c) Energy is directly proportional to frequency according to the equation $E = h\nu$. Therefore, the trend in increasing energy matches the trend in increasing frequency: **radio wave < microwave < infrared < visible < ultraviolet < x-ray**.

7.3 Evidence for the wave model is seen in the phenomena of diffraction and refraction. Evidence for the particle model includes the photoelectric effect and blackbody radiation.

7.4 In order to explain the formula he developed for the energy vs. wavelength data of blackbody radiation, Max Planck assumed that only certain quantities of energy, called quanta, could be emitted or absorbed. The magnitude of these gains and losses were whole number multiples of the frequency: $\Delta E = nh\nu$.

7.5 a) Frequency: **C < B < A**
b) Energy: **C < B < A**
c) Amplitude: **B < C < A**
d) Since wave A has a higher energy and frequency than B, wave **A** is more likely to cause a current.
e) Wave **C** is more likely to be infrared radiation since wave C has a longer wavelength than B.

7.6 Radiation (light energy) occurs as quanta of electromagnetic radiation, where each packet of energy is called a photon. The energy associated with this photon is fixed by its frequency, $E = h\nu$. Since energy depends on frequency, a threshold (minimum) frequency is to be expected. A current will flow as soon as a photon of sufficient energy reaches the metal plate, so there is no time lag.

7.7 Plan: Wavelength is related to frequency through the equation $c = \lambda\nu$. Recall that a Hz is a reciprocal second, or $1/\text{s} = \text{s}^{-1}$. Assume that the number "950" has three significant figures.
Solution:
$c = \lambda\nu$

$$\lambda\ (\text{m}) = \frac{c}{\nu} = \frac{3.00\times10^{8}\ \text{m/s}}{(950.\ \text{kHz})\left(\frac{10^{3}\ \text{Hz}}{1\ \text{kHz}}\right)\left(\frac{\text{s}^{-1}}{\text{Hz}}\right)} = 315.789 = \mathbf{316\ m}$$

$$\lambda\ (\text{nm}) = \frac{c}{\nu} = (315.789\ \text{m})\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = 3.15789\times10^{11} = \mathbf{3.16\times10^{11}\ nm}$$

$$\lambda\ (\text{Å}) = \frac{c}{\nu} = (315.789\ \text{m})\left(\frac{1\ \text{Å}}{10^{-10}\ \text{m}}\right) = 3.158\text{x}10^{12} = \mathbf{3.16x10^{12}\ Å}$$

7.8 Wavelength and frequency relate through the equation $c = \lambda\nu$. Recall that a Hz is a reciprocal second, or $1/\text{s} = \text{s}^{-1}$.

$$\lambda\ (\text{m}) = \frac{c}{\nu} = \frac{3.00\text{x}10^8\ \text{m/s}}{(93.5\ \text{MHz})\left(\frac{10^6\ \text{Hz}}{1\ \text{MHz}}\right)\left(\frac{\text{s}^{-1}}{\text{Hz}}\right)} = 3.208556 = \mathbf{3.21\ m}$$

$$\lambda\ (\text{nm}) = \frac{c}{\nu} = (3.208556\ \text{m})\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = 3.208556\text{x}10^{9} = \mathbf{3.21x10^{9}\ nm}$$

$$\lambda\ (\text{Å}) = \frac{c}{\nu} = (3.208556\ \text{m})\left(\frac{1\ \text{Å}}{10^{-10}\ \text{m}}\right) = 3.208556\text{x}10^{10} = \mathbf{3.21x10^{10}\ Å}$$

7.9 <u>Plan:</u> Frequency is related to energy through the equation $E = h\nu$. Note that 1 Hz = 1 s^{-1}.

<u>Solution:</u>

$E = h\nu$

$E = (6.626\text{x}10^{-34}\ \text{J•s})(3.8\text{x}10^{10}\ \text{s}^{-1}) = 2.51788\text{x}10^{-23} = \mathbf{2.5x10^{-23}\ J}$

7.10 $$E = \frac{hc}{\lambda} = \frac{(6.626\text{x}10^{-34}\ \text{J•s})(3.00\text{x}10^8\ \text{m/s})}{1.3\ \text{Å}}\left(\frac{1\ \text{Å}}{10^{-10}\ \text{m}}\right) = 1.5291\text{x}10^{-15} = \mathbf{1.5x10^{-15}\ J}$$

7.11 <u>Plan:</u> Energy is inversely proportional to wavelength ($E = \frac{hc}{\lambda}$). As wavelength decreases, energy increases.

<u>Solution:</u>

In terms of increasing energy the order is **red < yellow < blue**.

7.12 Since energy is directly proportional to frequency ($E = h\nu$): UV ($\nu = 8.0\text{x}10^{15}\ \text{s}^{-1}$) > IR ($\nu = 6.5\text{x}10^{13}\ \text{s}^{-1}$) > microwave ($\nu = 9.8\text{x}10^{11}\ \text{s}^{-1}$) or **UV > IR > microwave**.

7.13 Frequency and energy are related by $E = h\nu$, and wavelength and energy are related by $E = hc/\lambda$.

$$\nu\ (\text{Hz}) = \frac{E}{h} = \frac{(1.33\ \text{MeV})\left(\frac{10^6\ \text{eV}}{1\ \text{MeV}}\right)\left(\frac{1.602\text{x}10^{-19}\ \text{J}}{1\ \text{eV}}\right)}{6.626\text{x}10^{-34}\ \text{J•s}}\left(\frac{\text{Hz}}{\text{s}^{-1}}\right) = 3.2156\text{x}10^{20} = \mathbf{3.22x10^{20}\ Hz}$$

$$\lambda\ (\text{m}) = \frac{hc}{E} = \frac{(6.626\text{x}10^{-34}\ \text{J•s})(3.00\text{x}10^8\ \text{m/s})}{(1.33\ \text{MeV})\left(\frac{10^6\ \text{eV}}{1\ \text{MeV}}\right)\left(\frac{1.602\text{x}10^{-19}\ \text{J}}{1\ \text{eV}}\right)} = 9.32950\text{x}10^{-13} = \mathbf{9.33x10^{-13}\ m}$$

The wavelength can also be found using the frequency calculated in the equation $c = \lambda\nu$.

7.14 <u>Plan:</u> The least energetic photon in part a) has the longest wavelength (242 nm). The most energetic photon in part b) has the shortest wavelength (2200 Å). Use the relationship $c = \lambda\nu$ to find the frequency of the photons and relationship $E = \frac{hc}{\lambda}$ to find the energy.

<u>Solution:</u>

a) $c = \lambda\nu$

$$\nu = \frac{c}{\lambda} = \frac{3.00\text{x}10^8\ \text{m/s}}{242\ \text{nm}}\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = 1.239669\text{x}10^{15} = \mathbf{1.24x10^{15}\ s^{-1}}$$

$$E = \frac{hc}{\lambda} = \frac{\left(6.626\text{x}10^{-34}\ \text{J•s}\right)\left(3.00\text{x}10^{8}\ \text{m/s}\right)}{242\ \text{nm}}\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = 8.2140\text{x}10^{-19} = \mathbf{8.21x10^{-19}\ J}$$

b) $$\nu = \frac{c}{\lambda} = \frac{3.00\text{x}10^{8}\ \text{m/s}}{2200\ \text{Å}}\left(\frac{1\ \text{Å}}{10^{-10}\ \text{m}}\right) = 1.3636\text{x}10^{15} = \mathbf{1.4x10^{15}\ s^{-1}}$$

$$E = \frac{hc}{\lambda} = \frac{\left(6.626\text{x}10^{-34}\ \text{J•s}\right)\left(3.00\text{x}10^{8}\ \text{m/s}\right)}{2200\ \text{Å}}\left(\frac{1\ \text{Å}}{10^{-10}\ \text{m}}\right) = 9.03545\text{x}10^{-19} = \mathbf{9.0x10^{-19}\ J}$$

7.15 “*n*” in the Rydberg equation is equal to a Bohr orbit of quantum number “*n*” where $n = 1, 2, 3, \ldots\infty$.

7.16 An absorption spectrum is produced when atoms absorb certain wavelengths of incoming light as electrons move from lower to higher energy levels and results in dark lines against a bright background. An emission spectrum is produced when atoms that have been excited to higher energy emit photons as their electrons return to lower energy levels and results in colored lines against a dark background. Bohr worked with emission spectra.

