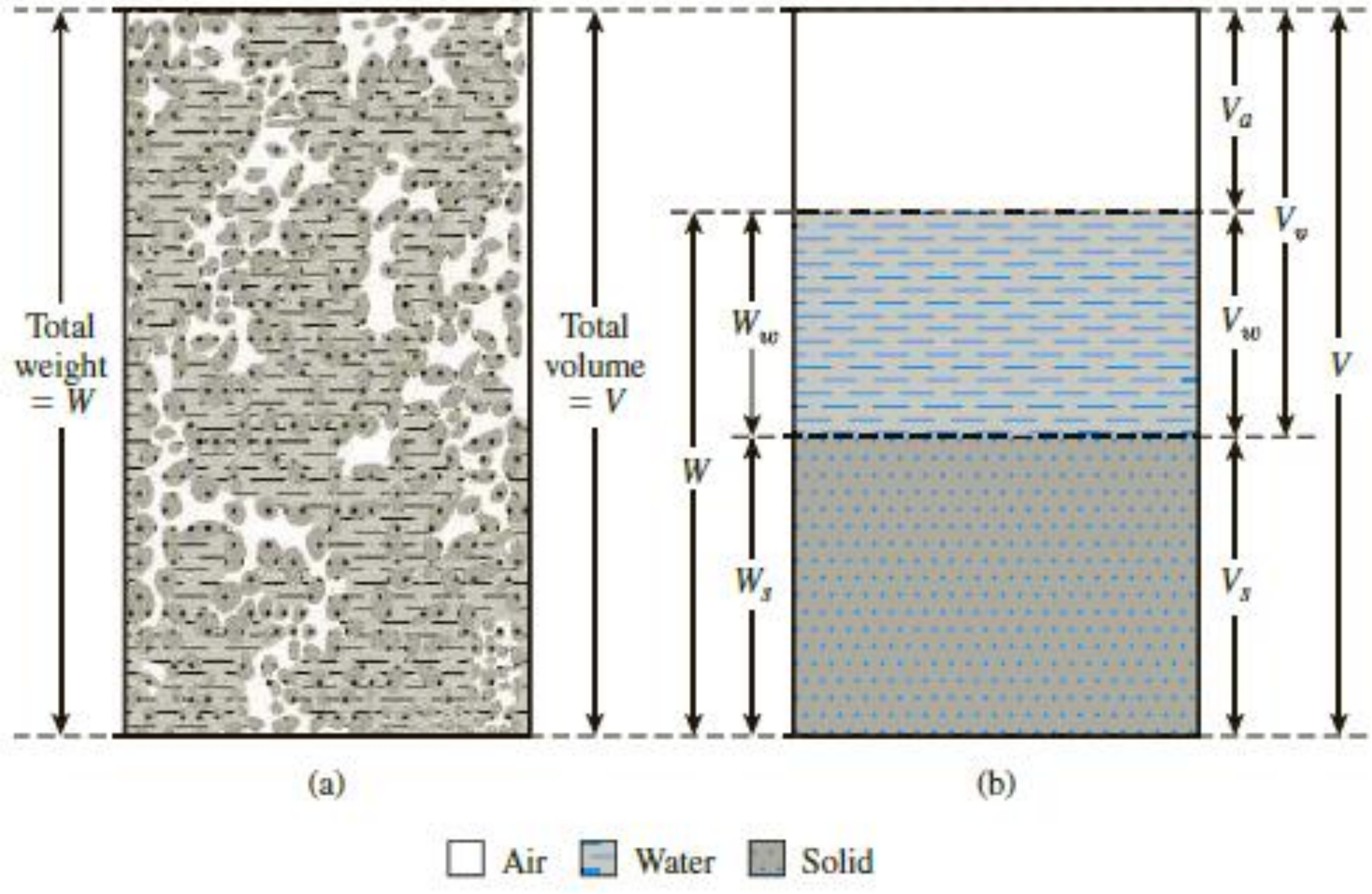
Dr. Khalil Qatu

ENCE 331: Weight – Volume relationships

Block Diagram (phase diagram)

- To develop the weight–volume relationships, we must separate the three phases (that is, solid, water, and air) as shown



$$V = V_s + V_v = V_s + V_w + V_a$$

where V_s = volume of soil solids
V_v = volume of voids
V_w = volume of water in the voids
V_a = volume of air in the voids

$$W = W_s + W_w$$

where W_s = weight of soil solids
W_w = weight of water

Volume relationships

- Void ratio: $e = \frac{V_v}{V_s}$ (indication of soil compaction) (can be larger than 1)

UNITS ??

- Porosity: $n = \frac{V_v}{V}$ (less confidence as indication of compaction) (less or more than 1 ???)

