Chapter 2

• Boolean Algebra and logic Gates

Boolean algebra

① set of elements $B \in \{0,1\}$.
② set of operators. (+, ·, AND, NOT, XOR)
③ set of postulates

Logic Gates

* used in all of todays computer.
* cost of the circuits that implement(تنفيذ) is an important addressed by designes.

Rules for Boolean algebra

1. closure ∀ operator +, · are closed for all $x, y \in B$.

$$x + y \in B^{\{0,1\}}$$
$$x \cdot y \in B^{\{0,1\}}$$

$0 \cdot 0 = 0$
$0 \cdot 1 = 0$
$1 \cdot 0 = 0$
$1 \cdot 1 = 1$

2. identity

* 0 is the identity element for + :

$0 + X = X$
$0 + 1 = 1$
$0 + 0 = 0$

* 1 is the identity element for . :

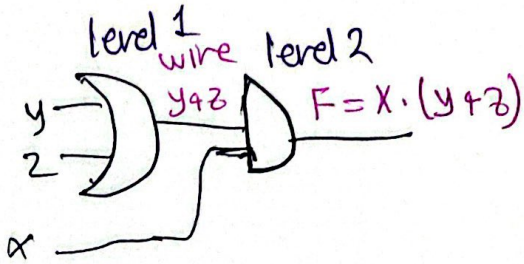
$1 \cdot X = X$
$1 \cdot 1 = 1$
$1 \cdot 0 = 0$

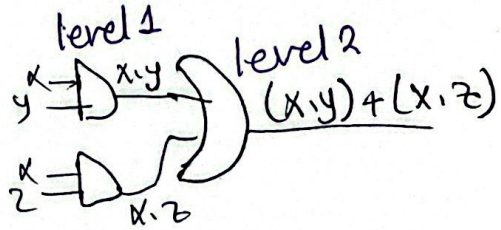
3. commutative

$X + y = y + X$

4. distributive

$X \cdot (y + z) = (X \cdot y) + (X \cdot z)$

① 

② 

5.

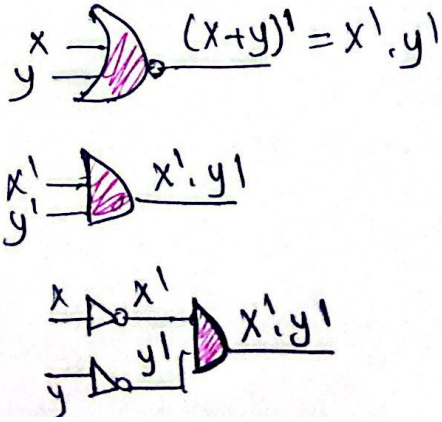
$X + X' = 1$
$X \cdot X' = 0$

6. complement

* comp of X is X^c, $\bar{X}$, X'
* comp of 0 is 1
* comp of 1 is 0

7. Demorgan law

* $(X+y)' = X' \cdot y'$
* $(X \cdot y)' = X' + y'$



Boolean function

expression formed with.

① boolean variables (0,1)
② operators.
③ sign (=)

$F = (x+y)\cdot z \longrightarrow (x\cdot z) + (y\cdot z)$

x	y	z	$(x+y)\cdot z$	$x\cdot z$	$y\cdot z$	$(x\cdot z)+(y\cdot z)$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	1	0	1	1
1	0	0	0	0	0	0
1	0	1	1	1	0	1
1	1	0	0	0	0	0
1	1	1	1	1	1	1

Note

- $x + x = x$
- $x \cdot x = x$
- $x + 1 = 1$
- $x \cdot 0 = 0$
- absorption (a) $x + xy = x$ (b) $x(x+y) = x$
- duality (a) $x\cdot(\bar{x}+y) = x\cdot y$; $x+(\bar{x}\cdot y) = x+y$ → تغيير الاشارات

- $A + \bar{A}B = A + B$
- $F = \bar{A}\cdot(\bar{B}+\bar{C})$
  $\bar{A}\cdot(\overline{BC}) \Rightarrow \overline{A+BC}$