7.17 Plan: The quantum number *n* is related to the energy level of the electron. An electron *absorbs* energy to change from lower energy (lower *n*) to higher energy (higher *n*), giving an absorption spectrum. An electron *emits* energy as it drops from a higher energy level (higher *n*) to a lower one (lower *n*), giving an emission spectrum.
Solution:
a) The electron is moving from a lower value of *n* (2) to a higher value of *n* (4): **absorption**
b) The electron is moving from a higher value of *n* (3) to a lower value of *n* (1): **emission**
c) The electron is moving from a higher value of *n* (5) to a lower value of *n* (2):**emission**
d) The electron is moving from a lower value of *n* (3) to a higher value of *n* (4): **absorption**

7.18 The Bohr model works only for a one-electron system. The additional attractions and repulsions in many-electron systems make it impossible to predict accurately the spectral lines.

7.19 Plan: Calculate wavelength by substituting the given values into Equation 7.3, where $n_1 = 2$ and $n_2 = 5$ because $n_2 > n_1$. Although more significant figures could be used, five significant figures are adequate for this calculation.
Solution:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \qquad R = 1.096776\text{x}10^{7}\ \text{m}^{-1}$$

$n_1 = 2 \quad n_2 = 5$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \left(1.096776\text{x}10^{7}\ \text{m}^{-1}\right)\left(\frac{1}{2^2} - \frac{1}{5^2}\right) = 2{,}303{,}229.6\ \text{m}^{-1}$$

$$\lambda\ (\text{nm}) = \left(\frac{1}{2{,}303{,}229.6\ \text{m}^{-1}}\right)\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = 434.1729544 = \mathbf{434.17\ nm}$$

7.20 Calculate wavelength by substituting the given values into the Rydberg equation, where $n_1 = 1$ and $n_2 = 3$ because $n_2 > n_1$. Although more significant figures could be used, five significant figures are adequate for this calculation.

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \left(1.096776\text{x}10^{7}\ \text{m}^{-1}\right)\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 9{,}749{,}120\ \text{m}^{-1}$$

$$\lambda\ (\text{Å}) = \left(\frac{1}{9{,}749{,}120\ \text{m}^{-1}}\right)\left(\frac{1\text{Å}}{10^{-10}\ \text{m}}\right) = 1025.7336 = \mathbf{1025.7\ Å}$$

7.21 Plan: To find the transition energy, use the equation for the energy of an electron transition and multiply by Avogadro's number to convert to energy per mole.
Solution:

$$\Delta E = \left(-2.18\text{x}10^{-18}\ \text{J}\right)\left(\frac{1}{n_{\text{final}}^2} - \frac{1}{n_{\text{initial}}^2}\right)$$

$$\Delta E = \left(-2.18\text{x}10^{-18}\ \text{J}\right)\left(\frac{1}{2^2} - \frac{1}{5^2}\right) = -4.578\text{x}10^{-19}\ \text{J/photon}$$

$$\Delta E = \left(\frac{-4.578\text{x}10^{-19}\ \text{J}}{\text{photon}}\right)\left(\frac{6.022\text{x}10^{23}\ \text{photons}}{1\ \text{mol}}\right) = -2.75687\text{x}10^{5} = \mathbf{-2.76x10^{5}\ J/mol}$$

The energy has a negative value since this electron transition to a lower n value is an emission of energy.

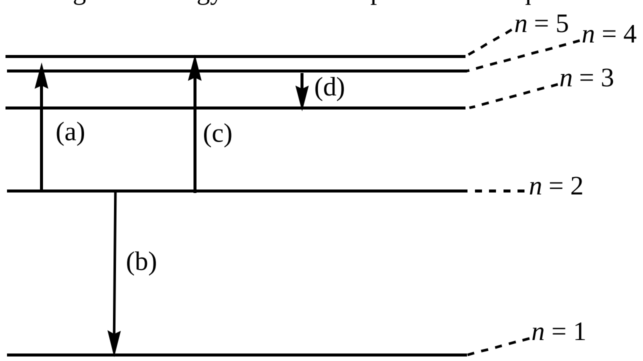
7.22 To find the transition energy, use the equation for the energy of an electron transition and multiply by Avogadro's number.

$$\Delta E = \left(-2.18\text{x}10^{-18}\ \text{J}\right)\left(\frac{1}{n_{\text{final}}^2} - \frac{1}{n_{\text{initial}}^2}\right)$$

$$\Delta E = \left(-2.18\text{x}10^{-18}\ \text{J}\right)\left(\frac{1}{3^2} - \frac{1}{1^2}\right) = 1.93778\text{x}10^{-18}\ \text{J/photon}$$

$$\Delta E = \left(\frac{1.93778\text{x}10^{-18}\ \text{J}}{\text{photon}}\right)\left(\frac{6.022\text{x}10^{23}\ \text{photons}}{1\ \text{mol}}\right) = 1.1669\text{x}10^{6} = \mathbf{1.17x10^{6}\ J/mol}$$

7.23 Plan: Determine the relative energy of the electron transitions. Remember that energy is directly proportional to frequency ($E = h\nu$).
Solution:
Looking at an energy chart will help answer this question.



Frequency is proportional to energy so the smallest frequency will be d) $n = 4$ to $n = 3$; levels 3 and 4 have a smaller ΔE than the levels in the other transitions. The largest frequency is b) $n = 2$ to $n = 1$ since levels 1 and 2 have a larger ΔE than the levels in the other transitions. Transition a) $n = 2$ to $n = 4$ will be smaller than transition c) $n = 2$ to $n = 5$ since level 5 is a higher energy than level 4. In order of increasing frequency the transitions are **d < a < c < b**.

7.24 **b > c > a > d**

7.25 Plan: Use the Rydberg equation. Since the electron is in the ground state (lowest energy level), $n_1 = 1$. Convert the wavelength from nm to units of meters.

Solution:

$\lambda = (97.20 \text{ nm})\left(\frac{10^{-9} \text{ m}}{1 \text{ nm}}\right) = 9.720\text{x}10^{-8} \text{ m}$ ground state: $n_1 = 1$; $n_2 = ?$

$$\frac{1}{\lambda} = (1.096776\text{x}10^{7} \text{ m}^{-1})\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{9.72\text{x}10^{-8} \text{ m}} = (1.096776\text{x}10^{7} \text{ m}^{-1})\left(\frac{1}{1^2} - \frac{1}{n_2^2}\right)$$

$$0.93803 = \left(\frac{1}{1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{n_2^2} = 1 - 0.93803 = 0.06197$$

$$n_2^2 = 16.14$$

$n_2 = \mathbf{4}$

7.26 $\lambda = (1281 \text{ nm})\left(\frac{10^{-9} \text{ m}}{1 \text{ nm}}\right) = 1.281\text{x}10^{-6} \text{ m}$

$$\frac{1}{\lambda} = (1.096776\text{x}10^{7} \text{m}^{-1})\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{1.281\text{x}10^{-6} \text{ m}} = (1.096776\text{x}10^{7} \text{ m}^{-1})\left(\frac{1}{n_1^2} - \frac{1}{5^2}\right)$$

$$0.07118 = \left(\frac{1}{n_1^2} - \frac{1}{5^2}\right)$$

$$\frac{1}{n_1^2} = 0.07118 + 0.04000 = 0.11118$$

$$n_1^2 = 8.9944$$

$n_1 = \mathbf{3}$

7.27 $E = \frac{hc}{\lambda} = \frac{(6.626\text{x}10^{-34} \text{ J•s})(3.00\text{x}10^{8} \text{ m/s})}{(436 \text{ nm})}\left(\frac{1 \text{ nm}}{10^{-9} \text{ m}}\right) = 4.55917\text{x}10^{-19} = \mathbf{4.56x10^{-19}\ J}$

7.28 a) Absorptions: **A, C, D**; Emissions: **B, E, F**
b) Energy of emissions: **E < F < B**
c) Wavelength of absorption: **D < A < C**

7.29 Macroscopic objects have significant mass. A large m in the denominator of $\lambda = h/mu$ will result in a very small wavelength. Macroscopic objects do exhibit a wavelike motion, but the wavelength is too small for humans to see it.

7.30 The Heisenberg uncertainty principle states that there is fundamental limit to the accuracy of measurements. This limit is not dependent on the precision of the measuring instruments, but is inherent in nature.

7.31 Plan: Use the de Broglie equation. Mass in lb must be converted to kg and velocity in mi/h must be converted to m/s because a joule is equivalent to kg•m^2/s^2.

