13.1 Types of Gears

Figure 13–1

Spur gears are used to transmit rotary motion between parallel shafts.

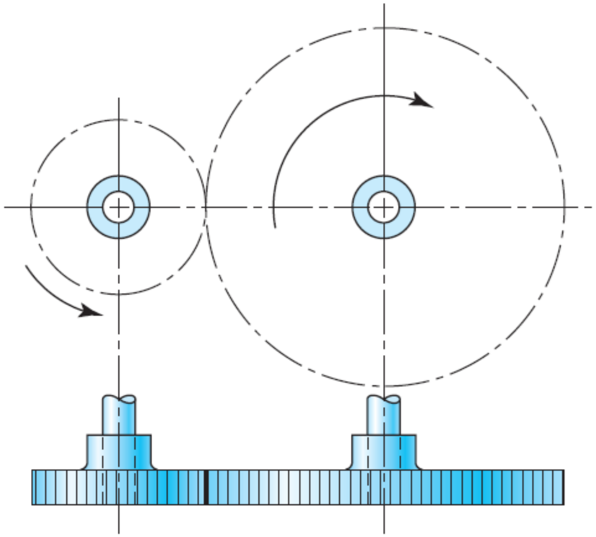
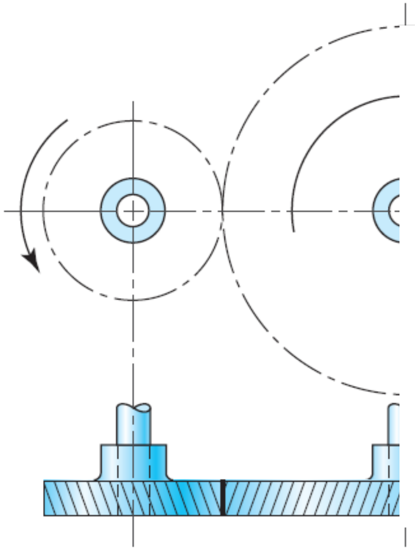


Figure 13–2

Helical gears are used to transmit motion between parallel or nonparallel shafts.



13.1 Types of Gears

Figure 13–3

Bevel gears are used to transmit rotary motion between intersecting shafts.

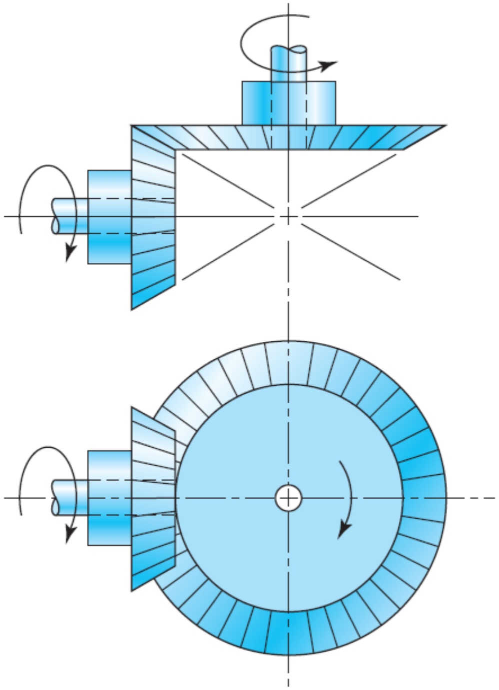
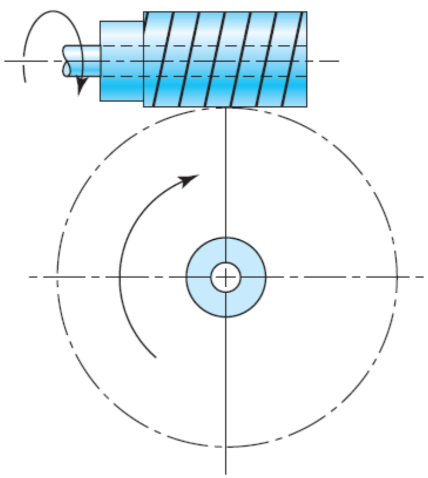
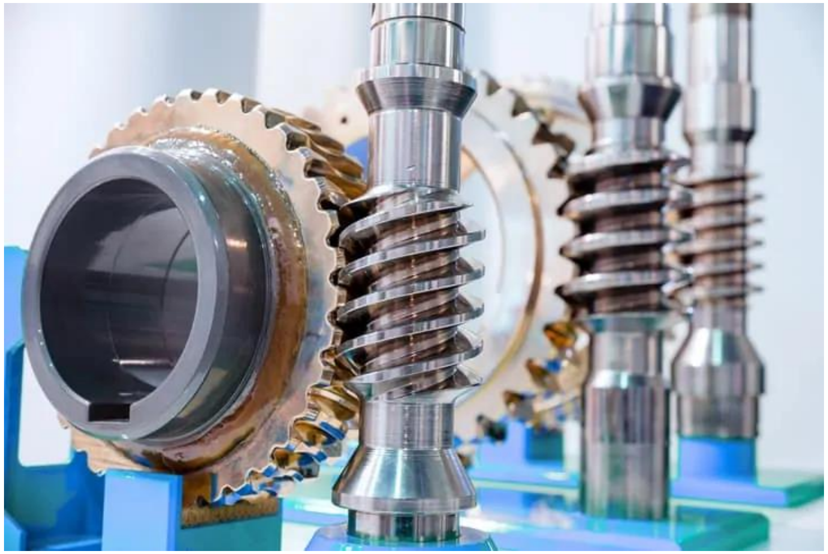


Figure 13–4

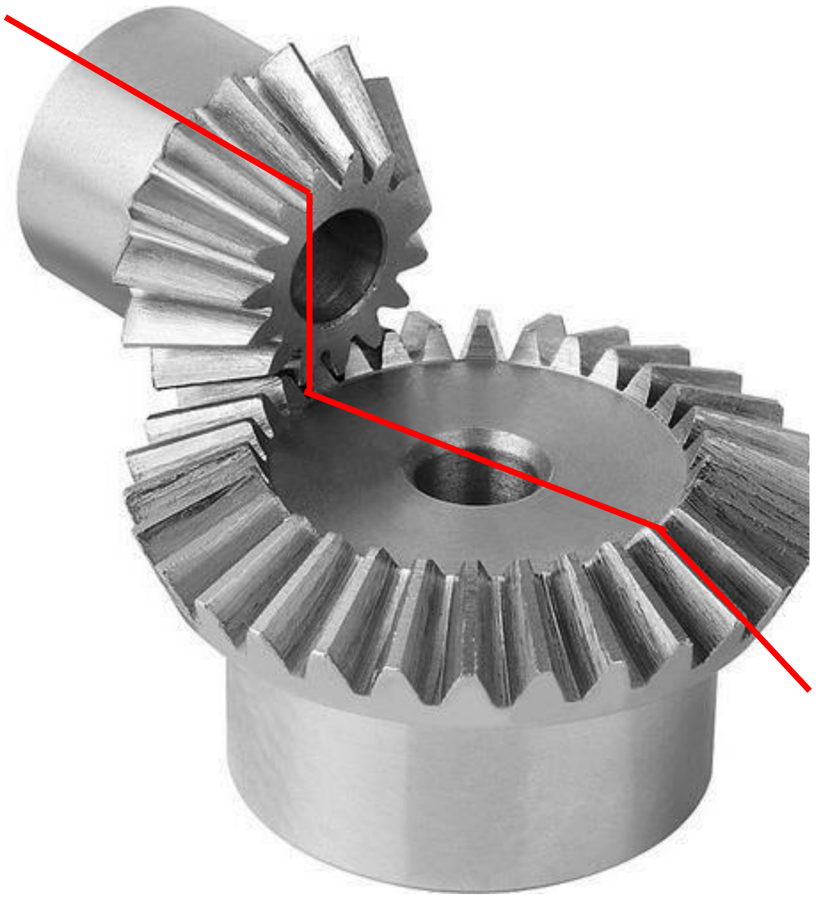
Worm gearsets are used to transmit rotary motion between nonparallel and nonintersecting shafts.

