Individual Demand Functions

Demand function: A representation of how quantity demanded depends on prices, income, and preferences.

يعرف منحنى الطلب بأنه علاقة بين الكمية المطلوبة والعوامل المؤثرة فيها (سعر السلعة، دخل المستهلك، وذوق المستهلك)

The quantities of X and Y that a person chooses depend on that person's preferences and on the details of his or her budget constraint. If we knew a person's preferences and all the economic forces that affect his or her choices, we could predict how much of each good would be chosen.

الكمية المطلوبة من السلعة X والسلعة Y التي يرغب المستهلك في شرائها تعتمد على ذوق المستهلك على تلك السلع وكذلك على دخل المستهلك. إذا تم تحديد ذوق المستهلك وتحديد جميع العوامل التي تؤثر على الكمية المطلوبة فإنه بإمكان المستهلك تحدد الكمية التي يرغب بشرائها من السلعتين .

We can summarize this conclusion using the demand function for some particular good, say, X:

ويمكن صياغة العلاقة بين الكمية المطلوبة والعوامل المؤثرة فيها بالصيغة الدالة التالية:

Quantity of X demanded = $d_x\ (P_X\ , P_y, I; \text{preferences})$

نقرأ الصيغة الرياضية لمنحنى الطلب بالشكل التالي:

تعتمد الكمية المطلوبة من السلعة X على سعر السلعة X و سعر السلعة ثانية وهي السلعة Yالتي لها علاقة مع السلعة X(يمكن تكون بديلة او مكملة)، وكذلك تعتمد على ذوق المستهلك (تفضيلة للسلعة).

تظهر ذوق المستهلك على يمين الفاصلة المنقوطة في المعادلة أعلاه لأننا نفترض أن ذوق الشخص للسلعة X لن يتغير أثناء التحليل.

This function contains the three elements that determine what the person can buy—the prices of X and Y and the person's income (I)—as well as a reminder that choices are also affected by preferences for the goods. These preferences appear to the right of the semicolon in above Equation because, for most of our discussion, we assume that preferences do not change.

The quantity demanded of good Y depends on these same general influences and can be summarized by:

Quantity of Y demanded = $d_y\ (P_X\ , P_y, I; \text{preferences})$

تعتمد الكمية المطلوبة من السلعى Y على سعر السلعة Y و سعر السلعة ثانية وهي السلعة X التي لها علاقة مع السلعة Y (يمكن تكون بديلة او مكملة)، وكذلك تعتمد على ذوق المستهلك (تفضيلة للسلعة).

Example (1):

A consumer purchases two goods, food (F) and clothing (C). Her utility function id given by $U(F, C) = FC + F$. The marginal utilities are $MUF = C + 1$ and $MUC = F$. The price of food is P_f, the price of clothing is P_c, and the consumer's income is *I*.

What is the equation for the demand curve for clothing?

لاشتقاق معادلة الطلب على السلعة يتم استخدام نفس الخطوات التي استخدمناها في تحديد Utility maximization

Budget line equation: $PF\,F + PC\,C = I$ .... (1)

To max utility: $\frac{MUF}{MUC} = \frac{P_F}{P_C}$

$\frac{C+1}{F} = \frac{P_F}{P_C}$ ⇨ $PF\,F = PC\,(C + 1)$ .... (2)

By solve equations (1) and (2) (بتعويض المعادلة 2 في المعادلة 1 ينتج)

$P_C\,(C + 1) + P_C\,C = I$ ⇨ $P_C\,C + P_C + P_C\,C = I$

⇨ $2\,P_C\,C + P_C = I$ ⇨ $2\,P_C\,C = I - P_C$

⇨ $C = \frac{I - P_C}{2P_C}$ .... (Demand for clothing)

Example (2):

Robert has utility function $U(X, Y) = X^2 Y$, where X is the quantity of apples and Y the quantity of banana he consume. The price of good X is P_x, the price of good Y P_y, and the consumer's income is I. Derive the demand curves for good Y.

Budget line equation: $Px\,X + Py\,Y = I$ .... (1)

To max utility: $\frac{MUX}{MUY} = \frac{P_X}{P_Y}$

MUX = 2XY MUY = X^2

$\frac{2XY}{X^2} = \frac{P_X}{P_Y}$ بإختصار قيمة اكس من البسط والمقام ينتج

$\frac{2Y}{X} = \frac{P_X}{P_Y}$ (ضرب تبادلي ينتج) ➔ $Px\,X = 2Py\,Y$ ....(2)

بتعويض المعادلة (2) في المعادلة (1) ينتج

$2Py\,Y + Py\,Y = I$ ➔ $3\,Py\,Y = I$ ➔ $Y = \frac{I}{3P_Y}$ .... Demand for good Y

Example (3):

The Jones family spends all its income on food and shelter. It derives maximum utility when it spends two-thirds of its income on shelter and one third on food. Use this information to calculate the demand function for shelter and food.

$P_F * F \equiv$ the amount of income spent on food
$P_S *S \equiv$ the amount of income spent on shelter

It spends two-thirds of its income on shelter ⇨ $P_S *S = \frac{2}{3} I$

⇨ $S = \frac{2I}{3P_S}$ .... demand equation for shelter

It spends one third of its income on food ⇨ $P_F\, F = \frac{1}{3}\, I$

⇨ $F = \frac{2\, I}{3\ PF}$ .... demand equation for food

Homogeneity

Homogeneous demand function: Quantity demanded does not change when prices and income increase in the same proportion.

تجانس منحنى الطلب : الكمية المطلوبة لا تتغيير عندما تزداد الاسعار والدخل بنفس النسبة (او القيمة). بمعنى ان الكمية المطلوبة من السلعة لا تتاثر عندما تتضاعف الاسعار والدخل.

If the prices of X and Y and income (I) were all to double (or to change by any identical percentage), the amounts of X and Y demanded by this person would not change.

The budget constraint $P_x\, X + P_y\, Y = I$ is the same as the budget constraint: $2P_x\, X + 2P_y\, Y = 2I$

Graphically, these are exactly the same lines. Consequently, both budget constraints are tangent to a person's indifference curve map at precisely the same point.

Example:

Maher's demand for Pizza is given by: $Q = \frac{0.3I}{P_p + P_c}$. Where P_p is the price of Pizza, and P_c the price of Coca-Cola. Is this function homogeneous in I and P_p, P_c?.

Homogeneous: when he doubles P_p, P_c, and I, Q does not change.

يتم في معادلة الطلب ضرب قيمة الدخل ب (2) و قيمة الاسعار ب (2) . وبعدها يتم عمل اختصارات ومقارنة الناتج مع معادلة الطلب الاصلية. اذا طلعت النتيجة نفس المعادلة الاصلية فإننا نستنتج بأن منحنى الطلب متجانس اما اذا حصلنا على نتيجة مختلفة عن معادلة الطلب الاصلية فإننا نقول بإن منحنى الطلب غير متجانس بمعنى انه عند مضاعفة الاسعار والدخل فإن الطلب على السلعة قد تعيير.

$$\frac{0.3\,(2I)}{2P_p + 2P_c} = \frac{2\,(0.3I)}{2(P_p + P_c)} = \frac{0.3I}{P_p + P_c} = Q$$ -----⟶ homogeneous

Example (2):

Robert has utility function U(X, Y) = 4X Y, where X is the quantity of apples and Y the quantity of banana he consume. The price of good X is P_x, the price of good Y P_y, and the consumer's income is I.

1. Derive the demand curves for good X.

   Budget line equation: $Px\,X + Py\,Y = I$ .... (1)

   To max utility: $\frac{MUX}{MUY} = \frac{P_X}{P_Y}$

   MUX = 4Y MUY = 4X

   $\frac{4Y}{4X} = \frac{P_X}{P_Y}$ بإختصار القيمة 4 من البسط والمقام ينتج

   $\frac{Y}{X} = \frac{P_X}{P_Y}$ (ضرب تبادلي ينتج) ➔ $Py\,Y = Px\,X$ ....(2)

   بتعويض المعادلة (2) في المعادلة (1) ينتج

   $Px\,X + Px\,X = I$ ➔ $2Px\,X = I$ ➔ $X = \frac{I}{2P_X}$ .... Demand for good X

2. Is demand for good x Homogenous? Explain?

   Homogeneous: when he doubles P_x, and I ➔ Q does not change.

   يتم في معادلة الطلب ضرب قيمة الدخل ب (2) و قيمة سعر اكس ب (2) . وبعدها يتم عمل اختصارات ومقارنة الناتج مع معادلة الطلب الاصلية. اذا طلعت النتيجة نفس المعادلة الاصلية فإننا نستنتج بأن منحنى الطلب متجانس اما اذا حصلنا على نتيجة مختلفة عن معادلة الطلب الاصلية فإننا نقول بإن منحنى الطلب غير متجانس بمعنى انه عند مضاعفة الاسعار والدخل فإن الطلب على السلعة قد تعيير.