Solution:

a) Mass (kg) = $(232 \text{ lb})\left(\frac{1 \text{ kg}}{2.205 \text{ lb}}\right)$ = 105.2154 kg

Velocity (m/s) = $\left(\frac{19.8 \text{ mi}}{\text{h}}\right)\left(\frac{1 \text{ km}}{0.62 \text{ mi}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)$ = 8.87097 m/s

$$\lambda = \frac{h}{mu} = \frac{(6.626\text{x}10^{-34} \text{ J•s})}{(105.2154 \text{ kg})\left(8.87097 \frac{\text{m}}{\text{s}}\right)}\left(\frac{\text{kg•m}^2/\text{s}^2}{\text{J}}\right) = 7.099063\text{x}10^{-37} = \mathbf{7.10x10^{-37}\ m}$$

b) Uncertainty in velocity (m/s) = $\left(\frac{0.1 \text{ mi}}{\text{h}}\right)\left(\frac{1 \text{ km}}{0.62 \text{ mi}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)$ = 0.0448029 m/s

$$\Delta x \text{•} m\Delta v \geq \frac{h}{4\pi}$$

$$\Delta x \geq \frac{h}{4\pi m\Delta v} \geq \frac{(6.626\text{x}10^{-34} \text{ J•s})}{4\pi(105.2154 \text{ kg})\left(\frac{0.0448029 \text{ m}}{\text{s}}\right)}\left(\frac{\text{kg•m}^2/\text{s}^2}{\text{J}}\right) \geq 1.11855\text{x}10^{-35} \geq \mathbf{1x10^{-35}\ m}$$

7.32 a) $$\lambda = \frac{h}{mu} = \frac{(6.626\text{x}10^{-34} \text{ J•s})}{(6.6\text{x}10^{-24} \text{ g})\left(3.4\text{x}10^7 \frac{\text{mi}}{\text{h}}\right)}\left(\frac{\text{kg•m}^2/\text{s}^2}{\text{J}}\right)\left(\frac{10^3 \text{ g}}{1 \text{ kg}}\right)\left(\frac{0.62 \text{ mi}}{1 \text{ km}}\right)\left(\frac{1 \text{ km}}{10^3 \text{ m}}\right)\left(\frac{3600 \text{ s}}{1 \text{ h}}\right)$$

$$= 6.59057\text{x}10^{-15} = \mathbf{6.6x10^{-15}\ m}$$

b) $$\Delta x \text{•} m\Delta v \geq \frac{h}{4\pi}$$

$$\Delta x \geq \frac{h}{4\pi m\Delta v} \geq \frac{(6.626\text{x}10^{-34} \text{ J•s})}{4\pi(6.6\text{x}10^{-24} \text{ g})\left(\frac{0.1\text{x}10^7 \text{ mi}}{\text{h}}\right)}\left(\frac{\text{kg•m}^2/\text{s}^2}{\text{J}}\right)\left(\frac{10^3 \text{ g}}{1 \text{ kg}}\right)\left(\frac{0.62 \text{ mi}}{1 \text{ km}}\right)\left(\frac{1 \text{ km}}{10^3 \text{ m}}\right)\left(\frac{3600 \text{ s}}{1 \text{ h}}\right)$$

$$\geq 1.783166\text{x}10^{-14} \geq \mathbf{2x10^{-14}\ m}$$

7.33 Plan: Use the de Broglie equation. Mass in g must be converted to kg and wavelength in Å must be converted to m because a joule is equivalent to kg•m^2/s^2.

Solution:

Mass (kg) = $(56.5 \text{ g})\left(\frac{1 \text{ kg}}{10^3 \text{ g}}\right)$ = 0.0565 kg

Wavelength (m) = $(5400 \text{ Å})\left(\frac{10^{-10} \text{ m}}{1 \text{ Å}}\right)$ = 5.4x10^{-7} m

$$\lambda = \frac{h}{mu}$$

$$u = \frac{h}{m\lambda} = \frac{(6.626\text{x}10^{-34} \text{ J•s})}{(0.0565 \text{ kg})(5.4\text{x}10^{-7} \text{ m})}\left(\frac{\text{kg•m}^2/\text{s}^2}{\text{J}}\right) = 2.1717\text{x}10^{-26} = \mathbf{2.2x10^{-26}\ m/s}$$

7.34 $\lambda = \dfrac{h}{mu}$

$$u = \frac{h}{m\lambda} = \frac{\left(6.626\text{x}10^{-34}\ \text{J•s}\right)}{\left(142\ \text{g}\right)\left(100.\ \text{pm}\right)}\left(\frac{\text{kg•m}^2/\text{s}^2}{\text{J}}\right)\left(\frac{10^3\ \text{g}}{1\ \text{kg}}\right)\left(\frac{1\ \text{pm}}{10^{-12}\ \text{m}}\right) = 4.666197\text{x}10^{-23} = \mathbf{4.67x10^{-23}\ m/s}$$

7.35 <u>Plan:</u> The de Broglie wavelength equation will give the mass equivalent of a photon with known wavelength and velocity. The term "mass equivalent" is used instead of "mass of photon" because photons are quanta of electromagnetic energy that have no mass. A light photon's velocity is the speed of light, $3.00\text{x}10^8$ m/s. Wavelength in nm must be converted to m.
<u>Solution:</u>

$$\text{Wavelength (m)} = \left(589\,\text{nm}\right)\left(\frac{10^{-9}\ \text{m}}{1\ \text{nm}}\right) = 5.89\text{x}10^{-7}\ \text{m}$$

$$\lambda = \frac{h}{mu}$$

$$m = \frac{h}{\lambda u} = \frac{\left(6.626\text{x}10^{-34}\,\text{J•s}\right)}{\left(5.89\text{x}10^{-7}\ \text{m}\right)\left(3.00\,\text{x}10^8\ \text{m/s}\right)}\left(\frac{\text{kg•m}^2/\text{s}^2}{\text{J}}\right) = 3.7499\text{x}10^{-36} = \mathbf{3.75x10^{-36}\ kg/photon}$$

7.36 $\lambda = \dfrac{h}{mu}$

$$m = \frac{h}{\lambda u} = \frac{\left(6.626\text{x}10^{-34}\ \text{J•s}\right)}{\left(671\ \text{nm}\right)\left(3.00\text{x}10^8\ \text{m/s}\right)}\left(\frac{\text{kg•m}^2/\text{s}^2}{\text{J}}\right)\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = 3.2916\text{x}10^{-36}\ \text{kg/photon}$$

$$\left(\frac{3.2916\text{x}10^{-36}\ \text{kg}}{\text{photon}}\right)\left(\frac{6.022\text{x}10^{23}\ \text{photons}}{\text{mol}}\right) = 1.9822\text{x}10^{-12} = \mathbf{1.98x10^{-12}\ kg/mol}$$

7.37 The quantity ψ^2 expresses the probability of finding an electron within a specified tiny region of space.

7.38 Since ψ^2 is the probability of finding an electron within a small region or volume, electron density would represent a probability per unit volume and would more accurately be called electron probability density.

7.39 a) Principal quantum number, *n*, relates to the size of the orbital. More specifically, it relates to the distance from the nucleus at which the probability of finding an electron is greatest. This distance is determined by the energy of the electron.
b) Angular momentum quantum number, *l*, relates to the shape of the orbital. It is also called the azimuthal quantum number.
c) Magnetic quantum number, m_l, relates to the orientation of the orbital in space in three-dimensional space.

7.40 <u>Plan:</u> The following letter designations correlate with the following *l* quantum numbers:
l = 0 = *s* orbital; *l* = 1 = *p* orbital; *l* = 2 = *d* orbital; *l* = 3 = *f* orbital. Remember that allowed m_l values are – *l* to + *l*.
The number of orbitals of a particular type is given by the number of possible m_l values.
<u>Solution:</u>
a) There is only a single *s* orbital in any shell. *l* = 1 and m_l = 0: one value of m_l = **one** *s* orbital.
b) There are five *d* orbitals in any shell. *l* = 2 and m_l = –2, –1, 0, +1, +2. Five values of m_l = **five** *d* orbitals.
c) There are three *p* orbitals in any shell. *l* = 1 and m_l = –1, 0, +1. Three values of m_l = **three** *p* orbitals.
d) If *n* = 3, *l* = 0(*s*), 1(*p*), and 2(*d*). There is a 3*s* (1 orbital), a 3*p* set (3 orbitals), and a 3*d* set (5 orbitals) for a total of **nine** orbitals (1 + 3 + 5 = 9).