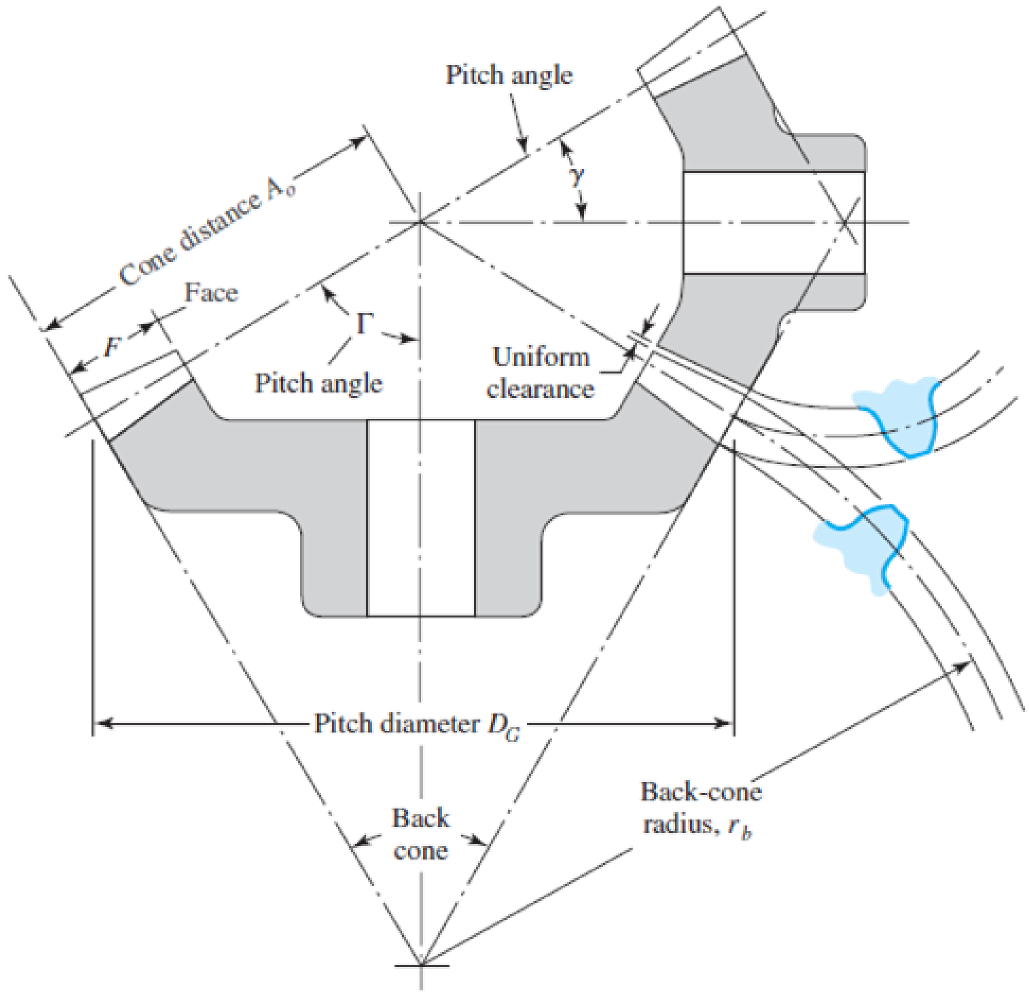




13.9 Straight Bevel Gears

Terminology of bevel gears



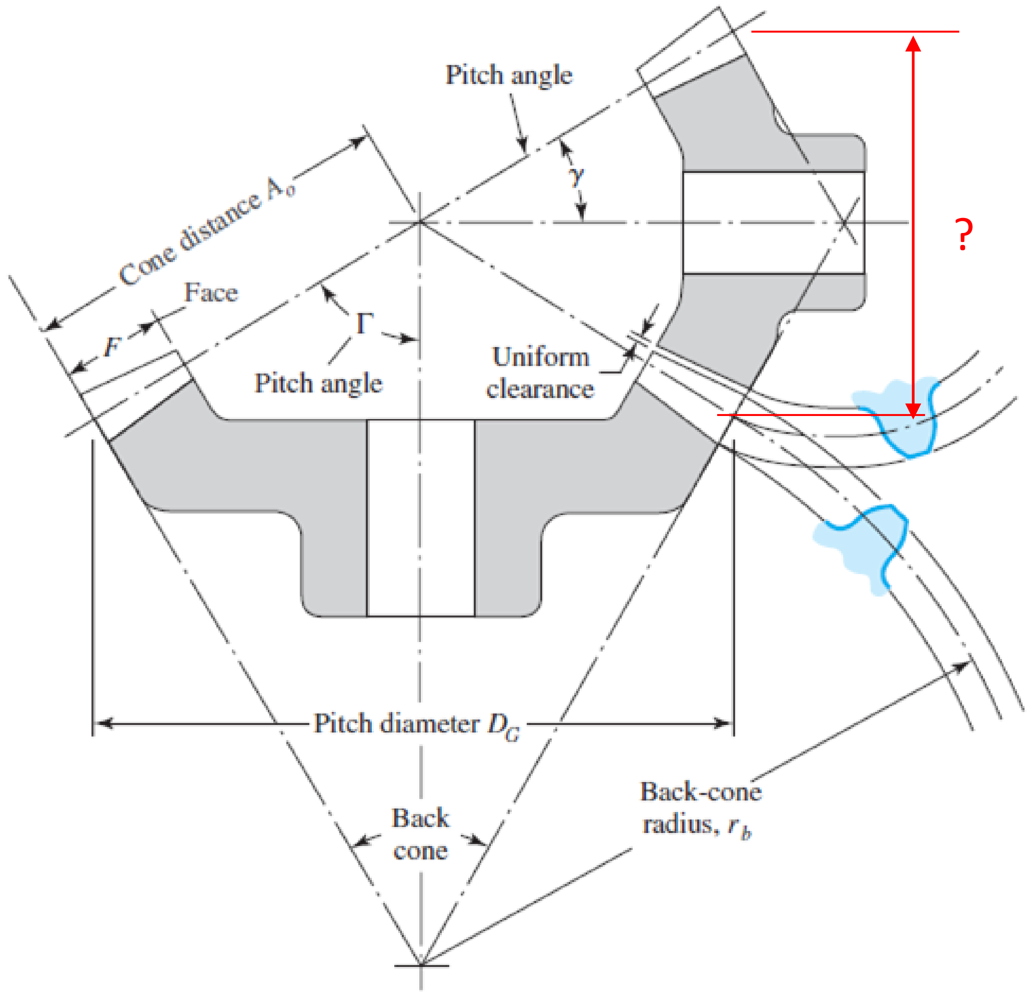


13.9 Straight Bevel Gears

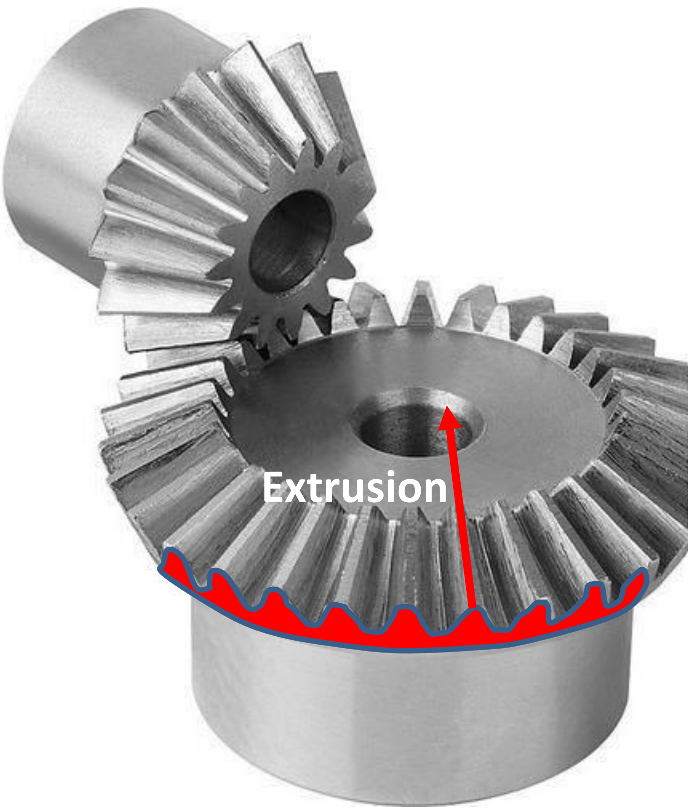
Terminology of bevel gears

$$N' = \frac{2\pi r_b}{p}$$

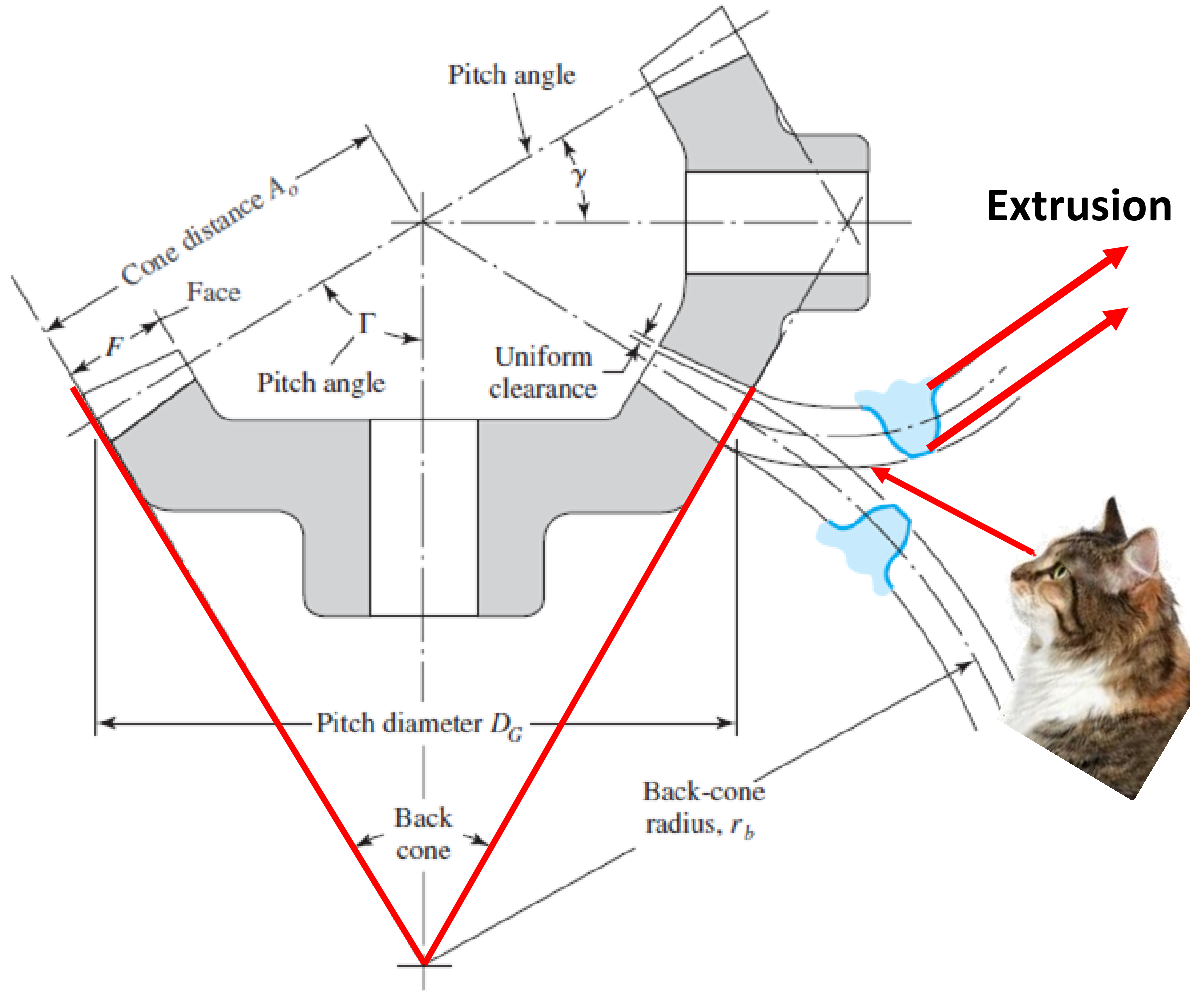