   $\frac{(2I)}{2(2P_X)}$ باستصار القيمة (2) من البسط والمقام ينتج ➔ $\frac{I}{2P_X} = X$ .... demand homogenous

   بعد مضاعفة كلاً من الدخل و سعر السلعة اكس فإن منحنى الطلب لم يتغيير (نفس المعادلة الاصلية) وبهذا نستنتج بإن منحنى الطلب متجانس .

Example (3):

Maher's demand for Pizza is given by: $Q = \frac{5I}{2P_p P_c}$. Where P_p is the price of Pizza, and P_c the price of cheese. Is this function homogeneous in I and P_p, P_c?

Homogeneous: when he doubles P_p, P_c, and I, Q does not change.

يتم في معادلة الطلب ضرب قيمة الدخل ب (2) و قيمة الاسعار ب (2) . وبعدها يتم عمل اختصارات ومقارنة الناتج مع معادلة الطلب الاصلية. اذا طلعت النتيجة نفس المعادلة الاصلية فإننا نستنتج بأن منحنى الطلب متجانس اما اذا حصلنا على نتيجة مختلفة عن معادلة الطلب الاصلية فإننا نقول بإن منحنى الطلب غير متجانس بمعنى انه عند مضاعفة الاسعار والدخل فإن الطلب على السلعة قد تعيير.

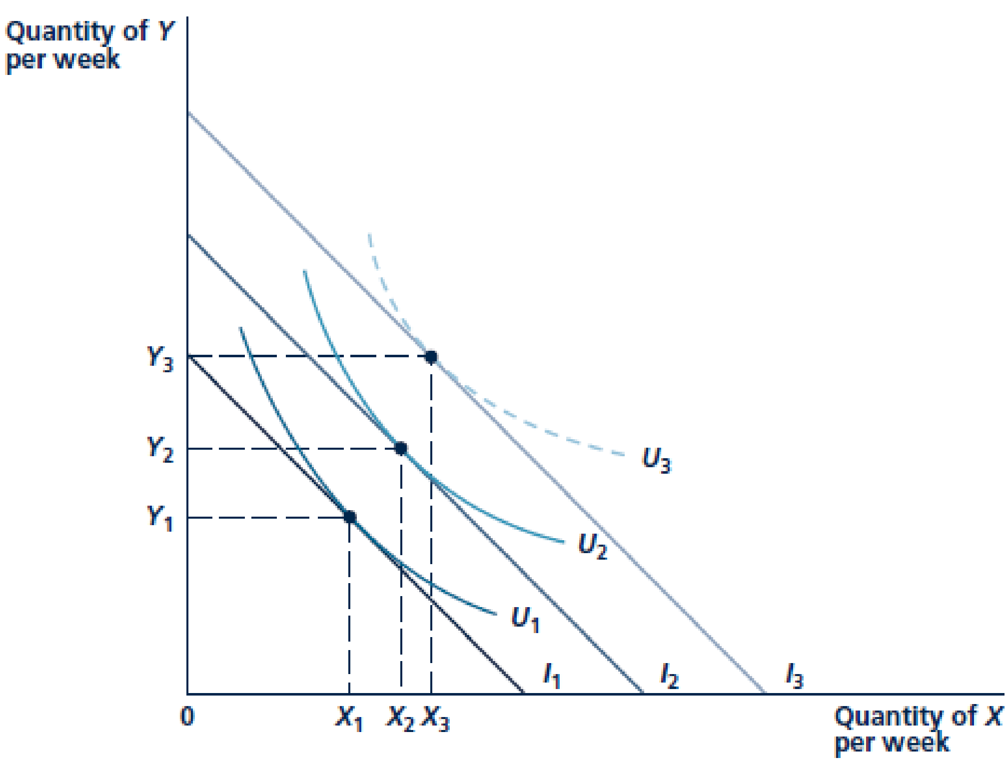
$\frac{5\,(2I)}{2(2P_p)(2P_c)} = \frac{5\,(2I)}{(4)\,2P_p + P_c}$ بإختصار القيمة (2) من البسط والمقام ➔ $\frac{5I}{4\,P_p + P_c} \neq Q$ -----→ not homogeneous

How a change in consumer income and good prices affect the consumer choice

Income Changes(Normal good):

As a person's total income rises, assuming prices do not change, we might expect the quantity purchased of each good also to increase. This situation is illustrated in Figure below. As income increases from I_1 to I_2 to I_3, the quantity of X demanded increases from X_1 to X_2 to X_3 and the quantity of Y demanded increases from Y_1 to Y_2 to Y_3. Budget lines I_1, I_2, and I_3 are all parallel because we are changing only income, not the relative prices of X and Y.

عندما يزداد دخل المستهلك ، مع افتراض أن الاسعار ثابتة لا تتغيير، فإن الطلب على السلعى يزداد. وهذا موضح في الشكل في الاسفل، عندما يزداد دخل المستهلك من دخل 1 الى دخل 2 الى دخل 3 فإن الكمية الطلب على السلعة اكس تزداد من اكس 1 الى اكس 2 الى أكس 3كذلك السلعة واي.



Normal Good:

Normal good: A good that is bought in greater quantities as income increases (I ↑ → D↑)

السلعة العادية هي سلعة يزداد الطلب عليها عندما يزداد دخل المستهلك.

In the figure above, both good X and good Y increase as income increases. Goods that follow this tendency are called normal goods. Most goods seem to be normal goods—as their incomes increase, people tend to buy more of this goods.

as Figure above shows, the demand for some ''luxury'' goods (such as Y) may increase rapidly when income rises, but the demand for ''necessities'' (such as X) may grow less rapidly.

Luxury Good. A luxury good means an increase in income causes a bigger percent increase in demand. For example, TV's would be luxury. When income rises, people spend a higher percent of their income on the luxury good. (Note: a luxury good is also a normal good, but a normal good isn't necessarily a luxury good).

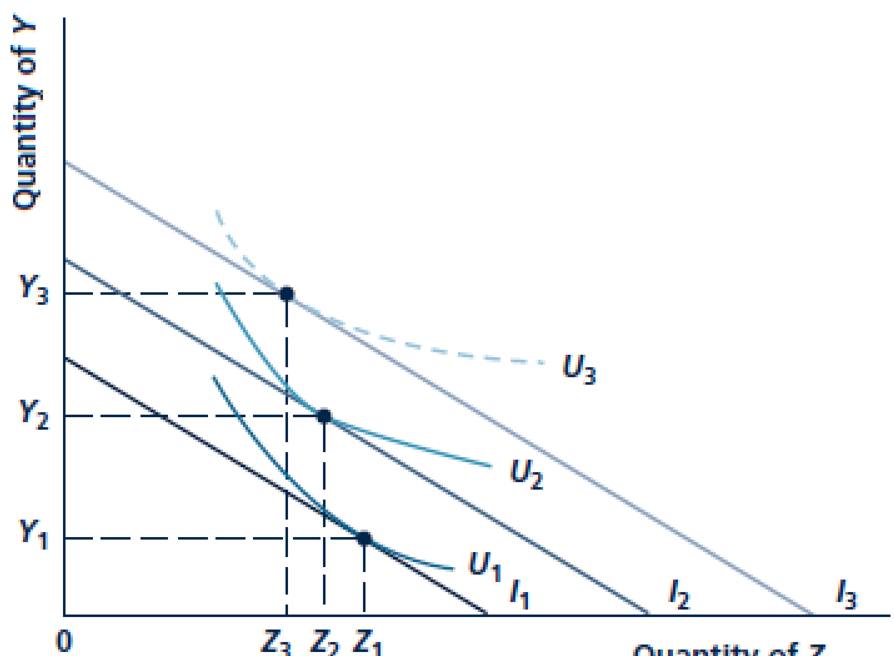
Income Changes(inferior good):

Inferior good: A good that is bought in smaller quantities as income increases.

السلعة الرديئة هي سلعة يقل الطلب عليها عندما يزداد دخل المستهلك.

How the demand for an inferior good responds to rising income is shown in the figure below. The good Z is inferior because the individual chooses less of it as his or her income increases. Although the curves in the figure continue to obey the assumption of a diminishing MRS, they exhibit inferiority. Good Z is inferior only because of the way it relates to the other goods available (good Y here), not because of its own qualities.

كما هو واضح في الشكل في الاسفل فإن السلعة Z هي سلعة رديئة لأن المستهلك يقلل الطلب عليها عندما يزداد دخلة. فعندما زاد دخل المستهلك من I1الى I2 فإن خط الميزانية انتقل من اليمين (ازاحة خط الميزانية الي اليمين)، وهذا ادي الى زيادة الطلب على السلعة y من y1 الى y2 وانخفاض الطلب على السلعة Z من القيمة Z1الى القيمة Z2 . وهذا يعني ان السلعة Yهي سلعة عادية والسلعة Z هي سلعة رديئة.