- Degree of saturation: $S = \frac{V_w}{V_v} * 100\%$ (indication of water presence)

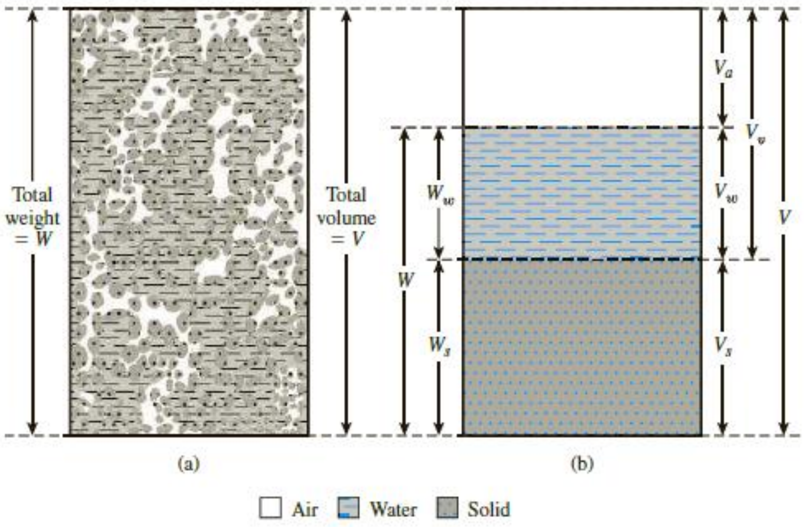
  - Relationship between e and n ??

$$W = W_s + W_w$$

where W_s = weight of soil solids
W_w = weight of water

$$V = V_s + V_v = V_s + V_w + V_a$$

where V_s = volume of soil solids
V_v = volume of voids
V_w = volume of water in the voids
V_a = volume of air in the voids



Weight relationships

- Moisture content (water content):

$$w\% = \frac{W_w}{W_s} * 100\%$$

UNITS ??

- Unit weight γ:
  - Bulk (moist) → $\gamma_B = \frac{W}{V}$
  - dry → $\gamma_d = \frac{W_s}{V}$
  - Saturated → $\gamma_{sat} = \frac{W_s + V_v\gamma_w}{V}$
  - Soil Solids → $\gamma_s = \frac{W_s}{V_s}$

$$W = W_s + W_w$$

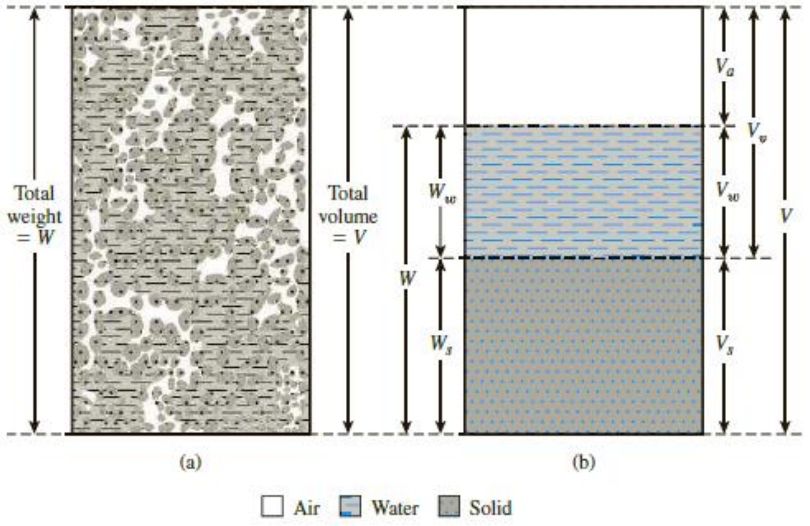
where W_s = weight of soil solids
W_w = weight of water

$$V = V_s + V_v = V_s + V_w + V_a$$

where V_s = volume of soil solids
V_v = volume of voids
V_w = volume of water in the voids
V_a = volume of air in the voids

- Specific Gravity

$$G = \frac{\gamma}{\gamma_w}, \qquad \gamma_w = 9.81\ kN/m^3$$
$$\gamma_w = 62.4\ lb/ft^3$$



- Specific gravity of soil solids

$$G_s = \frac{\gamma_s}{\gamma_w},$$

DOESN'T CHANGE

Volume - Weight relationships

Table 3.1 Void Ratio, Moisture Content, and Dry Unit Weight for Some Typical Soils in a Natural State

Type of soil	Void ratio, e	Natural moisture content in a saturated state (%)	Dry unit weight, γ_d	
			lb/ft^3	kN/m^3
Loose uniform sand	0.8	30	92	14.5
Dense uniform sand	0.45	16	115	18
Loose angular-grained silty sand	0.65	25	102	16
Dense angular-grained silty sand	0.4	15	121	19
Stiff clay	0.6	21	108	17
Soft clay	0.9–1.4	30–50	73–93	11.5–14.5
Loess	0.9	25	86	13.5
Soft organic clay	2.5–3.2	90–120	38–51	6–8
Glacial till	0.3	10	134	21