7.41 a) All *f* orbitals consist of sets of **seven** (l = 3 and m_l = –3, –2, –1, 0, +1, +2, +3).
b) All *p* orbitals consist of sets of **three** (l = 1 and m_l = –1, 0, +1).
c) All *d* orbitals consist of sets of **five** (l = 2 and m_l = –2, –1, 0, +1, +2).
d) If n = 2, then there is a 2*s* (1 orbital) and a 2*p* set (3 orbitals) for a total of **four** orbitals (1 + 3 = 4).

7.42 Plan: Magnetic quantum numbers (m_l) can have integer values from –l to + l. The l quantum number can have integer values from 0 to n – 1.
Solution:
a) l = 2 so m_l = **–2, –1, 0, +1, +2**
b) n = 1 so l = 1 – 1 = 0 and m_l = **0**
c) l = 3 so m_l = **–3, –2, –1, 0, +1, +2, +3**

7.43 Magnetic quantum numbers can have integer values from –l to +l. The l quantum number can have integer values from 0 to n – 1.
a) l = 3 so m_l = **–3, –2, –1, 0, +1, +2, +3**
b) n = 2 so l = 0 or 1; for l = 0, m_l = **0**; for l = l, m_l = **–1,0,+1**
c) l = 1 so m_l = **–1, 0, +1**

7.44 Plan: The following letter designations for the various sublevels (orbitals) correlate with the following l quantum numbers: l = 0 = *s* orbital; l = 1 = *p* orbital; l = 2 = *d* orbital; l = 3 = *f* orbital. Remember that allowed m_l values are – l to + l. The number of orbitals of a particular type is given by the number of possible m_l values.
Solution:

sublevel	allowable m_l	# of possible orbitals
a) *d* (l = 2)	–2, –1, 0, +1, +2	5
b) *p* (l = 1)	–1, 0, +1	3
c) *f* (l = 3)	–3, –2, –1, 0, +1, +2, +3	7

7.45

sublevel	allowable m_l	# of possible orbitals
a) *s* (l = 0)	0	1
b) *d* (l = 2)	–2, –1, 0, +1, +2	5
c) *p* (l = 1)	–1, 0, +1	3

7.46 Plan: The integer in front of the letter represents the n value. The letter designates the l value: l = 0 = *s* orbital; l = 1 = *p* orbital; l = 2 = *d* orbital; l = 3 = *f* orbital. Remember that allowed m_l values are – l to + l.
Solution:
a) For the 5*s* subshell, ***n* = 5** and ***l* = 0**. Since m_l = 0, there is **one** orbital.
b) For the 3*p* subshell, ***n* = 3** and ***l* = 1**. Since m_l = –1, 0, +1, there are **three** orbitals.
c) For the 4*f* subshell, ***n* = 4** and ***l* = 3**. Since m_l = –3, –2, –1, 0, +1, +2, +3, there are **seven** orbitals.

7.47 a) n = 6; l = 4; 9 orbitals (m_l = –4, –3, –2, –1, 0, +1, +2, +3, +4)
b) n = 4; l = 0; 1 orbital (m_l = 0)
c) n = 3; l = 2; 5 orbitals (m_l = –2, –1, 0, +1, +2)

7.48 Plan: Allowed values of quantum numbers: n = positive integers; l = integers from 0 to n – 1; m_l = integers from – l through 0 to + l.
Solution:
a) n = 2; l = 0; m_l = –1: With n = 2, l can be 0 or 1; with l = 0, the only allowable m_l value is 0. This combination is not allowed. To correct, either change the l or m_l value.
Correct: n = 2; l = 1; m_l = –1 or n = 2; l = 0; m_l = 0.
b) n = 4; l = 3; m_l = –1: With n = 4, l can be 0, 1, 2, or 3; with l = 3, the allowable m_l values are –3, –2, –1, 0, +1, +2, +3. Combination is allowed.
c) n = 3; l = 1; m_l = 0: With n = 3, l can be 0, 1, or 2; with l = 1, the allowable m_l values are –1, 0, +1. Combination is allowed.
d) n = 5; l = 2; m_l = +3: With n = 5, l can be 0, 1, 2, 3, or 4; with l = 2, the allowable m_l values are –2, –1, 0, +1, +2. +3 is not an allowable m_l value. To correct, either change l or m_l value.

Correct: $n = 5$; $l = 3$; $m_l = +3$ or $n = 5$; $l = 2$; $m_l = 0$.

7.49 a) Combination is allowed.
b) No; $n = 2$; $l = 1$; $m_l = +1$ or $n = 2$; $l = 1$; $m_l = 0$
c) No; $n = 7$; $l = 1$; $m_l = +1$ or $n = 7$; $l = 3$; $m_l = 0$
d) No; $n = 3$; $l = 1$; $m_l = -1$ or $n = 3$; $l = 2$; $m_l = -2$

7.50 Plan: When light of sufficient frequency (energy) shines on metal, electrons in the metal break free and a current flows.
Solution:
a) The lines do not begin at the origin because an electron must absorb a minimum amount of energy before it has enough energy to overcome the attraction of the nucleus and leave the atom. This minimum energy is the energy of photons of light at the threshold frequency.
b) The lines for K and Ag do not begin at the same point. The amount of energy that an electron must absorb to leave the K atom is less than the amount of energy that an electron must absorb to leave the Ag atom, where the attraction between the nucleus and outer electron is stronger than in a K atom.
c) Wavelength is inversely proportional to energy. Thus, the metal that requires a larger amount of energy to be absorbed before electrons are emitted will require a shorter wavelength of light. Electrons in Ag atoms require more energy to leave, so Ag requires a shorter wavelength of light than K to eject an electron.
d) The slopes of the line show an increase in kinetic energy as the frequency (or energy) of light is increased. Since the slopes are the same, this means that for an increase of one unit of frequency (or energy) of light, the increase in kinetic energy of an electron ejected from K is the same as the increase in the kinetic energy of an electron ejected from Ag. After an electron is ejected, the energy that it absorbs above the threshold energy becomes the kinetic energy of the electron. For the same increase in energy above the threshold energy, for either K or Ag, the kinetic energy of the ejected electron will be the same.

7.51 a) $E = \frac{hc}{\lambda} = \frac{(6.626\text{x}10^{-34}\ \text{J•s})(3.00\text{x}10^{8}\ \text{m/s})}{700.\ \text{nm}}\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = 2.8397\text{x}10^{-19}\ \text{J}$

This is the value for each photon, that is, J/photon.

Number of photons $= \left(2.0\text{x}10^{-17}\ \text{J}\right)\left(\frac{1\ \text{photon}}{2.8397\text{x}10^{-19}\ \text{J}}\right) = 70.430 = \mathbf{70.\ photons}$

b) $E = \frac{hc}{\lambda} = \frac{(6.626\text{x}10^{-34}\ \text{J•s})(3.00\text{x}10^{8}\ \text{m/s})}{475.\ \text{nm}}\left(\frac{1\,\text{nm}}{10^{-9}\,\text{m}}\right) = 4.18484\text{x}10^{-19}\ \text{J}$

This is the value for each photon, that is, J/photon.

Number of photons $= \left(2.0\text{x}10^{-17}\ \text{J}\right)\left(\frac{1\ \text{photon}}{4.18484\text{x}10^{-19}\ \text{J}}\right) = 47.7916 = \mathbf{48\ photons}$

7.52 Determine the wavelength:
$\lambda = 1/(1953\ \text{cm}^{-1}) = 5.1203277\text{x}10^{-4}\ \text{cm}$

$\lambda\ (\text{nm}) = \left(5.1203277\text{x}10^{-4}\ \text{cm}\right)\left(\frac{10^{-2}\ \text{m}}{1\ \text{cm}}\right)\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = 5120.3277 = \mathbf{5.120x10^{3}\ nm}$

$\lambda\ (\text{Å}) = \left(5.1203277\text{x}10^{-4}\ \text{cm}\right)\left(\frac{10^{-2}\ \text{m}}{1\ \text{cm}}\right)\left(\frac{1\ \text{Å}}{10^{-10}\ \text{m}}\right) = 51203.277 = \mathbf{5.120x10^{4}\ Å}$

$\nu = c/\lambda = \frac{2.9979\text{x}10^{8}\,\text{m/s}}{5.1203277\text{x}10^{-4}\,\text{cm}}\left(\frac{1\ \text{cm}}{10^{-2}\ \text{m}}\right)\left(\frac{1\ \text{Hz}}{1\ \text{s}^{-1}}\right) = 5.8548987\text{x}10^{13} = \mathbf{5.855x10^{13}\ Hz}$