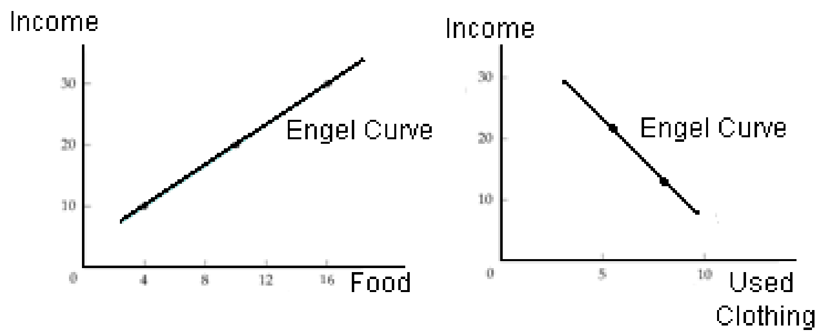


Engel Curve

Engel curve: Curve relating the quantity of a good consumed to income.

In figure (a), food is a normal good and the Engel curve is upward sloping.

In figure(b), Used clothing is an inferior good and the Engel curve is downward sloping.



Normal Good   Inferior Good

Example

A consumer purchases two goods, food (F) and clothing (C). Her utility function is given by U(F,C)= FC + C. Assume that the price of food is $1, and the price of clothing is $2, and the consumer's income is $15.

a. How much food and clothing should he buy to maximize his utility?

The budget line: $P_F F + P_C C = I$ ⇨ F + 2C = 15 .... (1)

To max utility: $\frac{MUF}{MUC} = \frac{P_F}{P_C}$ ⇨ $\frac{C}{F+1}$ = ½ ⇨ 2C = F + 1 .... (2)

By solve equations (1) and (2)

F + F + 1 = 15 ⇨ 2F = 14 ⇨ F =7

From equation (2): 2C = F + 1 ⇨ 2C = 7+1 ⇨ 2C = 8 ⇨ C = 4

The consumer should buy 7 units of food and 4 units of clothing in order to max utility

b. Suppose that the consumer's income increase to $25. How will this affect the demand for food and clothing? Is clothing a normal good in this case?

The new budget line: F + 2C = 25 .... (1)

To max utility: $\frac{MUF}{MUC} = \frac{P_F}{P_C}$ ⇨ $\frac{C}{F+1}$ = ½ ⇨ 2C = F + 1 .... (2)

By solve equations (1) and (2)

F + F + 1 = 25 ⇨ 2F = 24 ⇨ F = 12

From equation (2): 2C = F + 1 ⇨ 2C = 12+1 ⇨ 2C = 13 ⇨ C = 6.5

The consumer should buy 12 units of food and 6.5 units of clothing in order to max utility

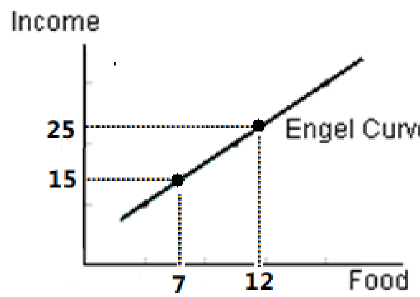
As income increase from $15 to $25 leads consumer to buy more of both goods food and clothing

→ both food and clothing are normal goods

c. Derive Engel curve for food.

من الفرع الاول عندما كان دخل المستهلك 15$ فإن الطلب على الطعام F سبع وحدات (F=7)
ولكن عندما زاد دخل المستهلك الى 25$ في الفرع الثاني فإن الطلب على الطعام اصبح 12 وحدة (F=12).

Income	F (quantity of food)
$15	7
$25	12



How a change in a good prices affect the consumer choice

سيتم هي هذا الجزء توضيح أثر تغيير سعر السلعة على الكمية المطلوبة من السلعة. بإعتماد على الرسم، رسم يضم خط الميزانية (budget line) و منحنى المنفعة (indifference curve). فعندما يتغير سعر السلعة فإن ميل خط الميزانية سيتغير (هذا يودي الى انحراق في رسم خط الميزانية).

Changing the price geometrically involves not only changing the intercept of the budget constraint but also changing its slope. Moving to the new utility-maximizing choice means moving to another indifference curve and to a point on that curve with a different MRS.

When a price changes, it has two different effects on people's choices. There is a substitution effect that occurs even if the individual stays on the same indifference curve because consumption has to be changed to equate the MRS to the new price ratio of the two goods. There is also an income effect because the price change also changes ''real'' purchasing power. People will have to move to a new indifference curve that is consistent with their new purchasing power. We now look at these two effects in several different situations.

Substitution effect: The part of the change in quantity demanded that is caused by substitution of one good for another. A movement along an indifference curve.

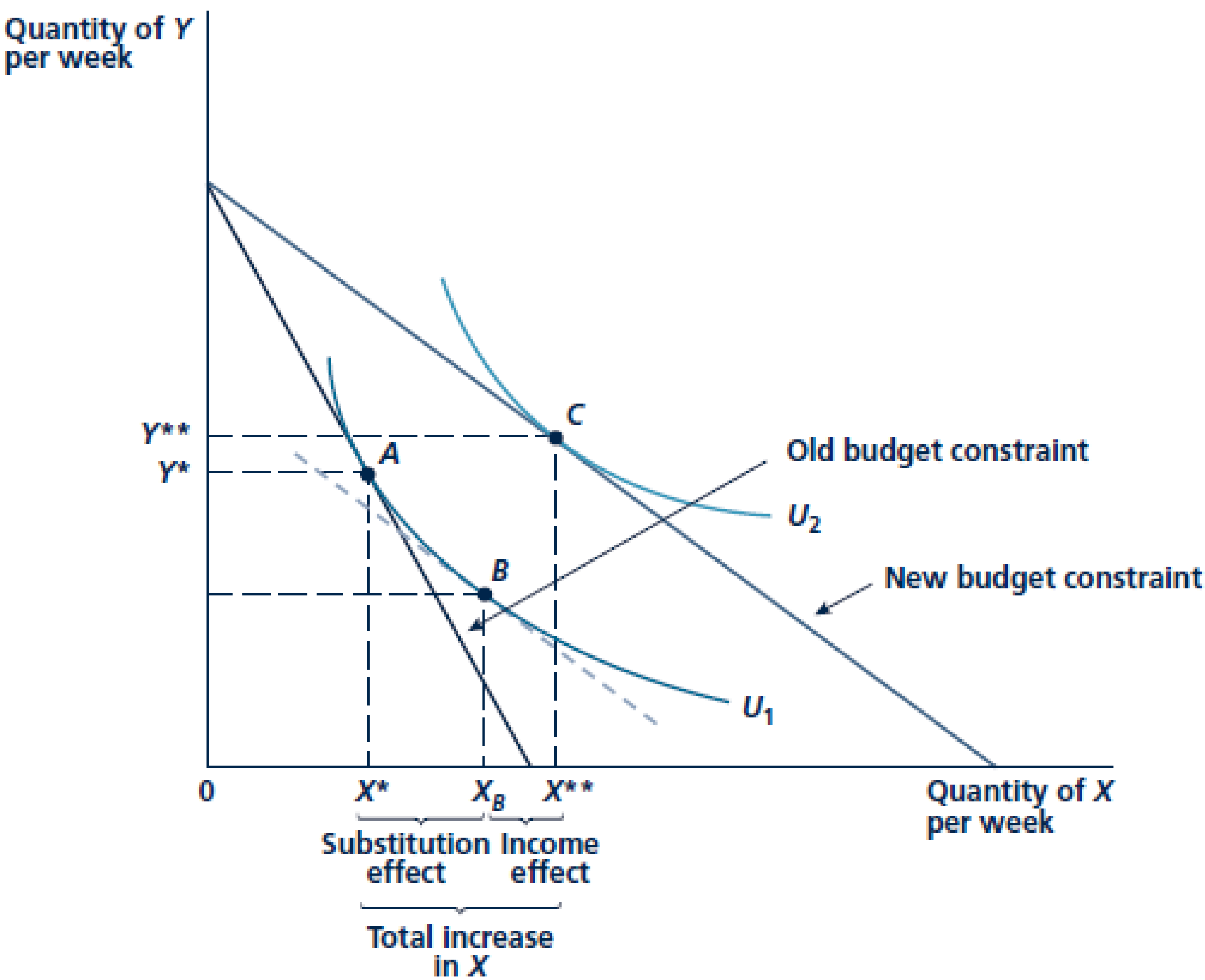
تاثير السلعة لبديلة: هو جزء من التاثير على الكمية المطلوبة ناتج عن قيام المستهلك بطلب السلعة البديلة عندما يزداد سعر السلعة. يتم تمثيلها على الرسم بالانتقال من نقطة الى نقط اخري على نفس منحنى المنفعة.

Income effect: The part of the change in quantity demanded that is caused by a change in real income. A movement to a new indifference curve.

تاثير الدخل: جزء من التاثير على الكمية المطلوبة ناتج عن تغيير في القوة الشرائية على السلعة عندما يتغيير سعرها. يتم تمثيلها على الرسم بالانتقال من نقطة على منحنى منفعة U1 الى منحنى منفعة مختلف U2.