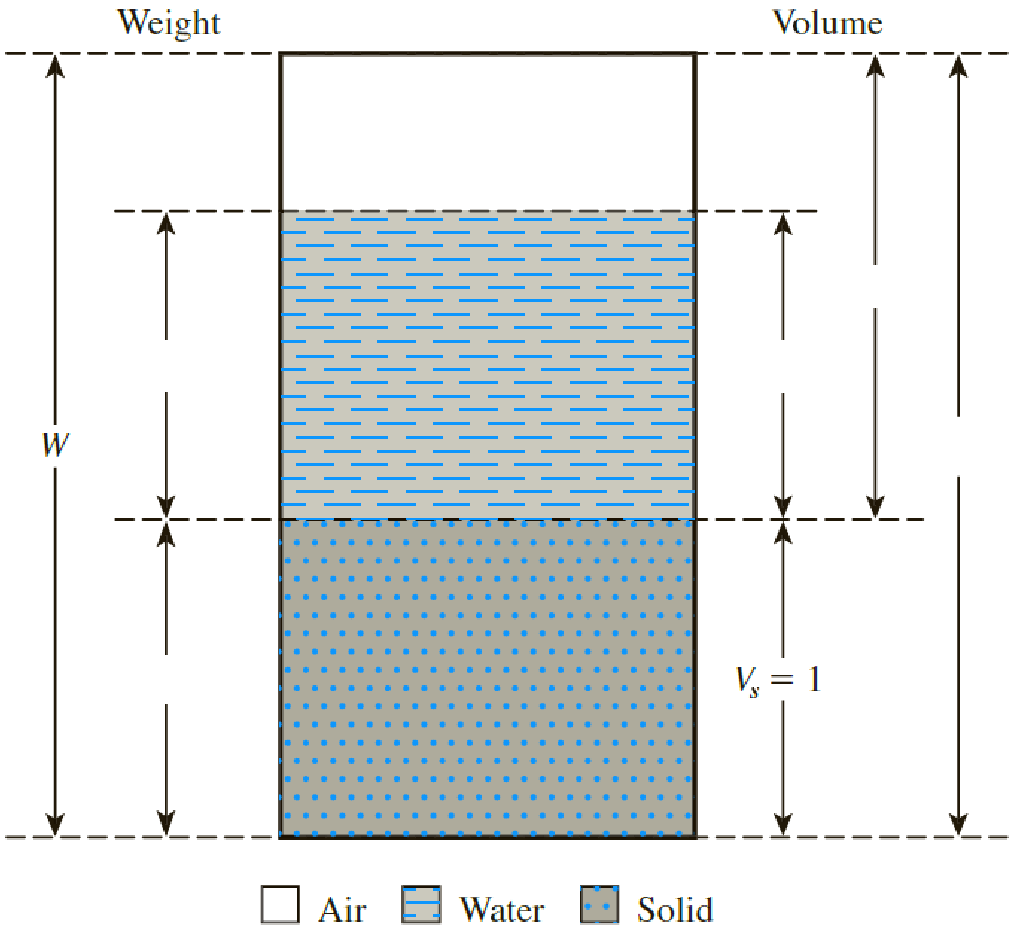
Volume - Weight relationships

- Relationships among Unit Weight, Void Ratio, Moisture Content, and Specific Gravity of soil solids

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{G_s\gamma_w + wG_s\gamma_w}{1+e} = \frac{(1+w)\,G_s\gamma_w}{1+e}$$