Minimize the following function

* $F = x \cdot (\bar{x} + y)$

$F = (x \cdot \bar{x}) + (x \cdot y) = 0 + (x \cdot y) = x \cdot y$

* $F = x + (\bar{x} \cdot y)$

$F = (x + \bar{x}) \cdot (x + y) = 1 \cdot (x + y) = x + y$

* $F = (x + y) \cdot (x + \bar{y})$

$F = x + (y \cdot \bar{y}) = x + 0 = x$

Complement of the function

* $F = x \cdot y + \bar{x} \cdot z + y \cdot z$

$F = x \cdot y + \bar{x} \cdot z + y \cdot z \cdot 1$

$= x \cdot y + \bar{x} \cdot z + y \cdot z \cdot (x + \bar{x})$

$= xy + \bar{x} \cdot z + x \cdot y \cdot z + \bar{x} \cdot y \cdot z$

$= x \cdot y (1 + z) + \bar{x} \cdot z (1 + y)$

$= x \cdot y + \bar{x} \cdot z$

* $F = (x + y)(x' + z)(y + z)$

{by duality

$(x + y) \cdot (x' + z)$

Identity

* 0 is the:

* 1

Minimize the following F...

Complement of the function

$F_1 = x'yz' + x'y'z$

The dual of F_1 is: $(x'+y+z') \cdot (x'+y'+z)$

Complement each literal: $(x+y'+z)(x+y+z') \Rightarrow F_1'$

$F_2 = x(y'z' + yz)$

The dual of F_2 is: $x + (y'+z') \cdot (y+z)$

Complement of each literal: $x' + (y+z) \cdot (y'+z') \Rightarrow F_2'$

- Canonical & standard form

$F_{(x,y,z)} = \bar{x} \cdot \bar{y} \cdot z + x \cdot y$ implementation

↳ Canonical ⇝ for standard langage for digital.

1) Sum of minterms (SOM)
2) product of Maxterms (POM)

Note: if we have function with 2 variables
$F_{(x,y)} \Rightarrow$ four combination (2^2)

- $x \cdot y \;/\; x' \cdot y \;/\; x \cdot y' \;/\; x' \cdot y'$
- $x + y \;/\; x' + y \;/\; x \cdot y' \;/\; x' + y'$

- function n variables $\Rightarrow 2^n$ combinations, each combination called minterm.

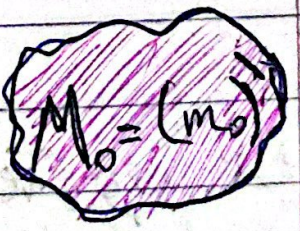
- each minterm is denoted by m_i; $0 \leq i \leq 2^n - 1$

ex $\Rightarrow$ 2 variables $\Rightarrow$ 4 combination $\Rightarrow m_0\ m_1\ m_2\ m_3 \Rightarrow 0 \leq i \leq$

Note: the complement of minterm is called Maxterms.

4 variable $\Rightarrow$ 16 combs.

m_0 (minterms) { M_0 (Maxterms)
$\vdots$
m_{15} { M_{15}



Example

$F = \underbrace{x \cdot y}_{\text{term}} + \underbrace{x \cdot z}_{\text{term}}$

$F_{(x,y,z)} =$

Input			minterms		maxterms	
x	y	z	Term	Designation	Term	Designation
0	0	0	$\bar{x}\cdot\bar{y}\cdot\bar{z}$	m_0	$(x+y+z)$	M_0
0	0	1	$\bar{x}\cdot\bar{y}\cdot z$	m_1	$x+y+\bar{z}$	M_1
0	1	0	$\bar{x}\cdot y\cdot\bar{z}$	m_2	$x+y'+z$	M_2
0	1	1	$\bar{x}\cdot y\cdot z$	m_3	$x+y'+z'$	M_3
1	0	0	$x\cdot\bar{y}\cdot\bar{z}$	m_4	$x'+y+z$	M_4
1	0	1	$x\cdot\bar{y}\cdot z$	m_5	$x'+y+z'$	M_5
1	1	0	$x\cdot y\cdot\bar{z}$	m_6	$x'+y'+z$	M_6
1	1	1	$x\cdot y\cdot z$	m_7	$x'+y'+z'$	M_7

* دائماً في minterm داخل الـ Term يوجد AND وجواب الـ Term واحد.

