

$$6) \sum_{n=2}^{\infty} \frac{3^{n+2}}{\ln n}$$

10.5

$$\lim_{n \rightarrow \infty} \frac{3^{n+3}}{\ln(n+1)} \cdot \frac{\ln n}{3^{n+2}} = 3 \lim_{n \rightarrow \infty} \frac{\ln n}{\ln(n+1)} = 3 \lim_{n \rightarrow \infty} \frac{1}{n} \cdot n+1 = 3 > 1 \text{ div by Ratio Test}$$

$$15) \sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^{n^2}$$

$$\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^{n^2 \cdot \frac{1}{n}} = \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^n = e^{-1} = \frac{1}{e} < 1 \text{ conv by Root test}$$

$$20) \sum_{n=1}^{\infty} \frac{n!}{10^{n^2}}$$

$$\lim_{n \rightarrow \infty} \frac{(n+1)!}{10^{n+1}} \cdot \frac{10^n}{n!} = \lim_{n \rightarrow \infty} \frac{(n+1)}{10} = \infty > 1 \text{ div by Ratio test}$$

$$30) \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2}\right)^n$$

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$$\lim_{n \rightarrow \infty} \left(\frac{1}{n} - \frac{1}{n^2}\right)^{n \cdot \frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{1}{n} - \frac{1}{n^2} = 0 < 1 \text{ conv by Root test}$$

$$7) \sum_{n=1}^{\infty} \frac{n^2(n+2)!}{n! \cdot 3^{2n}}$$

$$\lim_{n \rightarrow \infty} \frac{(n+1)^2 (n+3)!}{(n+1)! \cdot 3^{2n+2}} \cdot \frac{n! \cdot 3^{2n}}{n^2 (n+2)!} = \lim_{n \rightarrow \infty} \frac{(n+1)^2 (n+3)}{3^2 (n+1) n^2} = \lim_{n \rightarrow \infty} \frac{(n^2 + 2n + 1)(n+3)}{3n^3 + 3n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{n^3 + 2n^2 + n + 3n^2 + 6n + 3}{3n^3 + 3n^2} = \lim_{n \rightarrow \infty} \frac{n^3 + 5n^2 + 7n + 3}{3n^3 + 3n^2} = \frac{1}{3} < 1 \text{ conv by Ratio test}$$

$$12) \sum_{n=1}^{\infty} \left(\ln\left(e^2 + \frac{1}{n}\right)\right)^{n+1}$$

$$\ln \lim_{n \rightarrow \infty} \left(e^2 + \frac{1}{n}\right)^{\frac{n+1}{n}} = \ln \lim_{n \rightarrow \infty} \left(e^2 + \frac{1}{n}\right)^{1 + \frac{1}{n}} = \ln e^2 = 2 > 1 \text{ div by Root test}$$



$$16) \sum_{n=2}^{\infty} \frac{1}{n^{1+n}}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{1}{n^{1+n}}} = \lim_{n \rightarrow \infty} \frac{1}{n n^{\frac{1}{n}}} = 0 < 1 \quad \text{conv by Root test}$$

$$20) \sum_{n=1}^{\infty} \frac{n!}{10^n}$$

$$\lim_{n \rightarrow \infty} \frac{(n+1)!}{10^{n+1}} \cdot \frac{10^n}{n!} = \lim_{n \rightarrow \infty} \frac{n+1}{10} = \infty > 1 \quad \text{div by Ratio test}$$

$$28) \sum_{n=1}^{\infty} \frac{(\ln n)^n}{n^n}$$

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{\ln n}{n} = 0 < 1 \quad \text{conv by Root test}$$

$$38) \sum_{n=1}^{\infty} \frac{n!}{n^n}$$

$$\lim_{n \rightarrow \infty} \frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!} = \lim_{n \rightarrow \infty} \frac{(n+1)n^n}{(n+1)^n(n+1)} = \lim_{n \rightarrow \infty} \left(\frac{n}{n+1}\right)^n = \lim_{n \rightarrow \infty} \left(\frac{n+1-1}{n+1}\right)^n$$

$$\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n+1}\right)^n = \lim_{n \rightarrow \infty} \left(1 - \frac{1}{u}\right)^u = \lim_{n \rightarrow \infty} \left(1 - \frac{1}{u}\right)^u \left(1 - \frac{1}{u}\right)^{-1} = e^{-1} = \frac{1}{e} < 1 \quad u = n+1$$

conv by Ratio Test

$$43) \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$$

$$\lim_{n \rightarrow \infty} \frac{((n+1)!)^2}{(2n+2)!} \cdot \frac{(2n)!}{(n!)^2} = \lim_{n \rightarrow \infty} \frac{(n+1)^2}{(2n+2)(2n+1)} = \lim_{n \rightarrow \infty} \frac{n^2 + 2n + 1}{4n^2 + 2n + 4n + 2} = \frac{1}{4} < 1$$

conv by Ratio test

$$46) a_1 = 1 \quad a_{n+1} = \frac{1 + \tan^{-1} n}{n} a_n$$

$$\lim_{n \rightarrow \infty} \frac{1 + \tan^{-1} n}{n} = 0 < 1 \quad \text{conv by Ratio test}$$

$$60) \sum_{n=1}^{\infty} \frac{n^n}{(2^n)^2}$$

$$\lim_{n \rightarrow \infty} \frac{n}{4} = \infty > 1$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{a_n}$$

div by Root test