$$\gamma_d = \frac{W_s}{V} = \frac{G_s\gamma_w}{1+e}$$

$$e = \frac{G_s\gamma_w}{\gamma_d} - 1$$



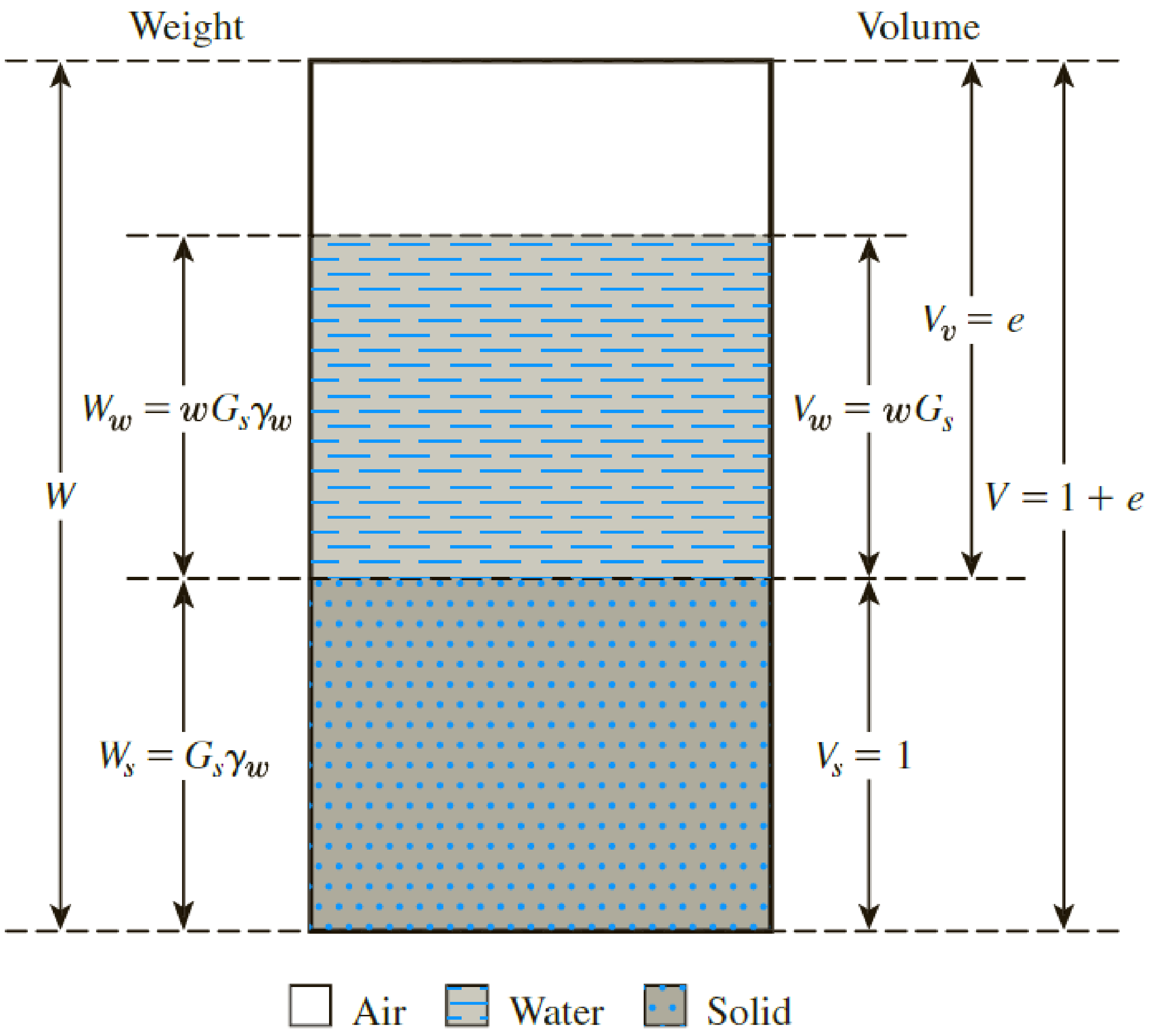
Volume - Weight relationships

- Relationships among Unit Weight, Void Ratio, Moisture Content, and Specific Gravity of soil solids