7.53 Plan: The Bohr model has been successfully applied to predict the spectral lines for one-electron species other than H. Common one-electron species are small cations with all but one electron removed. Since the problem specifies a metal ion, assume that the possible choices are Li^{2+} or Be^{3+}. Use the relationship $E = h\nu$ to convert the

frequency to energy and then solve Bohr's equation $E = \left(2.18\text{x}10^{-18}\ \text{J}\right)\left(\frac{Z^2}{n^2}\right)$ to verify if a whole number for Z can be calculated. Recall that the negative sign is a convention based on the zero point of the atom's energy; it is deleted in this calculation to avoid taking the square root of a negative number.
Solution:
The highest energy line corresponds to the transition from $n = 1$ to $n = \infty$.
$E = h\nu = (6.626\text{x}10^{-34}\ \text{J•s})(2.961\text{x}10^{16}\ \text{Hz})\ (\text{s}^{-1}/\text{Hz}) = 1.9619586\text{x}10^{-17}\ \text{J}$

$$E = \left(2.18\text{x}10^{-18}\ \text{J}\right)\left(\frac{Z^2}{n^2}\right) \qquad Z = \text{charge of the nucleus}$$

$$Z^2 = \frac{En^2}{2.18\text{x}10^{-18}\text{J}} = \frac{1.9619586\text{x}10^{-17}(1^2)}{2.18\text{x}10^{-18}\ \text{J}} = 8.99998$$

Then $Z^2 = 9$ and $Z = 3$.
Therefore, the ion is $\mathbf{Li^{2+}}$ with an atomic number of 3.

7.54 a) 59.5 MHz $\quad \lambda(\text{m}) = c/\nu = \dfrac{2.9979\ \text{x}\ 10^8\ \text{m/s}}{\left(59.5\ \text{MHz}\right)\left(\frac{10^6\ \text{Hz}}{1\ \text{MHz}}\right)\left(\frac{\text{s}^{-1}}{\text{Hz}}\right)} = 5.038487 = 5.04\ \text{m}$

215.8 MHz $\quad \lambda(\text{m}) = c/\nu = \dfrac{2.9979\ \text{x}\ 10^8\ \text{m/s}}{\left(215.8\ \text{MHz}\right)\left(\frac{10^6\ \text{Hz}}{1\ \text{MHz}}\right)\left(\frac{\text{s}^{-1}}{\text{Hz}}\right)} = 1.38920 = 1.389\ \text{m}$

Therefore, the VHF band overlaps with the 2.78–3.41 m FM band.

b) 550 kHz $\quad \lambda(\text{m}) = c/\nu = \dfrac{3.00\ \text{x}\ 10^8\ \text{m/s}}{\left(550\ \text{kHz}\right)\left(\frac{10^3\ \text{Hz}}{1\ \text{kHz}}\right)\left(\frac{\text{s}^{-1}}{\text{Hz}}\right)} = 545.45 = 550\ \text{m}$

1600 kHz $\quad \lambda(\text{m}) = c/\nu = \dfrac{3.00\ \text{x}\ 10^8\ \text{m/s}}{\left(1600\ \text{kHz}\right)\left(\frac{10^3\ \text{Hz}}{1\ \text{kHz}}\right)\left(\frac{\text{s}^{-1}}{\text{Hz}}\right)} = 187.5 = 190\ \text{m}$

FM width from 2.78 to 3.41 m gives 0.63 m, whereas AM width from 190 to 550 m gives 360 m.

7.55 $E = \dfrac{hc}{\lambda} \qquad$ thus $\lambda = \dfrac{hc}{E}$

a) $\lambda\ (\text{nm}) = \dfrac{hc}{E} = \dfrac{\left(6.626\text{x}10^{-34}\ \text{J•s}\right)\left(3.00\text{x}10^8\ \text{m/s}\right)}{4.60\text{x}10^{-19}\ \text{J}}\left(\dfrac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = 432.130 = \mathbf{432\ nm}$

b) $\lambda\ (\text{nm}) = \dfrac{hc}{E} = \dfrac{\left(6.626\text{x}10^{-34}\ \text{J•s}\right)\left(3.00\text{x}10^8\ \text{m/s}\right)}{6.94\text{x}10^{-19}\ \text{J}}\left(\dfrac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = 286.4265 = \mathbf{286\ nm}$

c) $\lambda\ (\text{nm}) = \dfrac{hc}{E} = \dfrac{\left(6.626\text{x}10^{-34}\ \text{J•s}\right)\left(3.00\text{x}10^8\ \text{m/s}\right)}{4.41\text{x}10^{-19}\ \text{J}}\left(\dfrac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = 450.748 = \mathbf{451\ nm}$

7.56 Plan: You are given the work function values of the three metals, which is the minimum energy required to remove an electron from the metal's surface. Use the relationship

$E = \dfrac{hc}{\lambda}$ to find the wavelength associated with each energy value (work function).

Solution:
a) The energy of visible light is lower than that of UV light. Thus, metal A must be **barium**, since of the three metals listed, barium has the smallest work function indicating the attraction between barium's nucleus and outer electron is less than the attraction in tantalum or tungsten. The longest wavelength corresponds to the lowest energy (work function). $E = hc/\lambda$ thus $\lambda = hc/E$

$$\text{Ta: } \lambda = \frac{hc}{E} = \frac{(6.626\text{x}10^{-34}\text{ J•s})(3.00\text{x}10^{8}\text{ m/s})}{6.81\text{x}10^{-19}\text{ J}}\left(\frac{1\text{ nm}}{10^{-9}\text{ m}}\right) = 291.894 = \mathbf{292\ nm}$$

$$\text{Ba: } \lambda = \frac{hc}{E} = \frac{(6.626\text{x}10^{-34}\text{ J•s})(3.00\text{x}10^{8}\text{ m/s})}{4.30\text{x}10^{-19}\text{ J}}\left(\frac{1\text{ nm}}{10^{-9}\text{ m}}\right) = 462.279 = \mathbf{462\ nm}$$

$$\text{W: } \lambda = \frac{hc}{E} = \frac{(6.626\text{x}10^{-34}\text{ J•s})(3.00\text{x}10^{8}\text{ m/s})}{7.16\text{x}10^{-19}\text{ J}}\left(\frac{1\text{ nm}}{10^{-9}\text{ m}}\right) = 277.6257 = \mathbf{278\ nm}$$

Metal A must be **barium**, because barium is the only metal that emits in the visible range (462 nm).
b) A UV range of **278 nm to 292 nm** is necessary to distinguish between tantalum and tungsten.

7.57 Extra significant figures are necessary because of the data presented in the problem.

He–Ne $\lambda = 632.8$ nm
Ar $\nu = 6.148\text{x}10^{14}\text{ s}^{-1}$
Ar–Kr $E = 3.499\text{x}10^{-19}$ J
Dye $\lambda = 663.7$ nm

Calculating missing λ values:
Ar $\lambda = c/\nu = (2.9979\text{x}10^{8}\text{ m/s})/(6.148\text{x}10^{14}\text{ s}^{-1}) = 4.8762199\text{x }10^{-7} = \mathbf{4.876x10^{-7}\ m}$
Ar–Kr $\lambda = hc/E = (6.626\text{x}10^{-34}\text{ J•s})\ (2.9979\text{x}10^{8}\text{ m/s})/(3.499\text{x}10^{-19}\text{ J}) = 5.67707\text{x}10^{-7} = \mathbf{5.677x10^{-7}\ m}$

Calculating missing ν values:
He–Ne $\nu = c/\lambda = (2.9979\text{x}10^{8}\text{ m/s})/[632.8\text{ nm }(10^{-9}\text{ m/nm})] = 4.7375\text{ x }10^{14} = \mathbf{4.738\ x\ 10^{14}\ s^{-1}}$
Ar–Kr $\nu = E/h = (3.499\text{x}10^{-19}\text{ J})/(6.626\text{x}10^{-34}\text{ J•s}) = 5.28071\text{x}10^{14} = \mathbf{5.281x10^{14}\ s^{-1}}$
Dye $\nu = c/\lambda = (2.9979\text{x}10^{8}\text{ m/s})/[663.7\text{ nm }(10^{-9}\text{ m/nm})] = 4.51695\text{x}10^{14} = \mathbf{4.517x10^{14}\ s^{-1}}$

Calculating missing E values:
He–Ne $E = hc/\lambda = [(6.626\text{x}10^{-34}\text{ J•s})(2.9979\text{x}10^{8}\text{ m/s})]/[632.8\text{ nm }(10^{-9}\text{ m/nm})]$
$= 3.13907797\text{x}10^{-19} = \mathbf{3.139x10^{-19}\ J}$
Ar $E = h\nu = (6.626\text{x}10^{-34}\text{ J•s})(6.148\text{x}10^{14}\text{ s}^{-1}) = 4.0736648\text{x}10^{-19} = \mathbf{4.074x10^{-19}\ J}$
Dye $E = hc/\lambda = [(6.626\text{x}10^{-34}\text{ J•s})(2.9979\text{x}10^{8}\text{ m/s})]/[663.7\text{ nm }(10^{-9}\text{ m/nm})]$
$= 2.99293\text{x}10^{-19} = \mathbf{2.993x10^{-19}\ J}$

The colors may be predicted from Figure 7.3 and the frequencies.