$$\tan \gamma = \frac{N_P}{N_G} \qquad \tan \Gamma = \frac{N_G}{N_P}$$



13.9 Straight Bevel Gears



Terminology of bevel gears



13.9 Straight Bevel Gears

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Table 13–3

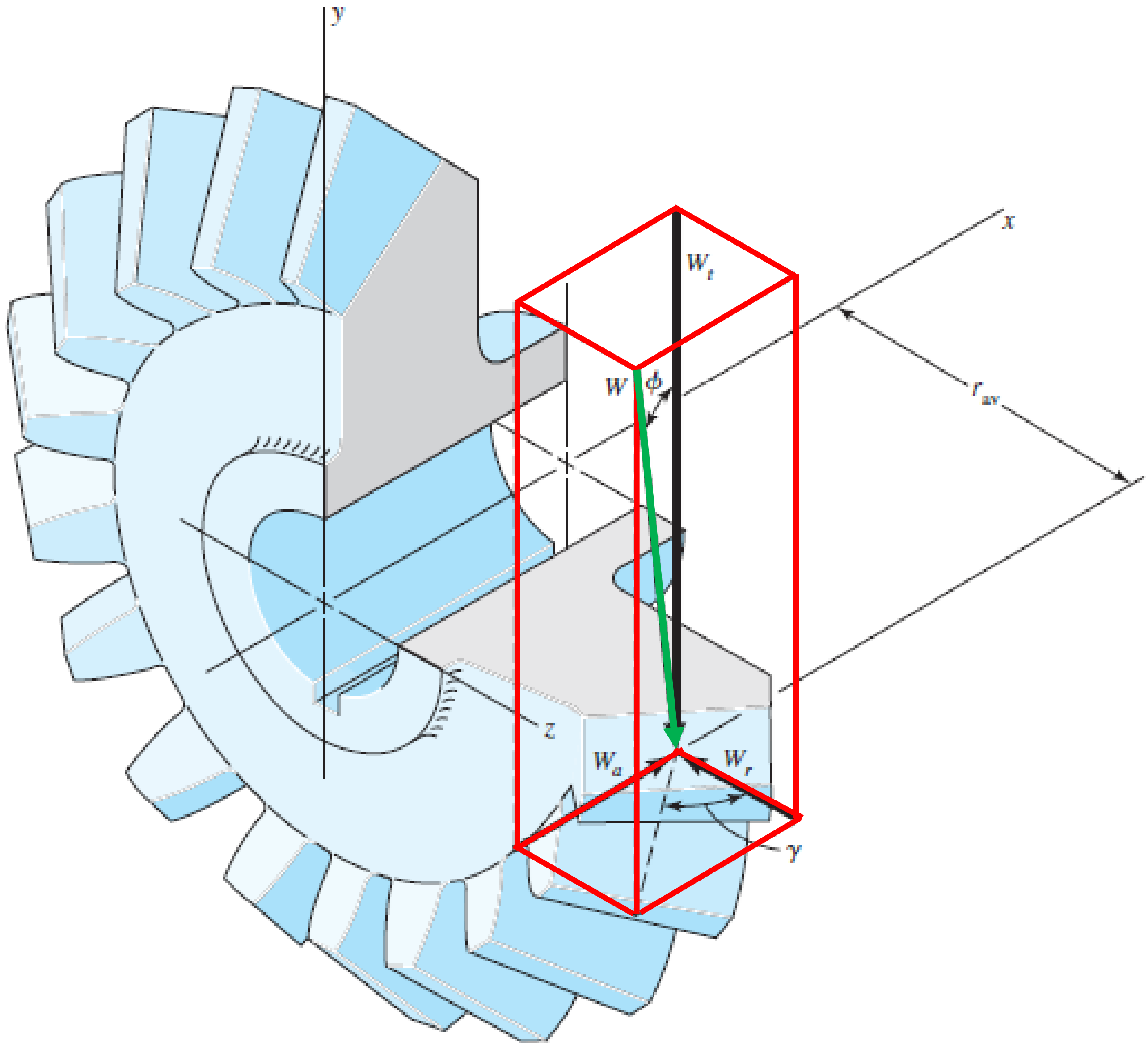
Tooth Proportions for 20° Straight Bevel-Gear Teeth