$$V_w = \frac{W_w}{\gamma_w} = \frac{wG_s\gamma_w}{\gamma_w} = wG_s$$

$$S = \frac{V_w}{V_v} = \frac{wG_s}{e}$$

$$Se = wG_s$$

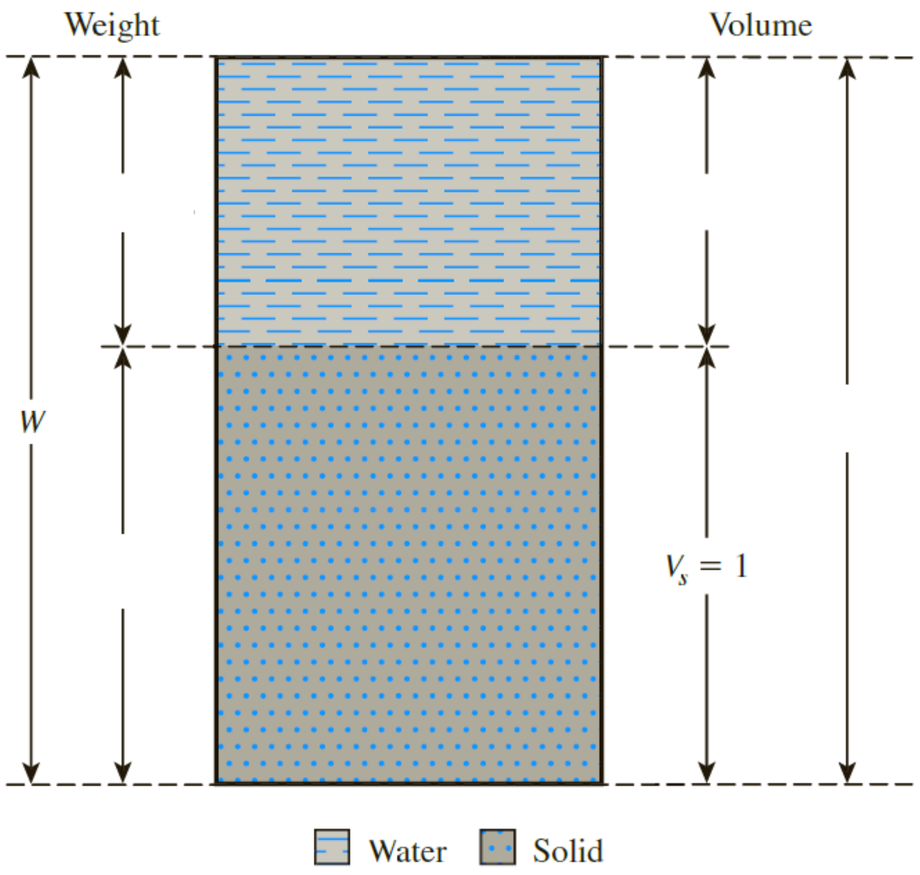


Volume - Weight relationships

- Relationships among Unit Weight, Void Ratio, Moisture Content, and Specific Gravity (saturated Case)

$$\gamma_{sat} = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{G_s\gamma_w + e\gamma_w}{1+e} = \frac{(G_s + e)\gamma_w}{1+e}$$