He–Ne	$\nu = 4.738\text{x}10^{14}\text{ s}^{-1}$	**Orange**
Ar	$\nu = 6.148\text{x}10^{14}\text{ s}^{-1}$	**Green**
Ar–Kr	$\nu = 5.281\text{x}10^{14}\text{ s}^{-1}$	**Yellow**
Dye	$\nu = 4.517\text{x}10^{14}\text{ s}^{-1}$	**Red**

7.58 Plan: Convert the distance in part a from km to m and use the speed of light to calculate the time needed for light to travel that distance. The frequency of the light is irrelevant. Use the given time in part b and the speed of light to calculate the distance.
Solution:

$$\text{a) Time} = (8.1\text{x}10^{7}\text{ km})\left(\frac{10^{3}\text{m}}{1\text{ km}}\right)\left(\frac{1\text{ s}}{3.00\text{x}10^{8}\text{ m}}\right) = 270 = \mathbf{2.7x10^{2}\ s}$$

$$\text{b) Distance} = (1.2\text{ s})\left(\frac{3.00\text{x}10^{8}\text{ m}}{\text{s}}\right) = \mathbf{3.6x10^{8}\ m}$$

7.59 $\frac{1}{\lambda} = \left(1.096776\text{x}10^7 \text{ m}^{-1}\right)\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

a) $\frac{1}{94.91 \text{ nm}}\left(\frac{1 \text{ nm}}{10^{-9}\text{m}}\right) = \left(1.096776\text{x}10^7 \text{m}^{-1}\right)\left(\frac{1}{1^2} - \frac{1}{n_2^2}\right)$

$0.9606608 = \left(\frac{1}{1^2} - \frac{1}{n_2^2}\right)$

$\boldsymbol{n_2 = 5}$

b) $\frac{1}{1281 \text{ nm}}\left(\frac{1 \text{ nm}}{10^{-9} \text{ m}}\right) = \left(1.096776\text{x}10^7 \text{m}^{-1}\right)\left(\frac{1}{n_1^2} - \frac{1}{5^2}\right)$

$0.071175894 = \left(\frac{1}{n_1^2} - \frac{1}{5^2}\right)$

$\boldsymbol{n_1 = 3}$

c) $\frac{1}{\lambda} = \left(1.096776\text{x}10^7 \text{m}^{-1}\right)\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$

$\frac{1}{\lambda} = 9.74912\text{x}10^6 \text{ m}^{-1}$

$\lambda = \left(1.02573\text{x}10^{-7} \text{ m}\right)\left(\frac{1 \text{ nm}}{10^{-9} \text{ m}}\right) = 102.573 = \mathbf{102.6 \text{ nm}}$

7.60 a) Orbital **D** has the largest value of *n*, given that it is the largest orbital.
b) $l = 1$ indicates a *p* orbital. Orbitals **A** and **C** are *p* orbitals. $l = 2$ indicates a *d* orbital. Orbitals **B** and **D** are *d* orbitals.
c) In an atom, there would be **four** other orbitals with the same value of *n* and the same shape as orbital B. There would be **two** other orbitals with the same value of *n* and the same shape as orbital C.
d) Orbital **D** has the highest energy and orbital **C** has the lowest energy.

7.61 The wavelengths of light responsible for the spectral lines (for the different series) for hydrogen are related by the Rydberg equation:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \text{ where } n = 1, 2, 3.... \text{ and } n_2 > n_1$$

As the values of *n* increase, the energies associated with *n* move closer. The result is that ΔE values within a series increase by continually **smaller** values and thus, the smaller wavelength associated with these ΔE values moves closer together. The spectral lines become closer and closer together in the short wavelength region of each series because the difference in energy associated with the transition from n_i to n_f becomes smaller and smaller with increasing distance from the nucleus.

7.62 Plan: Refer to Chapter 6 for the calculation of the amount of heat energy absorbed by a substance from its specific heat capacity and temperature change ($q = c$ x mass x ΔT). Using this equation, calculate the energy absorbed by the water. This energy equals the energy from the microwave photons. The energy of each photon can be calculated from its wavelength: $E = hc/\lambda$. Dividing the total energy by the energy of each photon gives the number of photons absorbed by the water.
Solution:
$q = c$ x mass x ΔT
$q = (4.184 \text{ J/g°C})(252 \text{ g})(98 - 20.)\text{°C} = 8.22407\text{x}10^4 \text{ J}$

$$E = \frac{hc}{\lambda} = \frac{\left(6.626\text{x}10^{-34} \text{ J•s}\right)\left(3.00\text{x}10^8 \text{ m/s}\right)}{1.55\text{x}10^{-2} \text{ m}} = 1.28245\text{x}10^{-23} \text{ J/photon}$$

$$\text{Number of photons} = \left(8.22407\text{x}10^{4}\ \text{J}\right)\left(\frac{1\ \text{photon}}{1.28245\text{x}10^{-23}\ \text{J}}\right) = 6.41278\text{x}10^{27} = \mathbf{6.4x10^{27}\ photons}$$

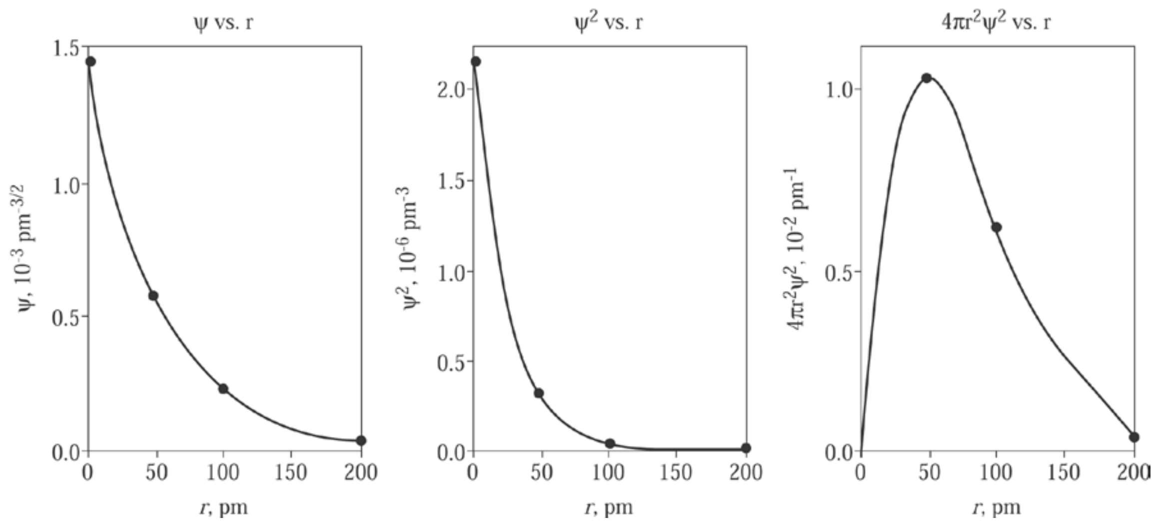
7.63 One sample calculation will be done using the equation in the book:

$$\psi = \left(\frac{1}{\sqrt{\pi}}\right)\left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0} = \left(\frac{1}{\sqrt{\pi}}\right)\left(\frac{1}{52.92\ \text{pm}}\right)^{3/2} e^{-r/a_0} = 1.465532\text{x}10^{-3}\ e^{-r/a_0}$$

For r = 50 pm:

$\psi = 1.465532\text{x}10^{-3}\ e^{-r/a_0} = 1.465532\text{x}10^{-3}\ e^{-50/52.92} = 5.69724\text{x}10^{-4}$
$\psi^2 = (5.69724\text{x}10^{-4})^2 = 3.24585\text{x}10^{-7}$
$4\pi r^2\psi^2 = 4\pi(50)^2(3.24585\text{x}10^{-7}) = 1.0197\text{x}10^{-2}$

r (pm)	ψ (pm$^{-3/2}$)	ψ^2 (pm^{-3})	$4\pi r^2\psi^2$ (pm^{-1})
0	$1.47\text{x}10^{-3}$	$2.15\text{x}10^{-6}$	0
50	$0.570\text{x}10^{-3}$	$0.325\text{x}10^{-6}$	$1.02\text{x}10^{-2}$
100	$0.221\text{x}10^{-3}$	$0.0491\text{x}10^{-6}$	$0.616\text{x}10^{-2}$
200	$0.0335\text{x}10^{-3}$	$0.00112\text{x}10^{-6}$	$0.0563\text{x}10^{-2}$



The plots are similar to Figure 7.16A in the text.