Item	Formula
Working depth	$h_k = 2.0/P$
Clearance	$c = (0.188/P) + 0.002$ in
Addendum of gear	$a_G = \dfrac{0.54}{P} + \dfrac{0.460}{P(m_{90})^2}$
Gear ratio	$m_G = N_G/N_P$
Equivalent 90° ratio	$m_{90} = m_G$ when $\Gamma = 90°$
	$m_{90} = \sqrt{m_G \dfrac{\cos\gamma}{\cos\Gamma}}$ when $\Gamma \neq 90°$
Face width	$F = 0.3A_0$ or $F = \dfrac{10}{P}$, whichever is smaller
Minimum number of teeth	Pinion: 16, 15, 14, 13
	Gear: 16, 17, 20, 30

13.15 Force Analysis - Bevel Gearing

Figure 13–35

Bevel-gear tooth forces.



13.15 Force Analysis - Bevel Gearing

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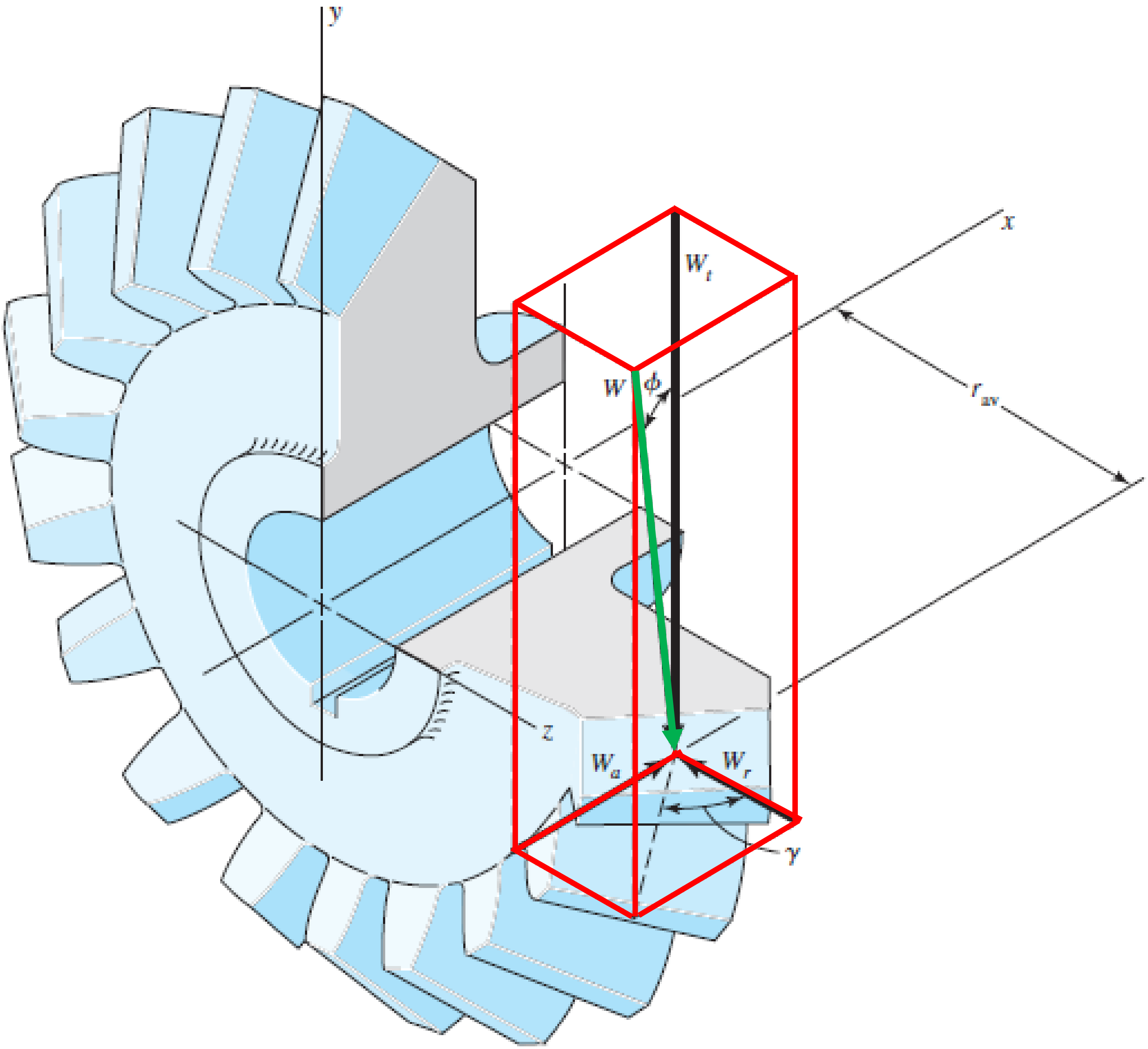
Figure 13–35

Bevel-gear tooth forces.