* دائماً في maxterm داخل الـ Term يوجد OR وجواب الـ Term صفر.

Example

$F_{(x,y,z)} = \sum(0,2,5)$ ← ممكن تشكيل

$F_{(x,y,z)} = \{ m_0 + m_2 + m_5 \}$ (SOM) ⇒ sum of minterm

زي كانة كل الدول بعرض هاي الصيغة اسم

AND داخله OR خارجيا

$= m_{000} + m_{010} + m_{101}$

$= \bar{x}.\bar{y}.\bar{z} + \bar{x}.y.\bar{z} + x.\bar{y}.z$

x	y	z	$m_0 + m_2 + m_5 = F$	
0	0	0	1	→ m_0
0	0	1	0	
0	1	0	1	→ m_2
0	1	1	0	
1	0	0	0	
1	0	1	1	→ m_5
1	1	0	0	
1	1	1	0	

$F(A,B,C) = \bar{A}.\bar{B}.\bar{C} + \bar{A}.\bar{B}.C + A.B.C$

$= \bar{0}.\bar{0}.\bar{0} + \bar{0}.\bar{0}.1 + 1.1.1$

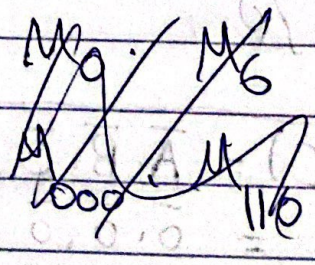
$= m_0 + m_1 + m_7$

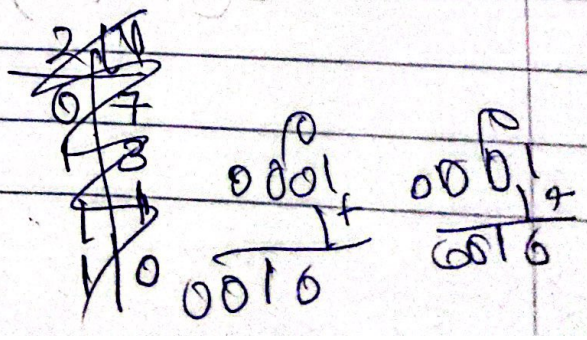
$= \Sigma(0,1,7)$

Find the truth table and mathematical expression.

$$F(w,x,y,z) = \Sigma 0, 1, 14, 15$$
$$= m_0 + m_1 + m_{14} + m_{15}$$
$$= m_{0000} + m_{0001} + m_{1110} + m_{1111}$$
$$= w'x'y'z' + w'x'y'z + wxyz' + wxyz$$

w	x	y	z	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	1
1	1	1	0	1
1	1	1	1	





Ex. Write the mathematical expression.

$$F_{(x,y,z)} = M_0 \cdot M_6$$
$$M_{000} \cdot M_{110}$$
$$= (x+y+z) \cdot (\bar{x}+\bar{y}+z)$$

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Express the following function in product of maxterms.

$$F(w,x,y,z) = \Sigma 0, 1, 2, 14$$
$$= M_0 \cdot M_1 \cdot M_2 \cdot M_{14}$$
$$= M_{0000} \cdot M_{0001} \cdot M_{0010} \cdot M_{1110}$$

$F_{(w,x,y,z)} = \Sigma\, 0, 1, 2, 14$

w	x	y	z	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

$F_{(w,x,y,z)} = M_3 \cdot M_4 \cdot M_5 \cdot M_6 \cdot M_7 \cdot M_8 \cdot M_9 \cdot M_{10} \cdot M_{11} \cdot M_{12} \cdot M_{13} \cdot M_{15}$

$\Pi\,(3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15)$

Express the complement of the following function using POM

$F_{(A,B,C)} = \Sigma 0, 2, 5, 7$

A	B	C	F	$\bar{F}$
0	0	0	1	0
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	1	0
1	1	0	0	1
1	1	1	1	0