Substitution and Income Effects from a Fall in Price (Normal good)

Let's look first at how the quantity consumed of good X changes in response to a fall in its price. This situation is illustrated in Figure below. Initially, the person maximizes utility by choosing the combination X*, Y* at point A. When the price of X falls, the budget line shifts outward to the new budget constraint, as shown in the figure. Remember that the budget constraint meets the Y-axis at the point where all available income is spent on good Y. Because neither the person's income nor the price of good Y has changed here, this Y-intercept is the same for both constraints. The new X-intercept is to the right of the old one because the lower price of X means that, with the lower price, this person could buy more X if he or she devoted all income to that purpose. The flatter slope of the budget constraint shows us that the relative price of X to Y (that is, P_X/P_Y) has fallen.



Substitution Effect

With this change in the budget constraint, the new position of maximum utility is at X**, Y** (point C). There, the new budget line is tangent to the indifference curve U_2. The movement to this new set of choices is the result of two different effects. First, the change in the slope of the budget constraint would have motivated this person to move to point B even if the person had stayed on the original indifference curve U_1. The dashed line in Figure has the same slope as the new budget constraint, but it is tangent to U_1 because we are holding ‘‘real’’ income constant. A relatively lower price for X causes a move from A to B if this person does not become better off as a result of the lower price. This movement is a graphic demonstration of the substitution effect. Even though the individual is no better off, the change in price still causes a change in consumption choices.

Given the budget line, the consumer chooses market basket C and consume X_B units of good X. *The distance X* X_B represent the substitution effect.*

Income Effect

Because the price of X has fallen but nominal income (I) has stayed the same, this person has a greater ‘‘real’’ income and can afford a higher utility level (U2). If X is a normal good, he or she will now demand more of it. This is the income effect. Notice that for normal goods this effect also causes price and quantity to move in opposite directions. When the price of X falls, this person’s real income is increased and he or she buys more X because X is a normal good. A similar statement applies when the price of X rises. Such a price rise reduces real income and, because X is a normal good, less of it is demanded.

The increases in good X consumption from X_B to X** is the measure of income effect, which is positive, because good X is a normal good (consumers will buy more of it as their income increase). Because it reflects a movement from one indifference curve to another (from U_1 to U_2), the income effect measures the change in the consumer’s purchasing power.

The total effect of a change in the price is given by the sum of substitution effect and the income effect:

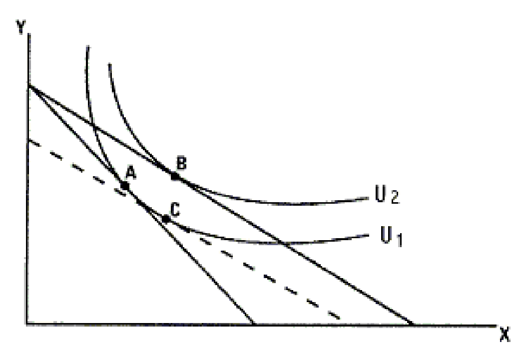
Total effect (X* X**) = Substitution effect (X* X_B) + Income effect (X_B X**)

For the normal good: the direction of the substitution and income effects always the same.

Example: Choose the correct answer

Consider the diagram, which depicts the change in a consumer's optimal consumption after a decrease in the price of good X. Which of the following statements is correct?

A. the substitution effect is given by C to B and the income effect is given by A to C
B. the substitution effect is given by A to B and the income effect is given by B to C
C. the substitution effect is given by C to A and the income effect is given by A to B
D. the substitution effect is given by A to C and the income effect is given by C to B

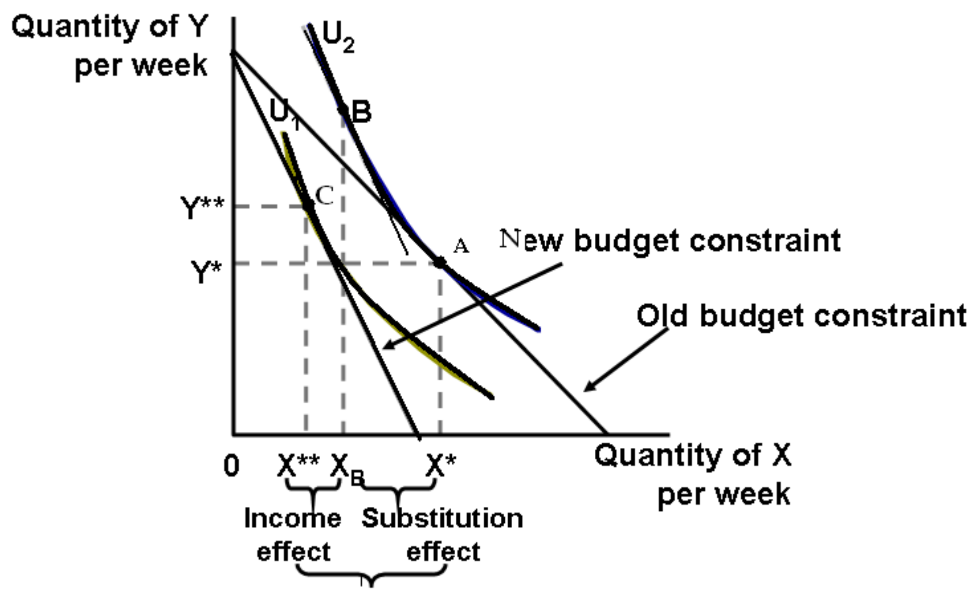


Substitution and Income Effects from an Increase in Price:

When the price of good X increases, the budget constraint shifts inward. The movement from the initial utility maximization point A to a new point B can again be analyzed as two separate effects. The substitution effect causes a movement to point B on the initial indifference curve U_2. The price increase also creates a loss of purchasing power. This income effect causes a consequent movement to a lower indifference curve U_1. The income and substitution effects together cause the quantity demanded of X to fall as s result of the increase in its price.

Total effect (X^* X^{**}) = Substitution effect (X^* X_B) + Income effect (X_B X^{**})

The substitution effect: decrease demand from X^* to X_B, and the income effect decrease demand from X_B to X^{**}. The total effects decrease demand for good X from X^* to X^{**}.



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By solve equations (1) and (2)

F + F + 1 = 15 ⇨ 2F = 14 ⇨ F =7

From equation (2): 2C = F + 1 ⇨ 2C = 7+1 ⇨ 2C = 8 ⇨ C = 4

The consumer should buy 7 units of food and 4 units of clothing in order to max utility

b. If the price of clothing decreases to \$1, how much food and clothing should he buy to maximize his utility?

New budget line equation: $P_F F + P'_C C = I$ ⇨ F + C = 15 …………… (1)

To max utility: $\frac{MUF}{MUC} = \frac{P_F}{P'_C}$ ⇨ $\frac{C}{F+1} = \frac{1}{1}$ ⇨ C = F + 1 …………… (2)

By solve equations (1) and (2)

F + F + 1 = 15 ⇨ 2F = 14 ⇨ F =7

From equation (2): C = F + 1 ⇨ C = 7+1 ⇨ C = 8

The consumer should buy 7 units of food and 8 units of clothing in order to max utility

When the price of clothing decrease from \$2 to \$1, the demand for clothing increase from 4 to 8 units.

Substitution and Income Effects for Inferior Goods

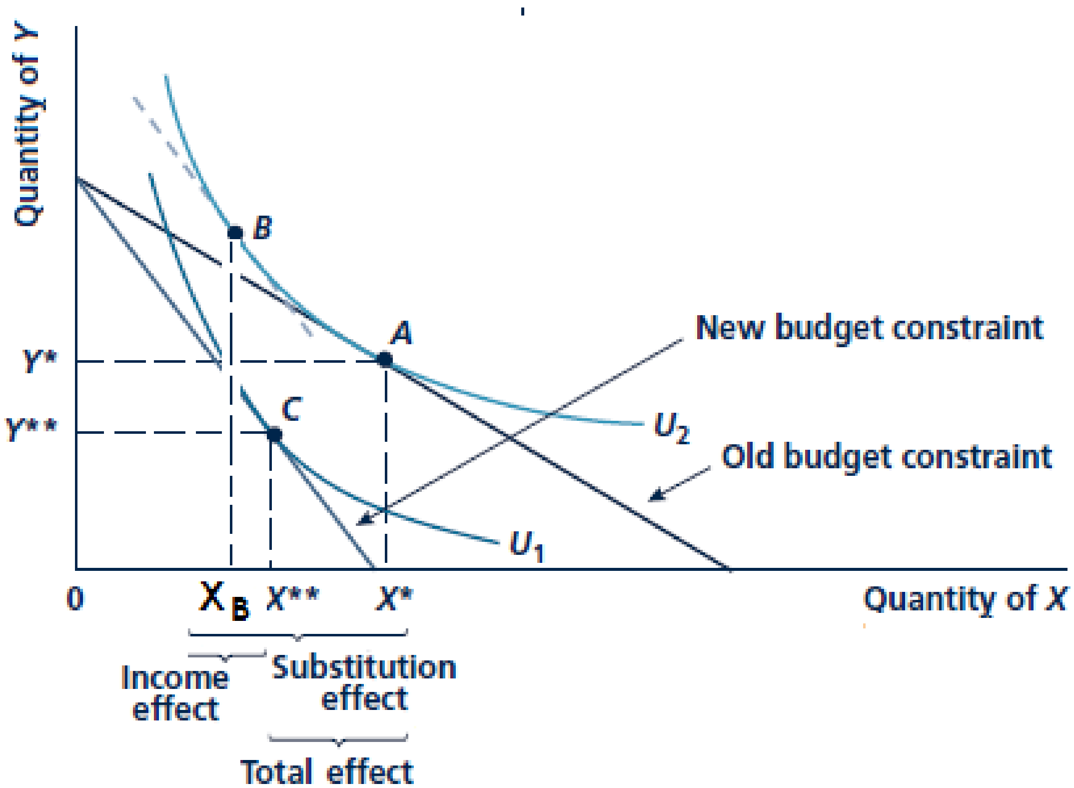
For the case of inferior goods, substitution and income effects work in opposite directions. The net effect of a price change on quantity demanded will be ambiguous. Here we show that ambiguity for the case of an increase in price.