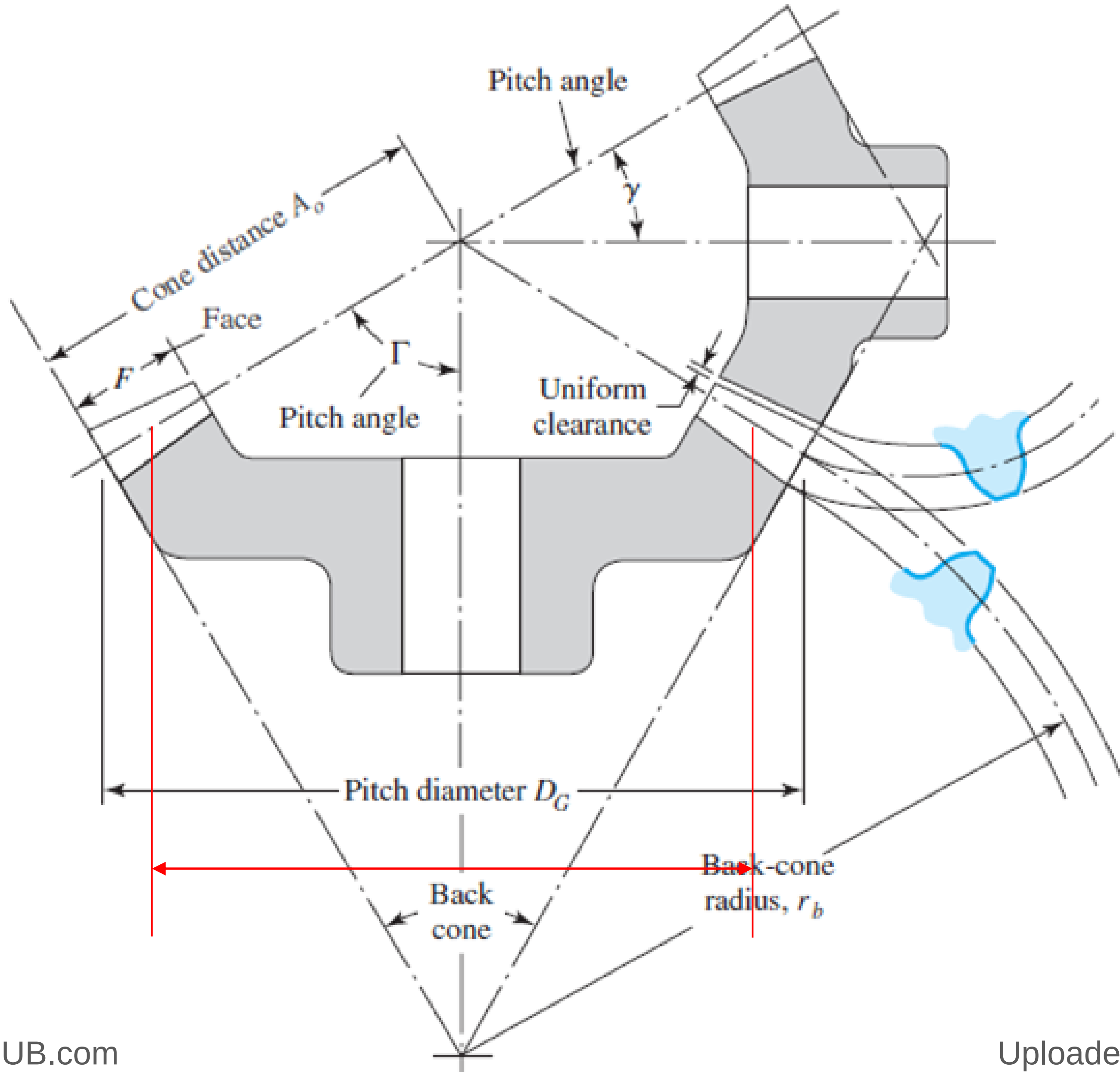
$$W_t = \frac{T}{r_{av}}$$

$$W_r = W_t \tan\phi \cos\gamma$$

$$W_a = W_t \tan\phi \sin\gamma$$

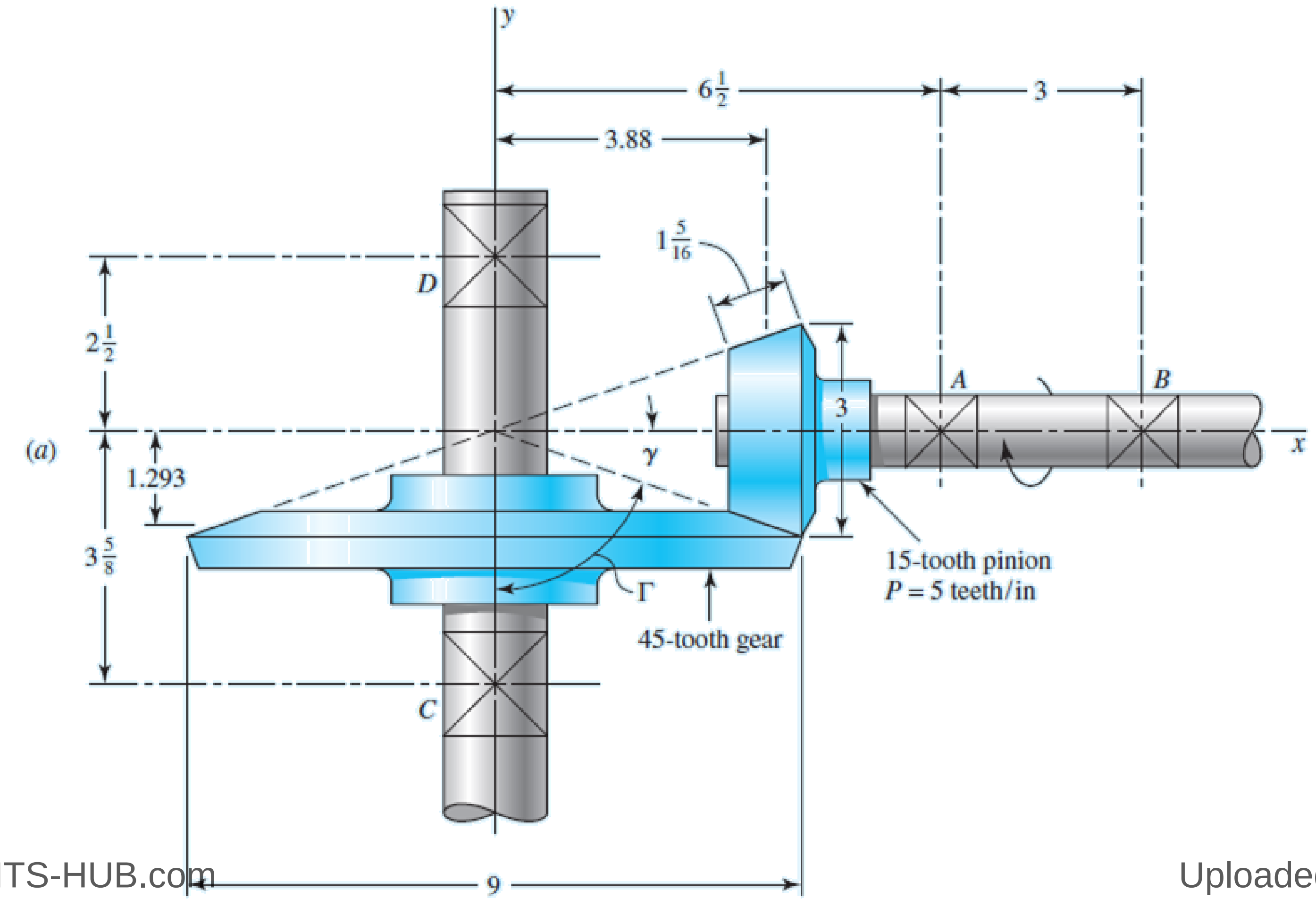


13.15 Force Analysis - Bevel Gearing

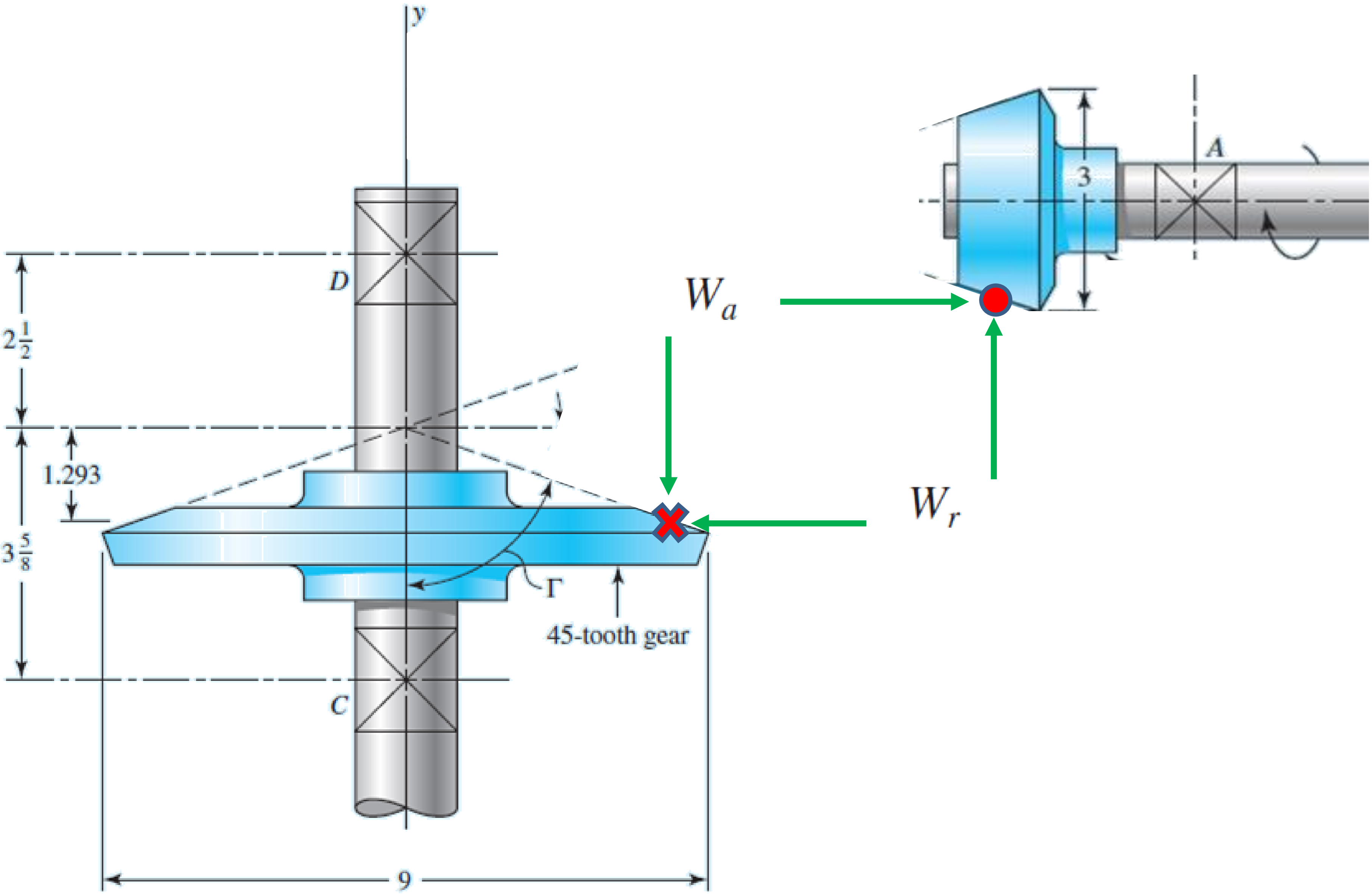


13.15 Force Analysis - Bevel Gearing – Example 13.8

The bevel pinion in Fig. 13–36*a* rotates at 600 rev/min in the direction shown and transmits 5 hp to the gear. The mounting distances, the location of all bearings, and the average pitch radii of the pinion and gear are shown in the figure. For simplicity, the teeth have been replaced by pitch cones. Bearings *A* and *C* should take the thrust loads. Find the bearing forces on the gearshaft.