$$e = wG_s$$

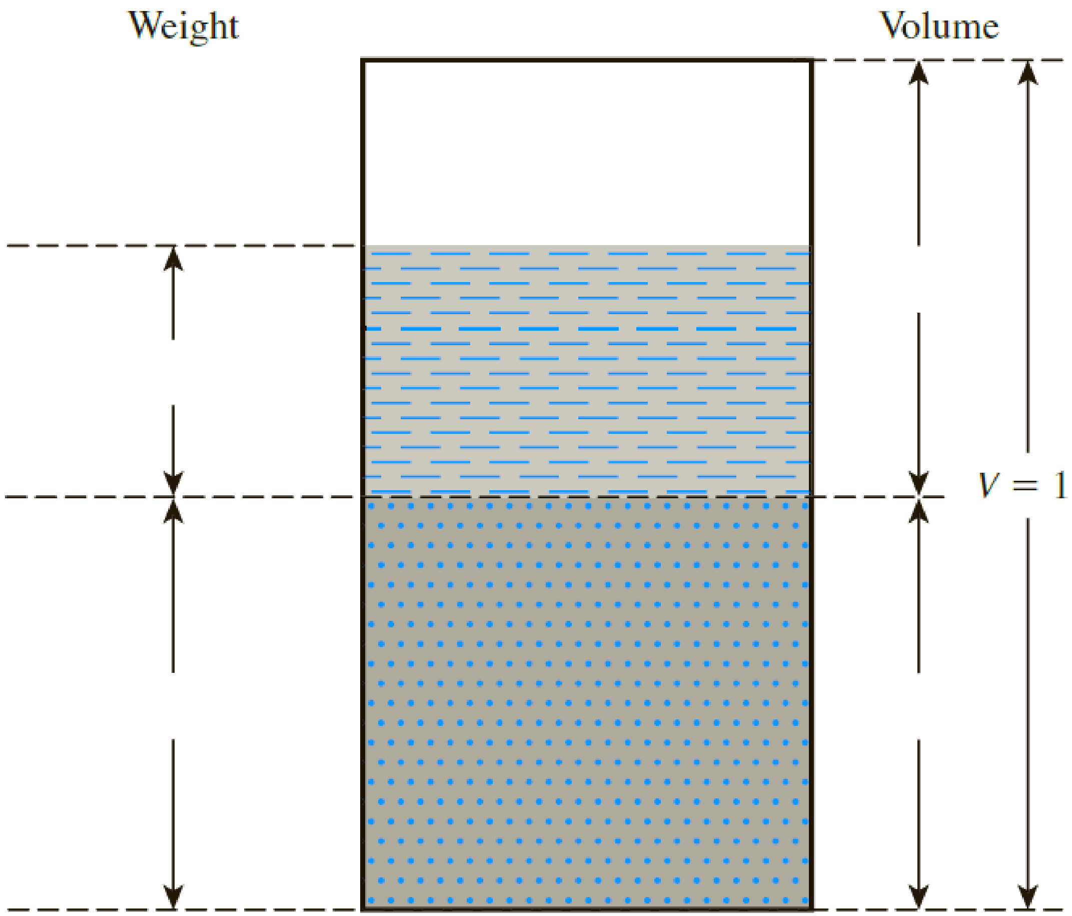


Volume - Weight relationships

- Relationships among Unit Weight, Porosity, and Moisture Content

$$\gamma_d = \frac{W_s}{V} = \frac{G_s\gamma_w(1-n)}{1} = G_s\gamma_w(1-n)$$

$$\gamma = \frac{W_s + W_w}{V} = G_s\gamma_w(1-n)(1+w)$$

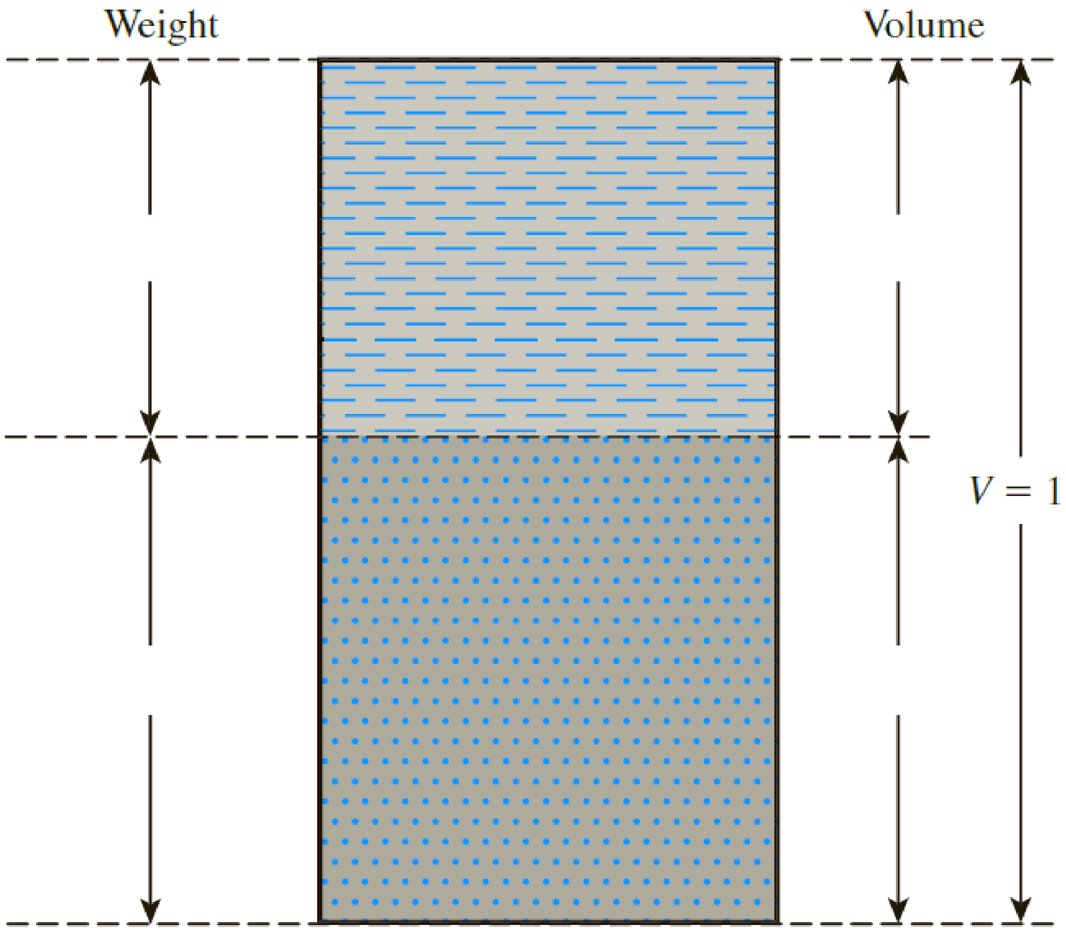


Volume - Weight relationships

- Relationships among Unit Weight, Porosity, and Moisture Content (Saturated case)

$$\gamma_{sat} = \frac{W_s + W_w}{V} = \frac{(1-n)G_s\gamma_w + n\gamma_w}{1} = [(1-n)G_s + n]\gamma_w$$

$$w_{sat} = \frac{W_w}{W_s} = \frac{n\gamma_w}{(1-n)\gamma_w G_s} = \frac{n}{(1-n)G_s}$$