في حالة السلعة الردئية، تاثير البديل وتاثير الدخل على الكمية المطلوبة يعملون عكس الاتجاه. بمعنى عند زيادة سعر السلعة فإن تاثير البديل يؤدي بالمستهلك بزيادة طلبه على السلعة البديلة وبتالي انخفاض الكمية المطلوبة من السلعة. اما تأثير الدخل عند ارتفاع السعر يؤدي الى انخفاض القوة الشرائية على السلعة ، بما ان السلعة سلعة ردئية فإن الطلب عليها يزداد بنخفاض الدخل (القوة الشرائية).

نلاحظ انه عندما ارتفع سعر السلعة الردئية: تأثير البديل ادى الى انخفاض الكمية المطلوبة من السلعة بينما تأثير الدخل ادي الى زيادة الكمية المطلوبة من السلعة، (عكس الاتجاه). التأثير الكلي لزيادة السعر في حالة السلعة الرديئة يكون غير محدد.

Figure below shows the income and substitution effects from an increase in price when X is an inferior good.

As the price of X rises, the substitution effect causes this person to choose less X. This substitution effect is represented by a movement from A to B in the initial indifference curve, U_2. Because price has increased, however, this person now has a lower real income and must move to a lower indifference curve, U_1. The individual will choose combination C. At C, more X is chosen than at point B. This happens because good X is an inferior good: As real income falls, the quantity demanded of X increases rather than declines as it would for a normal good. In our example here, the substitution effect is strong enough to outweigh the ‘‘perverse’’ income effect from the price change of this inferior good–so quantity demanded still falls as a result of the price rise.



The effect of an increase in the price of good X:

The substitution effect: decrease the quantity of good X demanded from X* to X_B

The income effect: increase in the quantity demanded of good X from X_B to X**

Total effect: decrease the quantity demanded of good X from X* to X**

Total effect (X* X**) = Substitution effect (X* X_B) + Income effect (X_B X**)

For the inferior good: the direction of the substitution and income effects work in the opposite direction

A Special Case: The Giffen Good

Giffen good: Good whose demand curve slopes upward because the income effect is larger than the substitution effect.

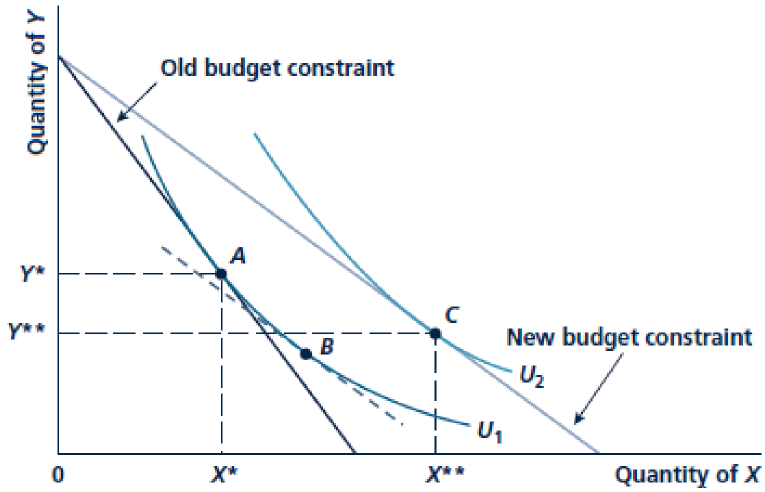
When good X is an inferior good, and when the income effect is larger enough to dominate the substitution effect, the demand curve will be upward-sloping. Because the income effect is larger than the substitution effect, an increase in the price of good X leads to an increase in the quantity of good X demanded.

If the income effect of a price change is greater than the substitution effects → the inferior good is a Giffen's good.

Changes in the Price of another Good

A change in the price of X will also affect the quantity demanded of the other good (Y). In Figure below, for example, a decrease in the price of X causes not only the quantity demanded of X to increase but the quantity demanded of Y to increase as well.

تغيير سعر السلعة X يؤثر على الطلب على السلعة Y. في اشكل في الاسفل، عند انخفاض سعر السلعة X أدي لتغيير كمية الطلب على السلعة Y. وفي هذه الحالة نستطيع ان نقول بأن السلعتين X و Y هما سلعتين بديلتين (انخفاض سعر السلعة X ادى الى انخفاض الطلب على السلعة Y من *Y الى **Y).



Complements good:

Two goods are complements if an increase in the price of one causes a decrease in the demanded of the other or a decrease in the price of one good cause an increase in the demand for the other.

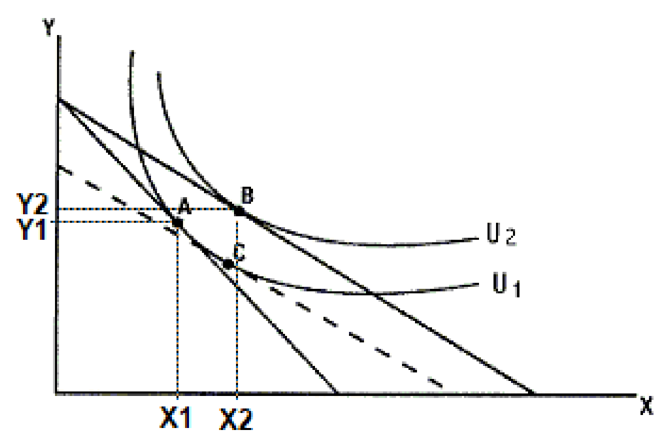
Substitutes good:

Two goods such that if the price of one increase, the demand for the other rises are called substitutes. If the price of one good decreases and the demand for the other good decreases, they are also substitutes.

Example: Choose the correct answer

Consider the diagram, which depicts the change in a consumer's optimal consumption after a DECREASE in the price of good X. Which of the following statement is/are correct?

A. goods X any Y are substitutes
B. **goods X any Y are complements**
C. goods X any Y are inferior
D. goods X any Y are Giffen goods



Example:

Taleen consumes only grapefruits and grapes. Her utility function is U(X,Y) = XY-2Y, where X is the number of grapefruits consumed and Y is the number of grapes consumed. Taleen's income is 48, and the prices of grapefruits and grapes are 1 and 3, respectively.

a. How many grapefruits and grapes will she consume in order to max utility?

Taleen budget line: X + 3Y = 48 .... (1)

To max utility: $\frac{MUX}{MUY} = \frac{P_X}{P_Y}$ ⇨ $\frac{Y}{X-2} = \frac{1}{3}$ ⇨ 3Y= X - 2 .... (2)

By solve equations (1) and (2)

X + X - 2 = 48 ⇨ 2X = 46 ⇨ X =23

From equation (2): 3Y= X - 2 ⇨ 3Y = 23 - 2 ⇨ 3Y = 21 ⇨ Y = 7

The consumer should consume 23 units of grapefruits and 7 units of grapes in order to max utility

b. If the price of grapefruits rises to $2, how many grapefruits and grapes will she consume in order to max utility? Is grapefruits and grapes are complements or substitutes in this case?