13.15 Force Analysis - Bevel Gearing – Example 13.8



13.15 Force Analysis - Bevel Gearing – Example 13.8

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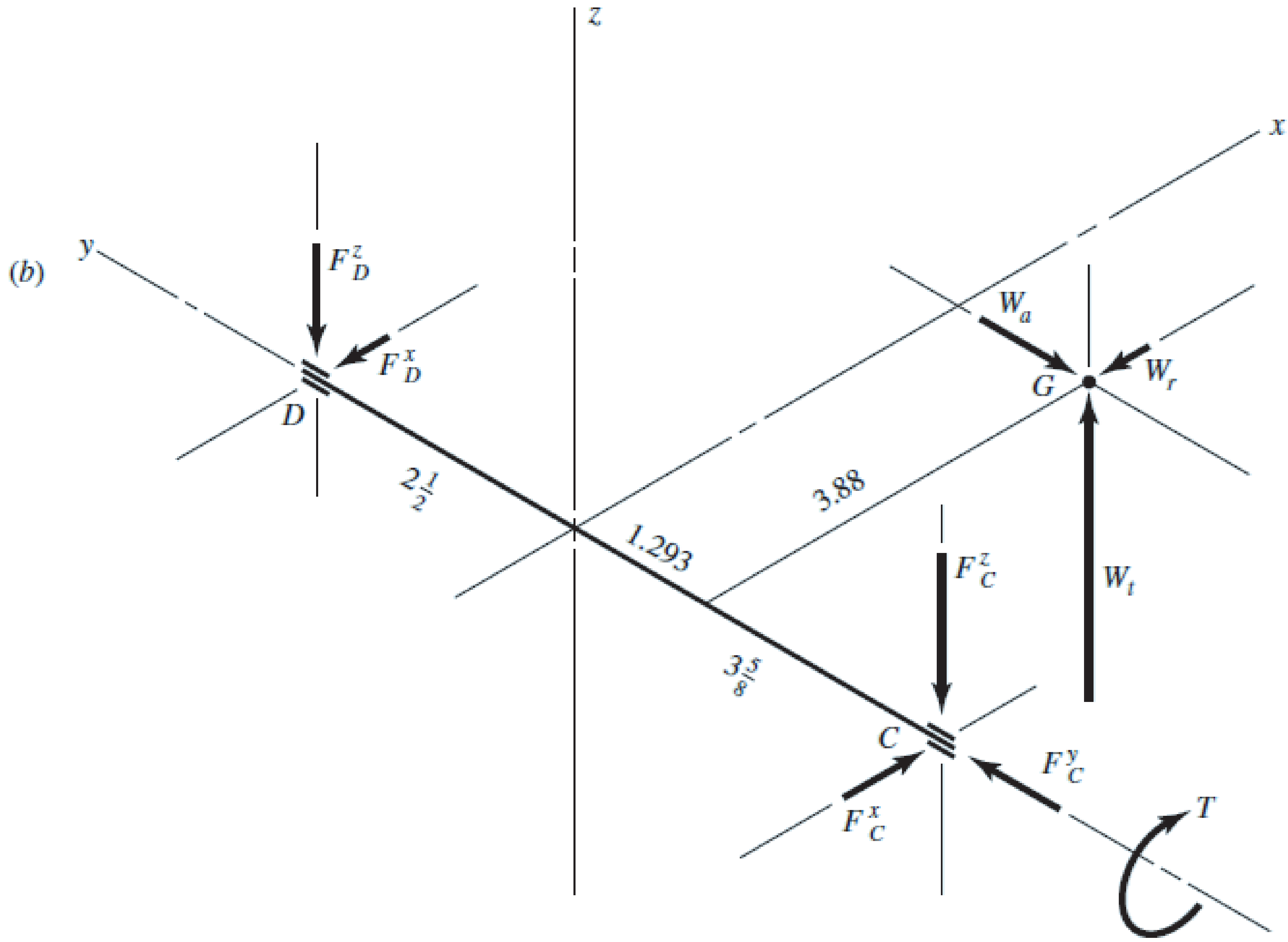


STRAIGHT-BEVEL GEAR WEAR BASED ON ANSI /AGMA 2003-B97 (U.S. customary units)

Geometry	Force Analysis	Strength Analysis
$d_P = \frac{N_P}{P_d}$	$W^t = \frac{2T}{d_{av}}$	$W^t = \frac{2T}{d_P}$
$\gamma = \tan^{-1}\frac{N_P}{N_G}$	$W^r = W^t \tan\phi \cos\gamma$	$W^r = W^t \tan\phi \cos\gamma$
$\Gamma = \tan^{-1}\frac{N_G}{N_P}$	$W^a = W^t \tan\phi \sin\gamma$	$W^a = W^t \tan\phi \sin\gamma$
$d_{av} = d_P - F\cos\Gamma$		

Gear contact stress

$$S_c = \sigma_c = C_p\left(\frac{W^t}{Fd_P I} K_o K_v K_m C_s C_{xc}\right)^{1/2}$$