Volume - Weight relationships

Moist unit weight (γ)		Dry unit weight (γ_d)		Saturated unit weight (γ_{sat})	
Given	Relationship	Given	Relationship	Given	Relationship
w, G_s, e	$\dfrac{(1+w)G_s\gamma_w}{1+e}$	γ, w	$\dfrac{\gamma}{1+w}$	G_s, e	$\dfrac{(G_s+e)\gamma_w}{1+e}$
S, G_s, e	$\dfrac{(G_s+Se)\gamma_w}{1+e}$	G_s, e	$\dfrac{G_s\gamma_w}{1+e}$	G_s, n	$[(1-n)G_s+n]\gamma_w$
w, G_s, S	$\dfrac{(1+w)G_s\gamma_w}{1+\dfrac{wG_s}{S}}$	G_s, n	$G_s\gamma_w(1-n)$	G_s, w_{sat}	$\left(\dfrac{1+w_{sat}}{1+w_{sat}G_s}\right)G_s\gamma_w$
w, G_s, n	$G_s\gamma_w(1-n)(1+w)$	G_s, w, S	$\dfrac{G_s\gamma_w}{1+\left(\dfrac{wG_s}{S}\right)}$	e, w_{sat}	$\left(\dfrac{e}{w_{sat}}\right)\left(\dfrac{1+w_{sat}}{1+e}\right)\gamma_w$
S, G_s, n	$G_s\gamma_w(1-n)+nS\gamma_w$	e, w, S	$\dfrac{eS\gamma_w}{(1+e)w}$	n, w_{sat}	$n\left(\dfrac{1+w_{sat}}{w_{sat}}\right)\gamma_w$
		γ_{sat}, e	$\gamma_{sat}-\dfrac{e\gamma_w}{1+e}$	γ_d, e	$\gamma_d+\left(\dfrac{e}{1+e}\right)\gamma_w$
		γ_{sat}, n	$\gamma_{sat}-n\gamma_w$	γ_d, n	$\gamma_d+n\gamma_w$
		γ_{sat}, G_s	$\dfrac{(\gamma_{sat}-\gamma_w)G_s}{(G_s-1)}$	γ_d, S	$\left(1-\dfrac{1}{G_s}\right)\gamma_d+\gamma_w$
				γ_d, w_{sat}	$\gamma_d(1+w_{sat})$

Volume - Weight relationships

- Example:

  For a moist soil sample, the following are given.

  Total volume: $V = 1.2\ m^3$

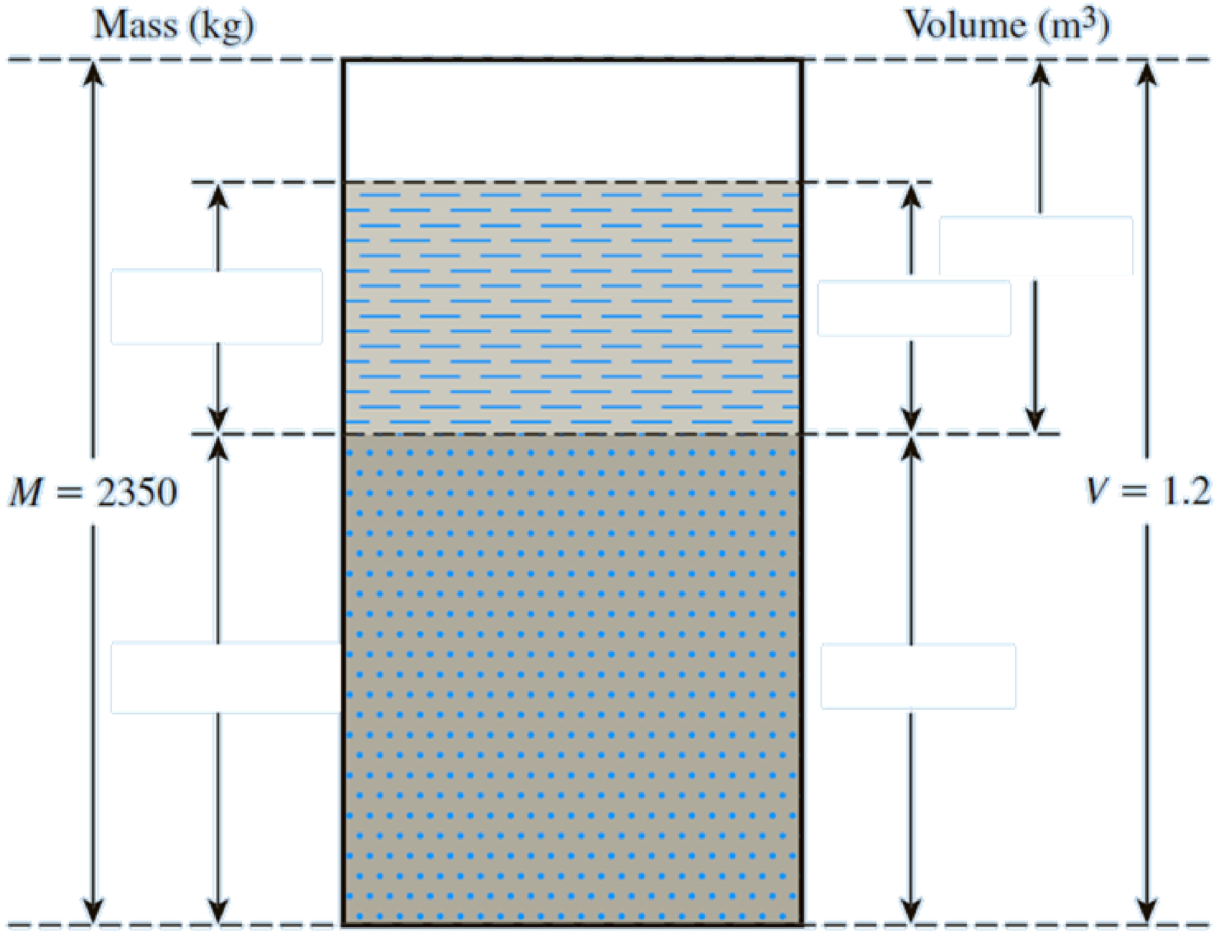
  Total mass: $M = 2350$ kg

  Moisture content: $w = 8.6\%$

  Specific gravity of soil solids: $G_s = 2.71$

  Determine the following.