$\bar{F}(A,B,C) = \Pi (0, 2, 5, 7)$

Write the following function using SOM,

$F_{(A,B,C)} = A + \bar{A}C$

(1) truth table

A	B	C	F	$A + \bar{A}C$
0	0	0	0	$0 + \bar{0}\cdot 0$
0	0	1	1	
0	1	0	0	
0	1	1	1	
1	0	0	1	
1	0	1	1	
1	1	0	1	
1	1	1	1	

$F_{(A,B,C)} = \{1, 3, 4, 5, 6, 7$

$= m_1 + m_3 + m_4 + m_5 + m_6 + m_7$

$$F = A + \bar{A}C$$
$$= A\cdot[B+\bar{B}] + \bar{A}\cdot C\cdot(B+\bar{B})$$
$$= AB + A\bar{B} + \bar{A}BC + \bar{A}\bar{B}C$$
$$= AB(C+\bar{C}) + A\bar{B}(C+\bar{C}) + \bar{A}BC + \bar{A}\bar{B}C$$
$$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC + \bar{A}\bar{B}C$$
$$= m_7 + m_6 + m_5 + m_4 + m_3 + m_1$$
$$= \Sigma\, 1, 3, 4, 5, 6, 7$$

(EX) (#) $F_{(x,y,z)} = x\cdot y + \bar{x}\cdot z$ POH

$\bar{x}\cdot z\,[y+\bar{y}]$

$$F_{(x,y,z)} = x\cdot y(z+\bar{z}) + \bar{x}\cdot z(y+\bar{y})$$
$$= x\cdot y\cdot z + x\cdot y\cdot\bar{z} + \bar{x}\cdot y\cdot z + \bar{x}\bar{y}z$$
$$m_7 + m_6 + m_3 + m_4$$
$$= \Sigma\,(1, 3, 6, 7)$$

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$F_{(A,B,C)} = \Pi\,(0, 2, 4, 5)$$

0.0+1.0
0 to

mint → 0
mint → 1

الطريقة الثانية

$F = x \cdot y + \bar{x} \cdot z$

$= w + (\bar{x} \cdot z)$

$F = (w + \bar{x}) \cdot (w + z)$

$F = ((x \cdot y) + \bar{x}) \cdot (x \cdot y + z)$

$F = (\bar{x} + (x \cdot y)) \cdot (z + x \cdot y)$

$F = (\bar{x} + x) \cdot (\bar{x} + y) \cdot (x + z) \cdot (y + z)$

$F = (\bar{x} + y) \cdot (x + z) \cdot (y + z)$

$F = (\bar{x} + y + z \cdot \bar{z}) \cdot (x + z + y \cdot y') \cdot (y + z + x \cdot x')$

$F = (w_1 + z \cdot \bar{z}) \cdot (w_2 + y \cdot y') \cdot (w_3 + x \cdot \bar{x})$

Minterm / maxterm ↓ term which is variable

$F = (w_1 + z) \cdot (w_1 + \bar{z}) \cdot (w_2 + y) \cdot (w_2 + \bar{y}) \cdot (w_3 + x) \cdot (w_3 + \bar{x})$

$F = (\bar{x} + y + z) \cdot (\bar{x} + y + \hat{z}) \cdot (x + y + z) \cdot (x + z + \bar{y}) \cdot (y + z + x) \cdot (y + z + \bar{x})$

$F = (\bar{x} + y + z) \cdot (\bar{x} + y + \hat{z}) \cdot (x + y + z) \cdot (x + \bar{y} + z) \cdot (x + y + z) \cdot (\bar{x} + y + z)$

$F = M_4 \cdot M_5 \cdot M_0 \cdot M_2 \cdot M_6 \cdot M_4$

$\Pi (0, 2, 4, 5)$

Canonical Form

SOM

$F_{(A,B,C)} = A \cdot B \cdot \bar{C} + A \cdot B \cdot C$

$= A \cdot B \cdot \bar{C} + A \cdot B$

Not SOM

POM

$F_{(A,B,C)} = (A+B+C) \cdot (\bar{A}+\bar{B}+\bar{C})$

$F = (A+B+C) \cdot (\bar{A}+\bar{B})$

Not POM

Standard Form

Sum of Product (SOP)