New budget line: 2X + 3Y = 48 .... (1)

To max utility: $\frac{MUX}{MUY} = \frac{P'_X}{P_Y}$ ⇨ $\frac{Y}{X-2} = \frac{2}{3}$ ⇨ 3Y= 2X - 4 .... (2)

By solve equations (1) and (2)

2X + 2X - 4 = 48 ⇨ 4X = 44 ⇨ X =11

From equation (2): 3Y= 2X - 4 ⇨ 3Y = 22 - 4 ⇨ 3Y = 18 ⇨ Y = 6

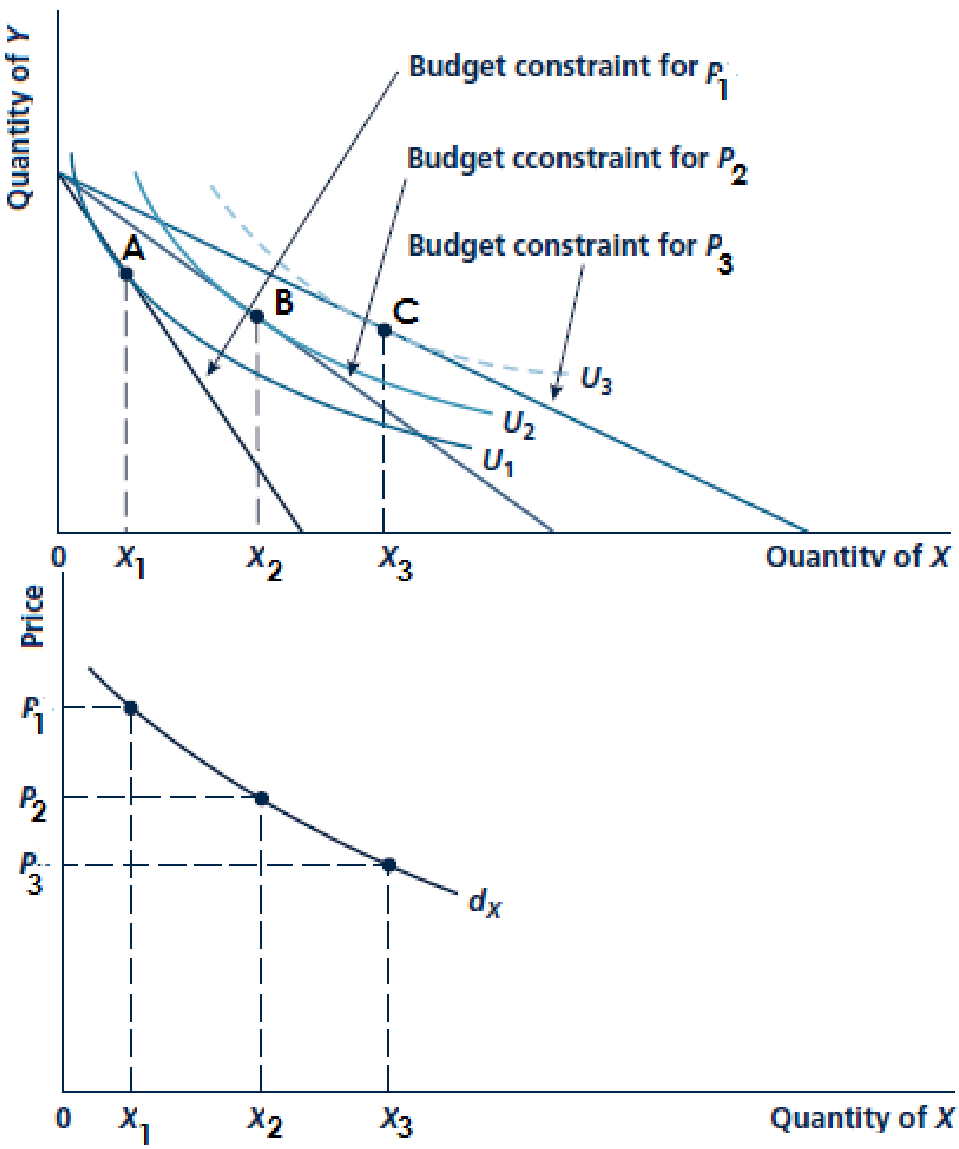
⇳ When the price of grapefruit increase the demand for grapes decrease ⇨ grapefruit and grapes are complements goods

Construction of Individual Demand Curves:

An individual demand curve is a graphic representation between the price of a good and the quantity of it demanded by a person holding all other factors (preferences, the prices of other goods, and income) constant.

Suppose that the price level is P_3, at this price level the consumer achieves the maximum utility at point A, with X_1 units of good X demanded. If the price decrease to P_2, the budget line rotates outward, the consumer now maximum utility at point B, with X_2 units of good X demanded.

If the price level decrease to P_1, the budget line rotates outward, the consumer can achieve the higher level of utility associated with indifference curve U_3 in the figure by selecting C, with X_3 units of good X demanded.



Example:

Robert has utility function U(X, Y) = X² Y², where X is the quantity of apples and Y the quantity of banana he consume. Robert's income is 36, and the prices of apples and bananas are 2 and 3, respectively.

1. How many apples and bananas will he consume in order to max utility?

   Robert budget line: 2X + 3Y = 36 .... (1)

   To max utility: $\frac{MUX}{MUY} = \frac{P_X}{P_Y}$ ⇨ $\frac{2XY^2}{2YX^2} = \frac{2}{3}$ ⇨ $\frac{Y}{X} = \frac{2}{3}$ ⇨ 3Y= 2X .... (2)

   By solve equations (1) and (2)

   2X + 2X = 36 ⇨ 4X = 36 ⇨ X =9

   From equation (2): 3Y= 2X ⇨ 3Y = 18 ⇨ Y = 6

   The consumer should consume 9 units of apples and 6 units of bananas in order to max utility

2. If the price of apples falls to $1, how many apples and bananas will he consume in order to max utility? Is apples and bananas are complements or substitutes in this case?

   New budget line: X + 3Y = 36 .... (1)

   To max utility: $\frac{MUX}{MUY} = \frac{P_X}{P_Y}$ ⇨ $\frac{2XY^2}{2YX^2} = \frac{1}{3}$ ⇨ $\frac{Y}{X} = \frac{1}{3}$ ⇨ 3Y= X .... (2)

   By solve equations (1) and (2)

   X + X = 36 ⇨ 2X = 36 ⇨ X =18

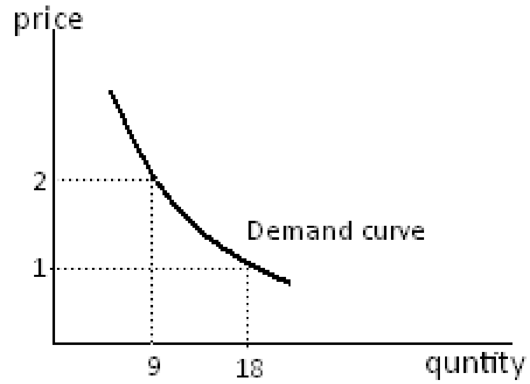
   From equation (2): 3Y= X ⇨ 3Y = 18 ⇨ Y = 6

   The consumer should consume 18 units of apples and 6 units of bananas in order to max utility

   When the price of apple decrease the quantity demand for banana does not change ⇨ apples and banana are unrelated (independent) goods

3. Construct the demand curve for grapefruits.

Price of apple	Quantity of apple demanded
$2	9
$1	18



Market Demand

Market demand curve: Curve relating the quantity of a good that all consumers in a market will buy to its price.

Market demand curve can be derived as the sum of the individual demand curves of all consumers in a particular market.

Assume that only three consumers (A, B, and C) are in the market for coffee. Table tabulates several points on each consumer's demand curve. The market demand is founded by adding the three individual demands at each price level.

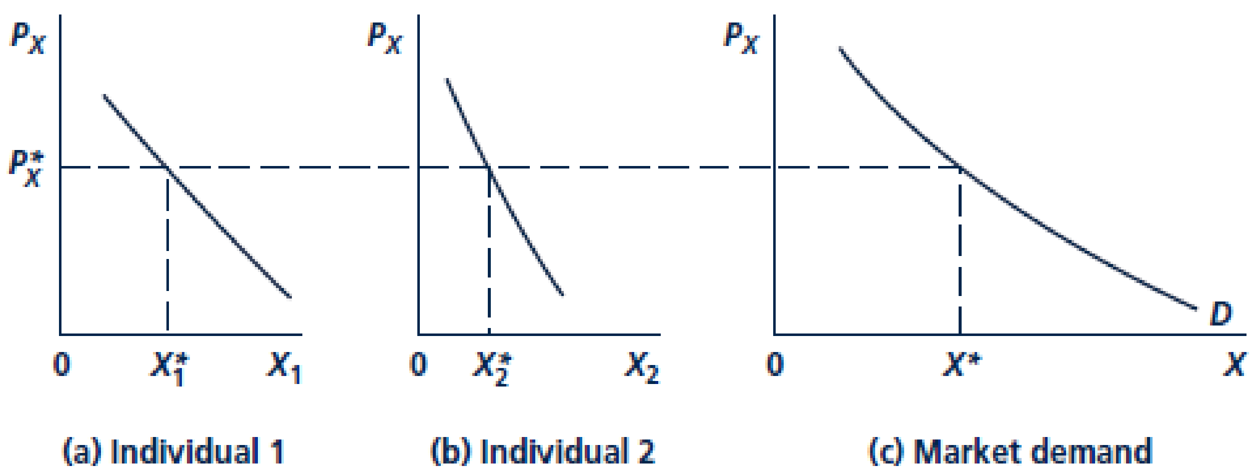
Price	Individual A (units)	Individual B (units)	Individual C (units)	Market (units)
1	6	10	16	6 + 10 + 16 = 32
2	4	8	13	4 + 8 + 13 = 25
3	2	6	10	2 + 6 + 10 = 18
4	0	4	7	0 + 4 + 7 = 11
5	0	2	4	0 + 2 + 4 = 6

Example

Suppose the quantity of good X demanded by individual 1 is given by: $X_1 = 10 - 2P_x + 0.01\,I_1 + 0.4P_y$.
And the quantity of X demanded by individual 2 is: $X_2 = 5 - P_x + 0.02\,I_2 + 0.2P_y$

What is the market demand function for good X?