  - Moist density
  - Dry density
  - Void ratio
  - Porosity
  - Degree of saturation
  - Volume of water in the soil sample



Volume - Weight relationships

- Example:

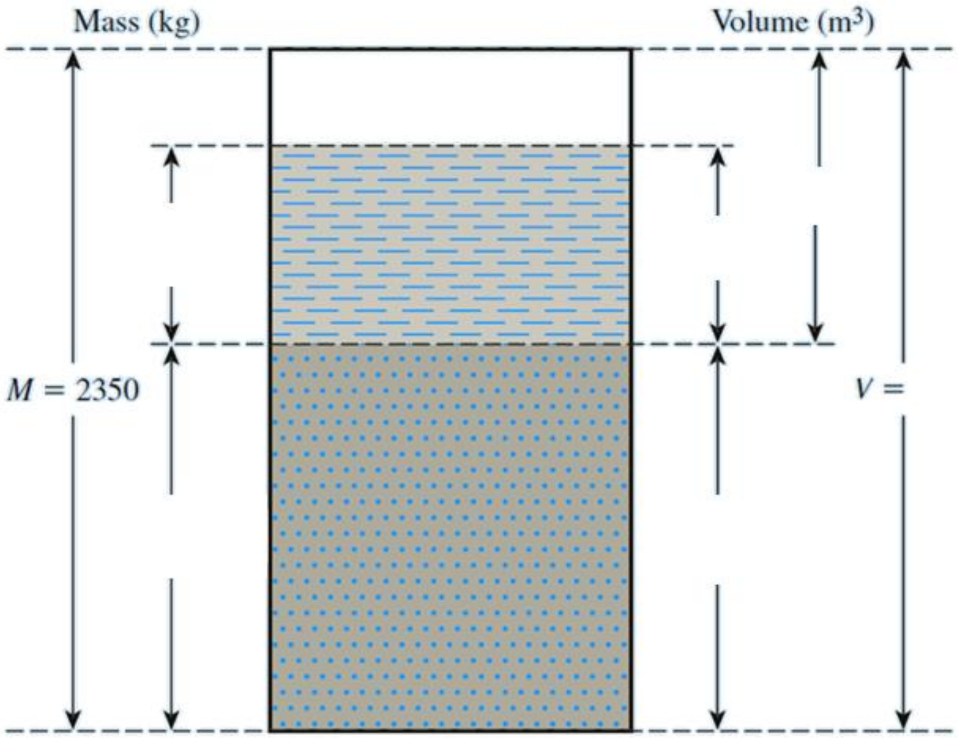
  The following data are given for a soil:

  Porosity: n = 0.4

  Specific gravity of the soil solids: Gs = 2.68

  Moisture content: w = 12%

  Determine the mass of water to be added to 10 m^3 of soil for full saturation.



Volume - Weight relationships

- Relative density (D_r): commonly used to indicate the in-situ denseness or looseness of granular soil

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}}$$

$$D_r = \left[\frac{\rho_d - \rho_{d(min)}}{\rho_{d(max)} - \rho_{d(min)}}\right]\frac{\rho_{d(max)}}{\rho_d}$$

Relative density (%)	Description of soil deposit
0–15	Very loose
15–50	Loose
50–70	Medium
70–85	Dense
85–100	Very dense

- ***Minimum dry unit weight***,

  sand is poured loosely into the mold from a funnel with a 12.7 mm (12 in.) diameter spout. The average height of the fall of sand into the mold is maintained at about 25.4 mm (1 in.).

- ***maximum dry unit weight***

  vibrating the sand in the mold for 8 min. A surcharge of 14 kN/m^2 (2 lb/in^2) is added to the top of the sand in the mold. The mold is placed on a table that vibrates at a frequency of 3600 cycles/min and that has an amplitude of vibration of 0.635 mm (0.025 in.)

LEGEND
1 - Mold
2 - Dial indicator
3 - Surcharge weight
4 - Guide sleeve
5 - Surcharge base plate
6 - Vibrating table

Volume - Weight relationships

- Borrow Pit:

Earth is required to be excavated from borrow pits for building an embankment as shown in the figure below.

The moist unit weight of the borrow pit is 18 kN/m^3 and its water content is 8%. The specific gravity of solids as 2.67

The dry unit weight required for the embankment is 15 kN/m^3 with a moisture content of 10%.

- Estimate the quantity of earth required to be excavated per meter length of embankment.
- Determine the degree of saturation of the embankment soil and the volume of water in the embankment