7.64 <u>Plan:</u> The energy differences sought may be determined by looking at the energy changes in steps. The wavelength is calculated from the relationship $\lambda = \frac{hc}{E}$.

<u>Solution:</u>

a) The difference between levels 3 and 2 (E_{32}) may be found by taking the difference in the energies for the 3 → 1 transition (E_{31}) and the 2 → 1 transition (E_{21}).

$E_{32} = E_{31} - E_{21} = (4.854\text{x}10^{-17}\ \text{J}) - (4.098\text{x}10^{-17}\ \text{J}) = \mathbf{7.56x10^{-18}\ J}$

$$\lambda = \frac{hc}{E} = \frac{\left(6.626\text{x}10^{-34}\ \text{J•s}\right)\left(3.00\text{x}10^{8}\ \text{m/s}\right)}{\left(7.56\text{x}10^{-18}\ \text{J}\right)} = 2.629365\text{x}10^{-8} = \mathbf{2.63x10^{-8}\ m}$$

b) The difference between levels 4 and 1 (E_{41}) may be found by adding the energies for the 4 → 2 transition (E_{42}) and the 2 → 1 transition (E_{21}).

$E_{41} = E_{42} + E_{21} = (1.024\text{x}10^{-17}\ \text{J}) + (4.098\text{x}10^{-17}\ \text{J}) = \mathbf{5.122x10^{-17}\ J}$

$$\lambda = \frac{hc}{E} = \frac{\left(6.626\text{x}10^{-34}\ \text{J•s}\right)\left(3.00\text{x}10^{8}\ \text{m/s}\right)}{\left(5.122\text{x}10^{-17}\ \text{J}\right)} = 3.88091\text{x}10^{-9} = \mathbf{3.881x10^{-9}\ m}$$

c) The difference between levels 5 and 4 (E_{54}) may be found by taking the difference in the energies for the 5 → 1 transition (E_{51}) and the 4 → 1 transition (see part b)).

$E_{54} = E_{51} - E_{41}$ = (5.242x10^{-17} J) – (5.122x10^{-17} J) = **1.2x10^{-18} J**

$$\lambda = \frac{hc}{E} = \frac{(6.626\text{x}10^{-34}\ \text{J}\bullet\text{s})(3.00\text{x}10^{8}\ \text{m/s})}{(1.2\text{x}10^{-18}\ \text{J})} = 1.6565\text{x}10^{-7} = \mathbf{1.66x10^{-7}\ m}$$

7.65 a) $$\lambda = h/mu = \frac{(6.626\text{x}10^{-34}\ \text{J}\bullet\text{s})}{(9.109\text{x}10^{-31}\ \text{kg})\left(5.5\text{x}10^{4}\ \frac{\text{m}}{\text{s}}\right)}\left(\frac{\text{kg}\bullet\text{m}^2/\text{s}^2}{\text{J}}\right) = 1.322568\text{x}10^{-8}\ \text{m}$$

Smallest object = λ/2 = (1.322568x10^{-8} m)/2 = 6.61284x10^{-9} = **6.6x10^{-9} m**

b) $$\lambda = h/mu = \frac{(6.626\text{x}10^{-34}\ \text{J}\bullet\text{s})}{(9.109\text{x}10^{-31}\ \text{kg})\left(3.0\text{x}10^{7}\ \frac{\text{m}}{\text{s}}\right)}\left(\frac{\text{kg}\bullet\text{m}^2/\text{s}^2}{\text{J}}\right) = 2.424708\text{x}10^{-11}\ \text{m}$$

Smallest object = λ/2 = (2.424708x10^{-11} m)/2 = 1.212354x10^{-11} = **1.2x10^{-11} m**

7.66 Plan: Examine Figure 7.3 and match the given wavelengths to their colors. For each salt, convert the mass of salt to moles and multiply by Avogadro's number to find the number of photons emitted by that amount of salt (assuming that each atom undergoes one-electron transition). Use the relationship $E = \frac{hc}{\lambda}$ to find the energy of one photon and multiply by the total number of photons for the total energy of emission.

Solution:

a) Figure 7.3 indicates that the 641 nm wavelength of Sr falls in the **red** region and the 493 nm wavelength of Ba falls in the **green** region.

b) $SrCl_2$

$$\text{Number of photons} = (5.00\ \text{g SrCl}_2)\left(\frac{1\ \text{mol SrCl}_2}{158.52\ \text{g SrCl}_2}\right)\left(\frac{6.022\text{x}10^{23}\ \text{photons}}{1\ \text{mol SrCl}_2}\right) = 1.8994449\text{x}10^{22}\ \text{photons}$$

$$\lambda\ (\text{m}) = (641\ \text{nm})\left(\frac{10^{-9}\ \text{m}}{1\text{nm}}\right) = 6.41\text{x}10^{-7}\ \text{m}$$

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{(6.626\text{x}10^{-34}\ \text{J}\bullet\text{s})(3.00\text{x}10^{8}\ \text{m/s})}{6.41\text{x}10^{-7}\ \text{m}}\left(\frac{1\ \text{kJ}}{10^3\ \text{J}}\right) = 3.10109\text{x}10^{-22}\ \text{kJ/photon}$$

$$E_{\text{total}} = (1.8994449\text{x}10^{22}\ \text{photons})\left(\frac{3.10109\text{x}10^{-22}\ \text{kJ}}{1\ \text{photon}}\right) = 5.89035 = \mathbf{5.89\ kJ}$$

$BaCl_2$

$$\text{Number of photons} = (5.00\ \text{g BaCl}_2)\left(\frac{1\ \text{mol BaCl}_2}{208.2\ \text{g BaCl}_2}\right)\left(\frac{6.022\text{x}10^{23}\ \text{photons}}{1\ \text{mol BaCl}_2}\right) = 1.44620557\text{x}10^{22}\ \text{photons}$$

$$\lambda\ (\text{m}) = (493\ \text{nm})\left(\frac{10^{-9}\ \text{m}}{1\text{nm}}\right) = 4.93\text{x}10^{-7}\ \text{m}$$

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{(6.626\text{x}10^{-34}\ \text{J}\bullet\text{s})(3.00\text{x}10^{8}\ \text{m/s})}{4.93\text{x}10^{-7}\ \text{m}}\left(\frac{1\ \text{kJ}}{10^3\ \text{J}}\right) = 4.0320487\text{x}10^{-22}\ \text{kJ/photon}$$

$$E_{\text{total}} = (1.44620557\text{x}10^{22}\ \text{photons})\left(\frac{4.0320487\text{x}10^{-22}\ \text{kJ}}{1\ \text{photon}}\right) = 5.83117 = \mathbf{5.83\ kJ}$$

7.67 a) The highest energy line corresponds to the shortest wavelength. The shortest wavelength line is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \left(1.096776\text{x}10^7\ \text{m}^{-1}\right)\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{3282\ \text{nm}}\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = \left(1.096776\text{x}10^7\ \text{m}^{-1}\right)\left(\frac{1}{n_1^2} - \frac{1}{\infty^2}\right)$$

304,692 m^{-1} = (1.096776x10^7 m^{-1}) ($1/n^2$)

$1/n^2$ = 0.0277807

***n* = 6**

b) The lowest energy line corresponds to the longest wavelength. The longest wavelength line is given by

$$\frac{1}{\lambda} = \left(1.096776\text{x}10^7\ \text{m}^{-1}\right)\left(\frac{1}{n_1^2} - \frac{1}{(n_1+1)^2}\right)$$

$$\frac{1}{7460\ \text{nm}}\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = \left(1.096776\text{x}10^7\ \text{m}^{-1}\right)\left(\frac{1}{n_1^2} - \frac{1}{(n_1+1)^2}\right)$$

$$134{,}048\ \text{m}^{-1} = \left(1.096776\text{x}10^7\ \text{m}^{-1}\right)\left(\frac{1}{n_1^2} - \frac{1}{(n_1+1)^2}\right)$$

$$0.0122220 = \left(\frac{1}{n_1^2} - \frac{1}{(n_1+1)^2}\right)$$

Rearranging and solving this equation for n_1 yields $\boldsymbol{n_1 = 5}$. (You and your students may well need to resort to trial-and-error solution of this equation!)