- W^t: At large end of tooth
- K_o: Table 15-2, p. 783
- K_v: Eqs. (15-5) to (15-8), p. 784
- K_m: Eq. (15-11), p. 785
- C_{xc}: Eq. (15-12), p. 785
- C_s: Eq. (15-9), p. 785
- I: Fig. 15-6, p. 786
- C_p: Eq. (15-21), p. 790

Gear wear strength

$$S_{wc} = (\sigma_c)_{all} = \frac{s_{ac} C_L C_H}{S_H K_T C_R}$$

- s_{ac}: Tables 15-4, 15-5, Fig. 15-12, Eq. (15-22), pp. 790–79
- C_L: Fig. 15-8, Eq. (15-14), p. 787
- C_H: Eqs. (15-16), (15-17), gear only, p. 788
- C_R: Eqs. (15-19), (15-20), Table 15-3, pp. 789, 790
- K_T: Eq. (15-18), p. 788

Wear factor of safety

$$S_H = \frac{(\sigma_c)_{all}}{\sigma_c}, \text{ based on strength}$$

$$n_w = \left(\frac{(\sigma_c)_{all}}{\sigma_c}\right)^2, \text{ based on } W^t\text{; can be compared directly with } S_F$$

STRAIGHT-BEVEL GEAR Bending BASED ON ANSI /AGMA 2003-B97 (U.S. customary units)

Geometry	Force Analysis	Strength Analysis
$d_P = \frac{N_P}{P_d}$	$W^t = \frac{2T}{d_{av}}$	$W^t = \frac{2T}{d_P}$
$\gamma = \tan^{-1}\frac{N_P}{N_G}$	$W^r = W^t \tan\phi \cos\gamma$	$W^r = W^t \tan\phi \cos\gamma$
$\Gamma = \tan^{-1}\frac{N_G}{N_P}$	$W^a = W^t \tan\phi \sin\gamma$	$W^a = W^t \tan\phi \sin\gamma$
$d_{av} = d_P - F\cos\Gamma$		

Gear bending stress

$$S_t = \sigma = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J}$$

- W^t: At large end of tooth
- K_o: Table 15-2, p. 783
- K_v: Eqs. (15-5) to (15-8), p. 784
- K_s: Eq. (15-10), p. 785
- K_m: Eq. (15-11), p. 785
- J: Fig. 15-7, p. 786
- K_x: Eq. (15-13), p. 785

Gear bending strength

$$S_{wt} = \sigma_{all} = \frac{s_{at} K_L}{S_F K_T K_R}$$

- s_{at}: Table 15-6 or 15-7, pp. 791, 792
- K_L: Fig. 15-9, Eq. (15-15), pp. 788, 787
- K_R: Eqs. (15-19), (15-20), Table 15-3, pp. 789, 790
- K_T: Eq. (15-18), p. 788

Bending factor of safety

$S_F = \frac{\sigma_{all}}{\sigma}$, based on strength

$n_B = \frac{\sigma_{all}}{\sigma}$, based on W^t, same as S_F

15.4 Straight Bevel Gear Analysis – Problem 15.10

A catalog of stock bevel gears lists a power rating of 5.2 hp at 1200 rev/min pinion speed for a straight-bevel gearset consisting of a 20-tooth pinion driving a 40-tooth gear. This gear pair has a 20° normal pressure angle, a face width of 0.71 in, a diametral pitch of 10 teeth/in, and is through-hardened to 300 BHN. Assume the gears are for general industrial use, are generated to a transmission accuracy number of 5, and are uncrowned. Also assume the gears are rated for a life of 3×10^6 revolutions with a 99 percent reliability. Given these data, what do you think about the stated catalog power rating?

15.4 Straight Bevel Gear Analysis – Problem 15.10

Bending Stress

$$s_t = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J}$$

Permissible Bending Stress Equation

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R}$$

Table 15–2

Overload Factors K_o (K_A)

Source: ANSI/AGMA 2003-B97.

Character of Prime Mover	Character of Load on Driven Machine: Uniform	Light Shock	Medium Shock	Heavy Shock
Uniform	1.00	1.25	1.50	1.75 or higher
Light shock	1.10	1.35	1.60	1.85 or higher
Medium shock	1.25	1.50	1.75	2.00 or higher
Heavy shock	1.50	1.75	2.00	2.25 or higher

Note: This table is for speed-decreasing drives. For speed-increasing drives, add $0.01(N/n)^2$ or $0.01(z_2/z_1)^2$ to the above factors.

15.4 Straight Bevel Gear Analysis – Problem 15.10

Bending Stress

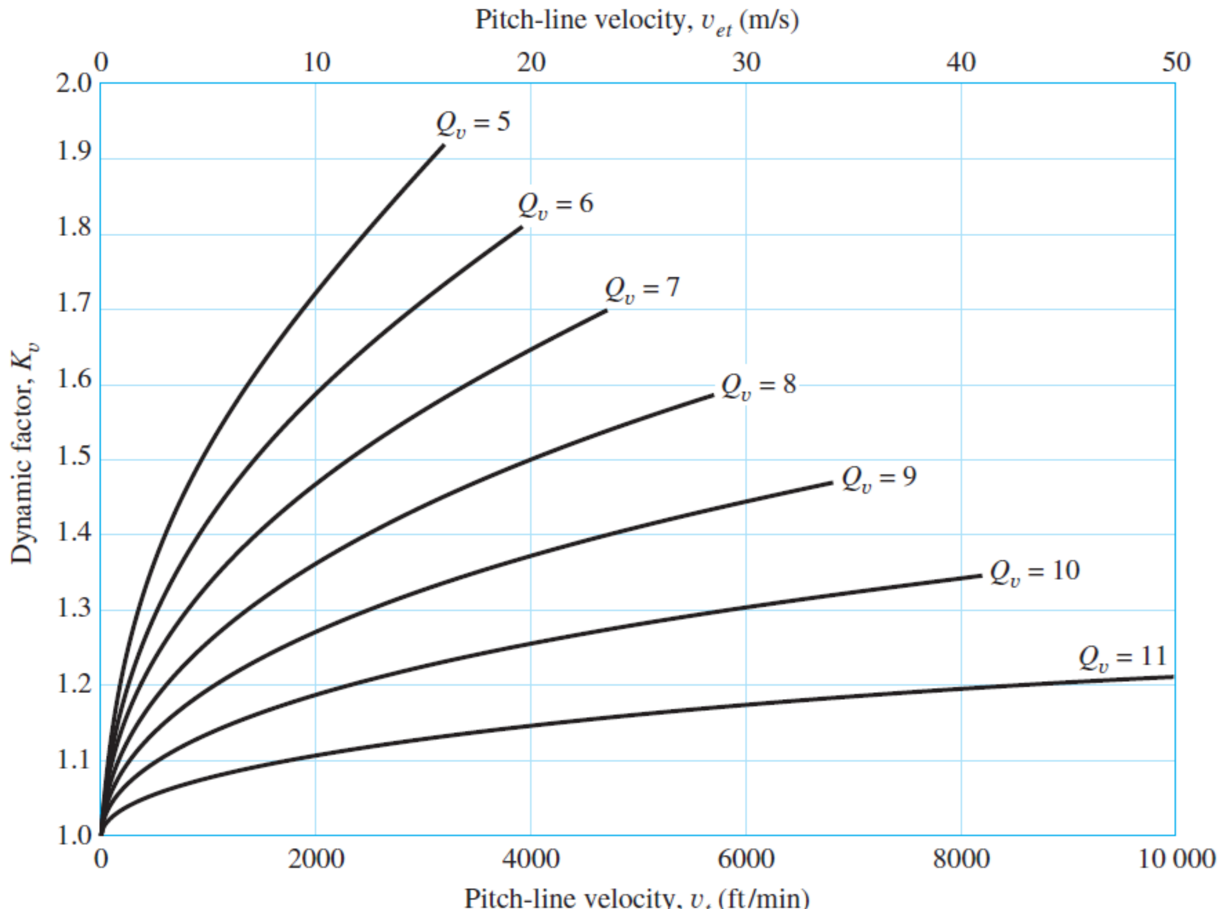
$$s_t = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J}$$