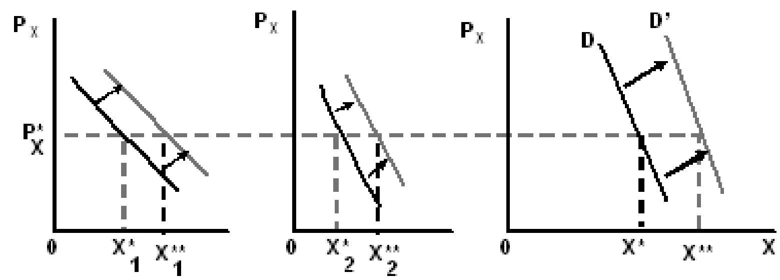
Market demand function $(X^*) = X1 + X2 = 15 - 3\,P_x + 0.01\,I_1 + 0.02\,I_2 + 0.6\,P_y$.



Shifts in the Market Demand Curve

For example, consider the two buyer case where both consumers regard X as a normal good. An increase in income for each consumer would shift their individual demand curves out so that the market demand curve would also shift out. This situation is shown in Figure

However, some events result in ambiguous outcomes. If one consumer's demand curve shifts out while another's shifts in, the net effect depends on the size of the relative shifts.



(a) Individual 1 (b) Individual 2 (c) Market Demand

Changes in the prices of related goods, substitutes or complements, will also shift the individual and market demand curves. If goods X and Y are substitutes, an increase in the price of Y will increase the demand for X. Similarly, a decrease in the price of Y will. If goods X and Y are complements, an increase in the price of Y will decrease the demand for X. A decrease in the price of Y will increase the demand for X.

Example:

Suppose the quantity of good X demanded by individual 1 is given by: $X_1 = 50 - 3P_x + 0.15\ I_1 + 0.2P_y$. And the quantity of good X demanded by individual 2 is given by: $X_2 = 10 - 2P_x + 0.01\ I_2 + 0.3P_y$

a. What is the market demand for good X if ($I_1 = 100$, $I_2 = 600$)?

$X^* = X_1 + X_2 = 60 - 5\ P_x + 0.15\ I_1 + 0.01\ I_2 + 0.5\ P_y$

At (I1 = 100, $I_2 = 600$) ⇨ $X^* = 60 - 5\ P_x + 0.15(100) + 0.01(600) + 0.5\ P_y$

$\underline{X^* = 81 - 5\ Px + 0.5\ Py}$

b. If I_1 increase to 110 and I_2 decrease to 590. Is the market demand shift? If yeas, it shift to the right or to the left?

At ($I_1 = 110$, $I_2 = 590$) ⇨ $X^* = 60 - 5\ P_x + 0.15(110) + 0.01(590) + 0.5\ P_y$

⇨ $X^* = 60 - 5\ P_x + 16.5 + 5.9 + 0.5\ P_y$

⇨ $\underline{X^* = 82.4 - 5\ P_x + 0.5\ P_y}$

⇨ Shift to the right (since the intercept increase).

Elasticity of demand

If we want to know how much the quantity supplied or demanded will rise or fall when the price change? How sensitive is the demand for coffee to its price? If price increases by 10 percent, how much will the quantities demanded change? How much will it change if income rises by 5 percent? We use elasticity's to answer questions like these.

Elasticity: the percentage change that will occur in one variable in response to a 1-percent increase in another variable.

The price elasticity of demand measures the sensitivity of quantity demanded to price changes.

The price elasticity of demand: the percentage change in the quantity demanded for a good will be following a 1-percent increase in the price of that good.

The price elasticity of demand (E_p) = $\frac{\%\Delta Q}{\%\Delta P} = \frac{\Delta Q/Q}{\Delta P/P}$

$$E_p = \frac{\Delta Q}{\Delta P} * \frac{P}{Q} = \frac{dQ}{dP} * \frac{P}{Q}$$

The price elasticity of demand is usually a negative number. When the price of a good increases, the quantity demanded usually falls. Thus $\frac{\Delta Q}{\Delta P}$ is negative.

When the price elasticity is greater than 1 in magnitude, we say that demand is price elastic. $|E_p| > 1 \rightarrow$ elastic demand

When demand elastic: $\%\Delta Q > \%\Delta P$

If the price elasticity is less than 1 in magnitude, demand is said to be price inelastic. $|E_p| < 1 \rightarrow$ inelastic demand

When demand inelastic: $\%\Delta Q < \%\Delta P$

Example:
Suppose that demand curve for a product is given by: $Q = 10 - 2P + P_s$
Where P is the product price and P_s is the price of a substitute good. The price of a substitute good is \$2.

a. Suppose P = \$1, what is the price elasticity of demand?

$$E_p = \frac{\Delta Q}{\Delta P} * \frac{P}{Q} = \frac{dQ}{dP} * \frac{P}{Q}$$

When P =\$1 ➔ Q = 10 – 2*1 + 2 = 10

$$E_p = \frac{dQ}{dP} * \frac{P}{Q} = -2 * \frac{1}{10} = \frac{-2}{10} = |-0.2| = 0.2 < 1 \text{ inelastic}$$

b. Suppose P = $3, what is the price elasticity of demand? is the elastic, inelastic or unit elastic

$E_p = \frac{\Delta Q}{\Delta P} * \frac{P}{Q} = \frac{dQ}{dP} * \frac{P}{Q}$

When P =$3 ⇨ Q = 10 - 2*3 + 2 = 6

$E_p = \frac{dQ}{dP} * \frac{P}{Q} = -2 * \frac{3}{6} = \frac{-6}{6} = |-1| = 1$ unit elastic

<u>Example:</u>

Consider the demand curve: $Q = \frac{100}{P}$, find E_p

$E_p = \frac{\Delta Q}{\Delta P} * \frac{P}{Q} = \frac{dQ}{dP} * \frac{P}{Q}$

$Q = 100P^{-1}$

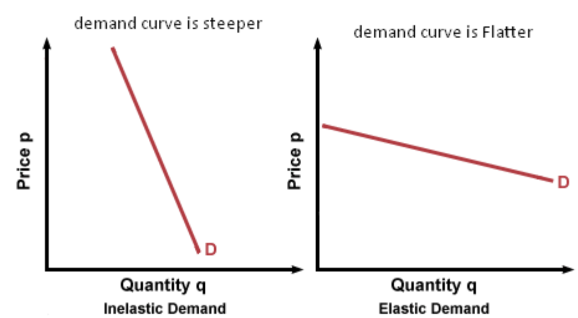
$E_p = -100\, P^{-2} * \frac{P}{Q}$

But $Q = 100\, P^{-1}$ → $E_p = -100P^{-2} * \frac{P}{100P-1} = \frac{100P^{-1}}{100P^{-1}}$ = 1 unit elastic

Price Elasticity and the Shape of the Demand Curve

Steeper and flatter demand curve

- When the demand curve is steeper: %Δ P > %Δ Q → demand less elastic
- When the demand curve is Flatter: %Δ in Q > %Δ P → demand more elastic



Price Elasticity and the Substitution Effect

- Goods which have many close substitutes are subject to large substitution effects from a price change so their market demand curve is likely to be relatively elastic.
- Goods with few close substitutes will likely be relatively inelastic.