$F = A \cdot B \cdot C + A \cdot B$

Not sum of Product

$= A \cdot B \cdot C + A \cdot B \cdot \bar{C}$

SOP هو SOM اي والعكس غير صحيح

Product of sum (POS)

$F = (A+B+C) \cdot (A+\bar{B})$

✓ Product of sum

✗ Product of max

$F = (A+B+C) \cdot (A+\bar{B}+C)$

prod of sum ✓

prod of max ✓

Ex) $F_{(x,y,z)} = \overbrace{x.y.z}^{\text{pro}} + \overbrace{x.y.z}^{\text{pro}}$ (sum)

① sum of minterms.

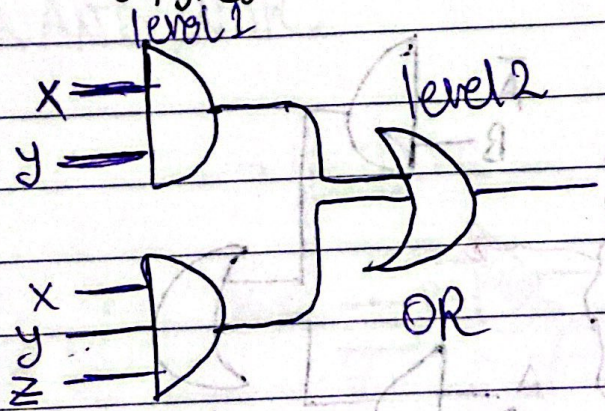
② sum of product.

$F_{(x,y,z)} = \overbrace{x.y}^{\text{pro}} + \overbrace{x.y.z}^{\text{pro}}$ (sum)

① sum of product.

SOM | SOP

$F_{(x,y,z)} = x.y + x.y.z$ (SOP)

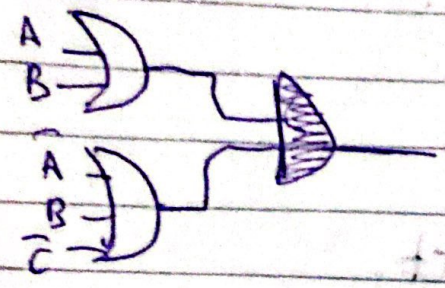


$F_{(A,B,C)} = (A+B+C).(\overline{A}+B+\overline{C})$

sum, pro, Sum

* Product of maxterm.
* product of sum.

* $F_{(A,B,C)} = (A+B) \cdot (\bar{A}+B+\bar{C})$

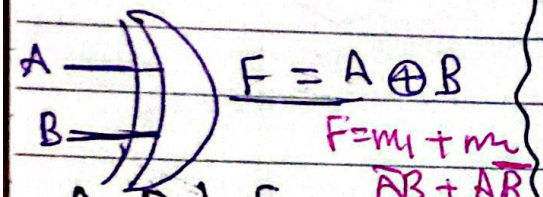


X ⇒ product of maxterm
✓ ⇒ product of sum.

OR — AND

Note

(odd) XOR gate

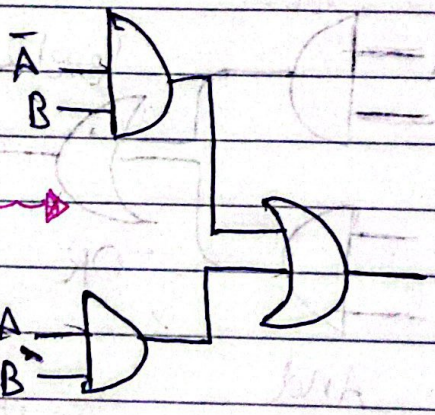


$F = A \oplus B$

$F = m_1 + m_2$

$\bar{A}B + A\bar{B}$

A	B	F	
0	0	0	
0	1	1	$m_1 = \bar{A}B$
1	0	1	$m_2 = A\bar{B}$
1	1	0	



(even) XNOR gate

$F = (A \oplus B)^{-1}$

$F = m_0 + m_3$

$= \bar{A}\bar{B} + AB$

A	B	F
0	0	1
0	1	0
1	0	0
1	1	1