7.68 Plan: Examine Figure 7.3 to find the region of the electromagnetic spectrum in which the wavelength lies. Compare the absorbance of the given concentration of Vitamin A to the absorbance of the given amount of fish-liver oil to find the concentration of Vitamin A in the oil.

Solution:

a) At this wavelength the sensitivity to absorbance of light by Vitamin A is maximized while minimizing interference due to the absorbance of light by other substances in the fish-liver oil.

b) The wavelength 329 nm lies in the **ultraviolet region** of the electromagnetic spectrum.

c) A known quantity of vitamin A (1.67x10^{-3} g) is dissolved in a known volume of solvent (250. mL) to give a *standard* concentration with a known response (1.018 units). This can be used to find the unknown quantity of Vitamin A that gives a response of 0.724 units. An equality can be made between the two concentration-to-absorbance ratios.

Concentration (C_1, g/mL) of Vitamin A = $\left(\frac{1.67\text{x}10^{-3}\ \text{g}}{250.\ \text{mL}}\right)$ = 6.68x10^{-6} g/mL Vitamin A

Absorbance (A_1) of Vitamin A = 1.018 units.

Absorbance (A_2) of fish-liver oil = 0.724 units

Concentration (g/mL) of Vitamin A in fish-liver oil sample = C_2

$$\frac{A_1}{C_1} = \frac{A_2}{C_2}$$

$$C_2 = \frac{A_2C_1}{A_1} = \frac{(0.724)\left(6.68\text{x}10^{-6}\ \text{g/mL}\right)}{(1.018)} = 4.7508\text{x}10^{-6}\ \text{g/mL Vitamin A}$$

Mass (g) of Vitamin A in oil sample = $(500.\ \text{mL oil})\left(\frac{4.7508\text{x}10^{-6}\ \text{g Vitamin A}}{1\ \text{mL oil}}\right)$ = 2.3754x10^{-3} g Vitamin A

Concentration of Vitamin A in oil sample = $\frac{(2.3754\text{x}10^{-3}\text{ g})}{(0.1232\text{ g Oil})}$ = 1.92808x10^{-2} = **1.93x10^{-2} g Vitamin A/g oil**

7.69 $\lambda = hc/E = \frac{(6.626\text{x}10^{-34}\text{ J•s})(3.00\text{x}10^{8}\text{ m/s})}{(7.59\text{x}10^{-19}\text{ J})}\left(\frac{1\text{ nm}}{10^{-9}\text{ m}}\right)$ = 261.897 = **262 nm**

Silver is not a good choice for a photocell that uses visible light because 262 nm is in the ultraviolet region.

7.70 Mr. Green must be in the dining room where green light (520 nm) is reflected. Lower frequency, longer wavelength light is reflected in the lounge and study. Both yellow and red light have longer wavelengths than green light. Therefore, Col. Mustard and Ms. Scarlet must be in either the lounge or study. The shortest wavelengths are violet. Prof. Plum must be in the library. **Ms. Peacock** must be the murderer.

7.71 $E_k = \frac{1}{2}mv^2$

$v = \sqrt{\frac{E_k}{\frac{1}{2}m}} = \sqrt{\frac{4.71\text{x}10^{-15}\text{ J}}{\frac{1}{2}(9.109\text{x}10^{-31}\text{ kg})}\left(\frac{\text{kg•m}^2/\text{s}^2}{\text{J}}\right)}$ = 1.01692775x10^{8} m/s

$\lambda = h/mv = \frac{(6.626\text{x}10^{-34}\text{ J•s})}{(9.109\text{x}10^{-31}\text{ kg})\left(1.01692775\text{x}10^{8}\ \frac{\text{m}}{\text{s}}\right)}\left(\frac{\text{kg•m}^2/\text{s}^2}{\text{J}}\right)$ = 7.15304x10^{-12} = **7.15x10^{-12} m**

7.72 Plan: First find the energy in joules from the light that shines on the text. Each watt is one joule/s for a total of 75 J; take 5% of that amount of joules and then 10% of that amount. Use $E = \frac{hc}{\lambda}$ to find the energy of one photon of light with a wavelength of 550 nm. Divide the energy that shines on the text by the energy of one photon to obtain the number of photons.

Solution:

The amount of energy is calculated from the wavelength of light:

λ (m) = $(550\text{ nm})\left(\frac{10^{-9}\text{ m}}{1\text{ nm}}\right)$ = 5.50x10^{-7} m

$E = \frac{hc}{\lambda} = \frac{(6.626\text{x}10^{-34}\text{ J•s})(3.00\text{x}10^{8}\text{ m/s})}{5.50\text{x}10^{-7}\text{ m}}$ = 3.614182x10^{-19} J/photon

Amount of power from the bulb = $(75\text{ W})\left(\frac{1\text{ J/s}}{1\text{ W}}\right)$ = 75 J/s

Amount of power converted to light = $(75\text{ J/s})\left(\frac{5\%}{100\%}\right)$ = 3.75 Js

Amount of light shining on book = $(3.75\text{ J/s})\left(\frac{10\%}{100\%}\right)$ = 0.375 J/s

Number of photons: $\left(\frac{0.375\text{ J}}{\text{s}}\right)\left(\frac{1\text{ photon}}{3.614182\text{x}10^{-19}\text{ J}}\right)$ = 1.0376x10^{18} = **1.0x10^{18} photons/s**

7.73 a) $6CO_2(g) + 6H_2O(l) \rightarrow C_6H_{12}O_6(s) + 6O_2(g)$

ΔH_{rxn} = {(1 mol) ΔH_f° [$C_6H_{12}O_6$] + (6 mol) ΔH_f° [O_2]} – {(6 mol) ΔH_f° [CO_2] + (6 mol) ΔH_f° [H_2O]}

ΔH_{rxn} = [–1273.3 kJ + 6(0.0 kJ)] – [6(–393.5 kJ) + 6(–285.840 kJ)] = 2802.74 = 2802.7 kJ

$6CO_2(g) + 6H_2O(l) \rightarrow C_6H_{12}O_6(s) + 6O_2(g)$ ΔH_{rxn} = 2802.7 kJ (for 1.00 mol $C_6H_{12}O_6$)

b) $E = hc/\lambda = \dfrac{(6.626\text{x}10^{-34}\ \text{J•s})(3.00\text{x}10^{8}\ \text{m/s})}{680.\ \text{nm}}\left(\dfrac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = 2.9232353\text{x}10^{-19}$ J/photon

Number of photons = $(2802.7\ \text{kJ})\left(\dfrac{10^{3}\ \text{J}}{1\ \text{kJ}}\right)\left(\dfrac{1\ \text{photon}}{2.9232353\text{x}10^{-19}\ \text{J}}\right) = 9.5877\text{x}10^{24}$ = **$9.59\text{x}10^{24}$ photons**

7.74 Use the equation $\Delta x \ \bullet \ m\Delta u \ \geq \ \dfrac{h}{4\pi}$. The uncertainty in the speed Δu is given as 1.00%.

Δu = 1.00% of u = 0.0100(100.0 mi/h) = 1.00 mi/h

$$\Delta u = \left(\frac{1.00\,\text{mi}}{\text{h}}\right)\left(\frac{1\ \text{h}}{60\ \text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{1.609\ \text{km}}{1\ \text{mi}}\right)\left(\frac{10^{3}\,\text{m}}{1\ \text{km}}\right) = 0.4469\ \text{m/s}$$

$$\Delta x \ \geq \ \frac{h}{4\pi m\Delta u} \geq \ \frac{6.626\text{x}10^{-34}\ \text{J s}}{4\pi(142\ \text{g})(0.4469\ \text{m/s})}\left(\frac{10^{3}\,\text{g}}{1\ \text{kg}}\right)\left(\frac{1\text{kg•m}^{2}/\text{s}^{2}}{1\ \text{J}}\right) = 8.3089\text{x}10^{-34} \ = \mathbf{8.31x10^{-34}\ m}$$