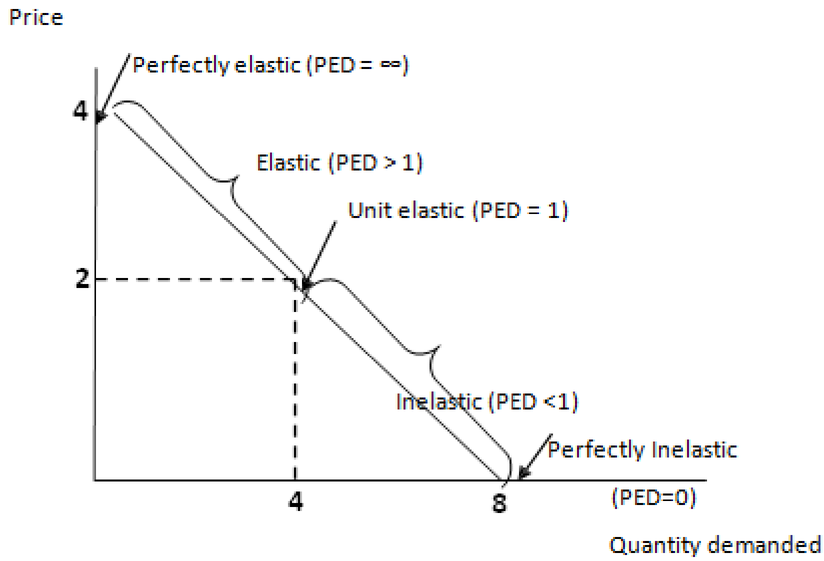
Linear Demand Curves and Price Elasticity:

The price elasticity of demand depends not only on the slope of the demand curve but also on the price and quantity. The elasticity, therefore, varies along the curve as price and quantity change.

Slope is constant for this linear demand curve. Near the top, because price is high and quantity is small, the elasticity is large in magnitude. The elasticity becomes smaller as we move down the curve.

As an example, consider the demand curve: Q = 8 - 2P

For this curve, Δ Q/Δ P is constant and equal to –2. However, the curve does not have a constant elasticity. Observe from Figure that as we move down the curve, the ratio P/Q falls; the elasticity therefore decreases in magnitude.



Near the intersection of the curve with the price axis, Q is very small, so E_p = –2(P/Q) is large in magnitude. When P = 2 and Q = 4, E, = –1. At the intersection with the quantity axis, P = so E = 0.

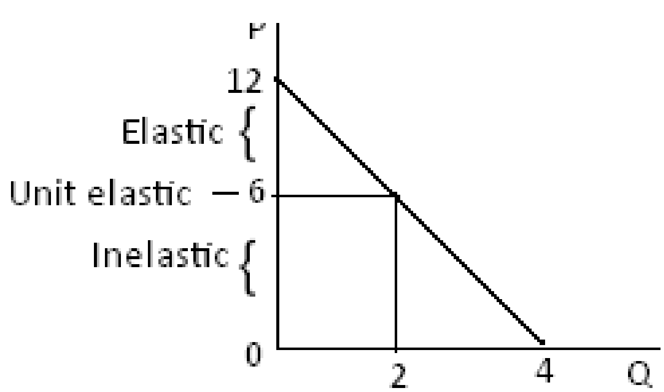
The price elasticity of demand is always changing along a straight line demand curve.

- Demand is elastic at prices above the midpoint price.
- Demand is unit elastic at the midpoint price.
- Demand is inelastic at prices below the midpoint price.

<u>Example</u>

If the demand curve is given by Q = 4 - $\frac{P}{3}$, at which prices (if any) the demand elastic?

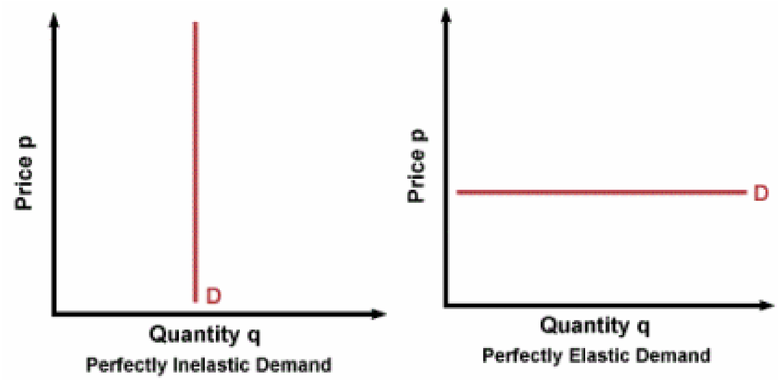
6 < P < 12 → Demand elastic
0 < P < 6 → Demand inelastic
P = 6 → Demand unit elastic



Infinitely Elastic and Completely Inelastic Demand

The Figure show a demand curve *reflecting infinitely elastic demand*: Consumers will buy much as they can at a single price P*. For even the smallest increase in price above this level, quantity demanded drops to zero,

For any decrease in price, quantity demanded increases without limit. The demand curve (2) in the Figure on the other hand, *reflects completely inelastic demand*: Consumers will buy fixed quantity Q*, no matter what the price.



Other Demand Elasticities

Income elasticity of demand: Percentage change in the quantity demanded resulting from a 1-percent increase in income.

The income elasticity of demand (E_I) = $\frac{\%\Delta Q}{\%\Delta I} = \frac{\Delta Q/Q}{\Delta I/I}$

$E_I = \frac{\Delta Q}{\Delta I} * \frac{I}{Q} = \frac{dQ}{dI} * \frac{I}{Q}$

- For normal goods, E_I is positive ($E_I > 0$), because increases in income lead to increases in purchases of the good.
- For inferior goods E_I is negative ($E_I < 0$), because increases in income lead to a decrease in purchases of the good.

Example:
Suppose Individual's demand for oranges is given by: $Q = 10 - 2P_x + 0.1 I + 0.5P_y$. If P_x =\$1 , P_y = \$4, and I = \$200. (Where: P_x is the price of orange, P_y is the price of grapefruit).
a. What is the income elasticity of demand?
b. Is good X a normal or inferior good?

Q = 10 – 2(1) + 0.1(200) + 0.5(4) = 10 – 2 + 20 + 2 = 30

$\frac{dQ}{dI} = 0.1$

$E_I = \frac{\Delta Q}{\Delta I} * \frac{I}{Q} = \frac{dQ}{dI} * \frac{I}{Q}$ = 0.1 x $\frac{200}{30} = \frac{2}{3}$; $E_I > 0$ → good X is a normal good.

Cross-Price Elasticity of Demand:

The cross-price elasticity of demand measures the percentage change in the quantity demanded of a good in response to a 1 percent change in the price of another good.

$$E_{QA\ PB} = \frac{\%\Delta Q_A}{\%\Delta P_B} = \frac{\Delta Q_A / Q_A}{\Delta P_B / P_B}$$

$$E_{QA\ PB} = \frac{dQ_A}{dP_B} * \frac{P_B}{Q_A}$$

- If the two goods are substitutes, an increase in the price of one will cause buyers to purchase more of the substitute, so the cross-price elasticity will be positive.
- If the two goods are complements, an increase in the price of one will cause buyers to buy less of that good and also less of the good they use with it, so the cross-price elasticity will be negative.
- If $E_{QA\ PB} = 0$, the two goods are unrelated (independent)

Example: -

Suppose demand for good A is given by $Q_A = 500 - 10\ P_A + 2\ P_B + 0.7\ I$. where P_A is the price of good A, P_B is the price of good B, and I is income. Assume that P_A = \$20, P_B = \$15, and I = \$100. What is the cross price elasticity of the demand for good A with respect to the price of good B at the current situation? Are good A and B complement or substitutes?

$$E_{QA\ PB} = \frac{dQ_A}{dP_B} * \frac{P_B}{Q_A}$$

$$\frac{dQ_A}{dP_B} = 2$$

P_A = \$20, P_B = \$15, and I = \$100

$Q_A = 500 - 10\ (20) + 2\ (15) + 0.7\ (100) = 400$

$E_{QA\ PB} = 2 \times \frac{15}{400} = \frac{30}{400} > 0$ (Positive) → good A and B are substitutes goods

Example:

Suppose demand for good A is given by $Q_A^d = 100 - 5PA - 3PB + 0.5I$. Where P_A is the price of good A, P_B is the price of some other good B, and I is income. Assume that P_A is currently \$8, P_B is currently \$5, and I is currently \$200.

a. What is the elasticity of demand for good A with respect to the price of good A at the current situation? Is demand for good A elastic or inelastic?

$$E_p = \frac{dQA}{dPA} * \frac{PA}{QA}$$

$$\frac{dQA}{dPA} = -5$$

$$Q_A = 100 - 5(8) - 3(5) + 0.5(200) = 145$$

$$E_p = \frac{dQA}{dPA} * \frac{PA}{QA} = -5 * \frac{8}{145} = \frac{-40}{145} = -0.275$$

$|Ep| = |-0.275| = 0.275 < 1$ ➔ inelastic

b. What is the cross-price elasticity of the demand for good A with respect to the price of good B at the current situation? Are good A and B complement or substitutes?

$$E_{QA\,PB} = \frac{dQ_A}{dP_B} * \frac{P_B}{Q_A}$$

$$\frac{dQ_A}{dP_B} = -3$$

$$E_{QA\,PB} = -3 * \frac{5}{145} = \frac{-15}{145} = -0.103$$

$E_{QA\,PB} = -0.103 < 0$ ➔ goods A and B are complements

c. What is the income elasticity of demand for good A at the current situation? Is good A a normal or inferior good?

$E_I = \frac{dQ}{dI} * \frac{I}{Q} = 0.5 \times \frac{200}{145} = \frac{100}{145} = 0.69$; $EI > 0$ → good X is a normal good.