Permissible Bending Stress Equation

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R}$$

Figure 15–5

Dynamic factor K_v. *(Source: ANSI/AGMA 2003-B97.)*



15.4 Straight Bevel Gear Analysis – Problem 15.10

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Bending Stress

$$s_t = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J}$$

Permissible Bending Stress Equation

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R}$$

$$K_v = \left(\frac{A + \sqrt{v_t}}{A}\right)^B \qquad \text{(U.S. customary units)}$$

where

$$A = 50 + 56(1 - B)$$

$$B = 0.25(12 - Q_v)^{2/3}$$

and $v_t(v_{et})$ is the pitch-line velocity at outside pitch diameter, expressed ir

$$v_t = \pi d_P n_P / 12 \qquad \text{(U.S. customary units)}$$

15.4 Straight Bevel Gear Analysis – Problem 15.10

Bending Stress

$$s_t = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J}$$

Permissible Bending Stress Equation

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R}$$

Size Factor for Bending K_s (Y_x)

$$K_S = \begin{cases} 0.4867 + 0.2132/P_d & 0.5 \le P_d \le 16 \text{ teeth/in} \\ 0.5 & P_d > 16 \text{ teeth/in} \end{cases} \quad \text{(U.S. customary units)}$$

(15–10)

$$Y_x = \begin{cases} 0.5 & m_{et} < 1.6 \text{ mm} \\ 0.4867 + 0.008\,339 m_{et} & 1.6 \le m_{et} \le 50 \text{ mm} \end{cases} \quad \text{(SI units)}$$

15.4 Straight Bevel Gear Analysis – Problem 15.10

Bending Stress

$$s_t = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J}$$

Permissible Bending Stress Equation

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R}$$

Load-Distribution Factor K_m ($K_{H\beta}$)

$$K_m = K_{mb} + 0.0036 F^2 \qquad \text{(U.S. customary units)}$$

$$K_{H\beta} = K_{mb} + 5.6(10^{-6}) b^2 \qquad \text{(SI units)} \qquad (15\text{–}11)$$

where

$$K_{mb} = \begin{cases} 1.00 & \text{both members straddle-mounted} \\ 1.10 & \text{one member straddle-mounted} \\ 1.25 & \text{neither member straddle-mounted} \end{cases}$$

15.4 Straight Bevel Gear Analysis – Problem 15.10

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Bending Stress

$$s_t = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J}$$

Permissible Bending Stress Equation

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R}$$

Lengthwise Curvature Factor for Bending Strength K_x (Y_β)

For straight-bevel gears,

$$K_x = Y_\beta = 1$$

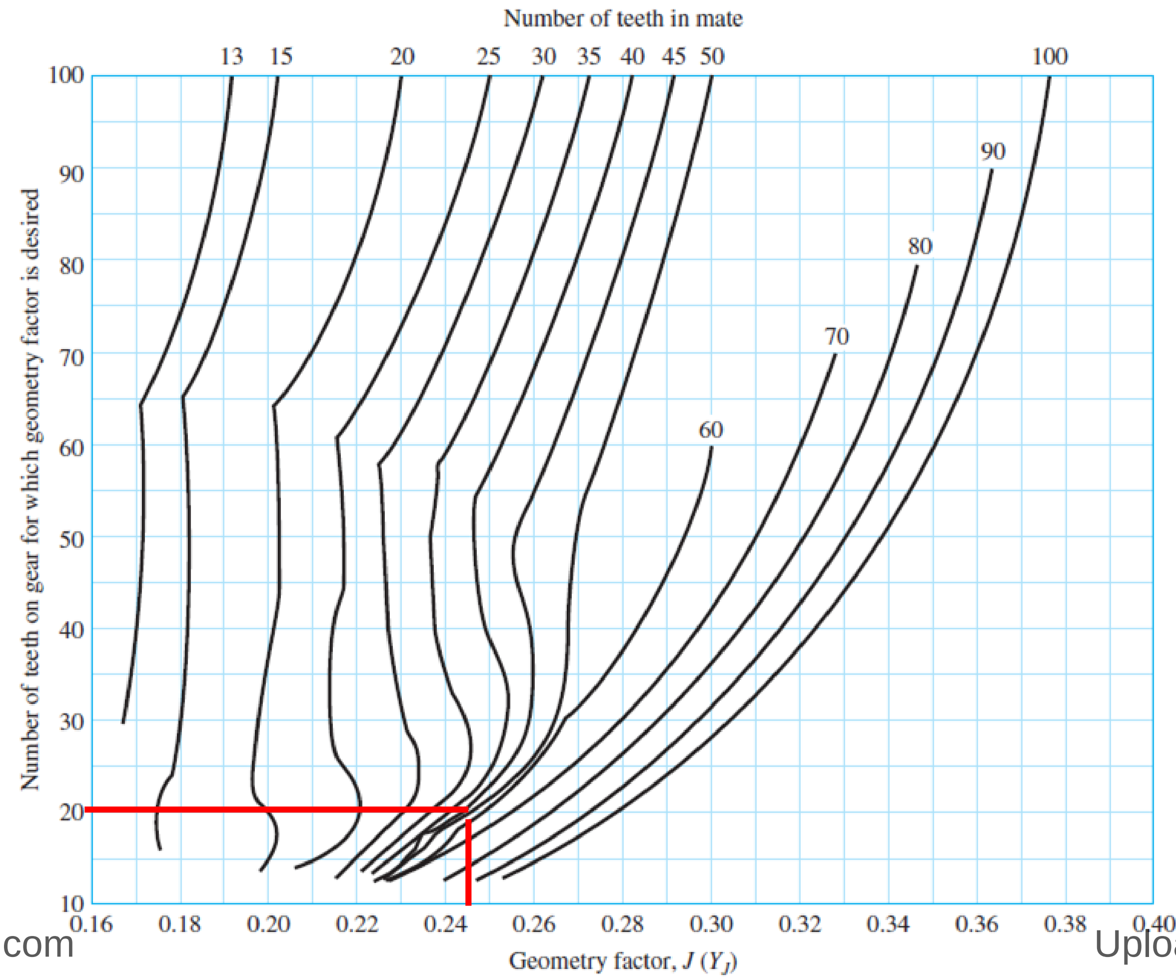
15.4 Straight Bevel Gear Analysis – Problem 15.10

Permissible Bending Stress Equation

Bending Stress

$$s_t = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J}$$

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R}$$



15.4 Straight Bevel Gear Analysis – Problem 15.10

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Bending Stress

$$s_t = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J}$$

Permissible Bending Stress Equation

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R}$$

Stress-Cycle Factor for Bending Strength K_L (Y_{NT})

$$K_L = \begin{cases} 2.7 & 10^2 \le N_L < 10^3 \\ 6.1514 N_L^{-0.1192} & 10^3 \le N_L < 3(10^6) \\ 1.683 N_L^{-0.0323} & 3(10^6) \le N_L \le 10^{10} \quad \text{critical} \\ 1.3558 N_L^{-0.0178} & 3(10^6) \le N_L \le 10^{10} \quad \text{general} \end{cases}$$

$$Y_{NT} = \begin{cases} 2.7 & 10^2 \le n_L < 10^3 \\ 6.1514 n_L^{-0.1192} & 10^3 \le n_L < 3(10^6) \\ 1.683 n_L^{-0.0323} & 3(10^6) \le n_L \le 10^{10} \quad \text{critical} \\ 1.3558 n_L^{-0.0178} & 3(10^6) \le n_L \le 10^{10} \quad \text{general} \end{cases}$$

15.4 Straight Bevel Gear Analysis – Problem 15.10

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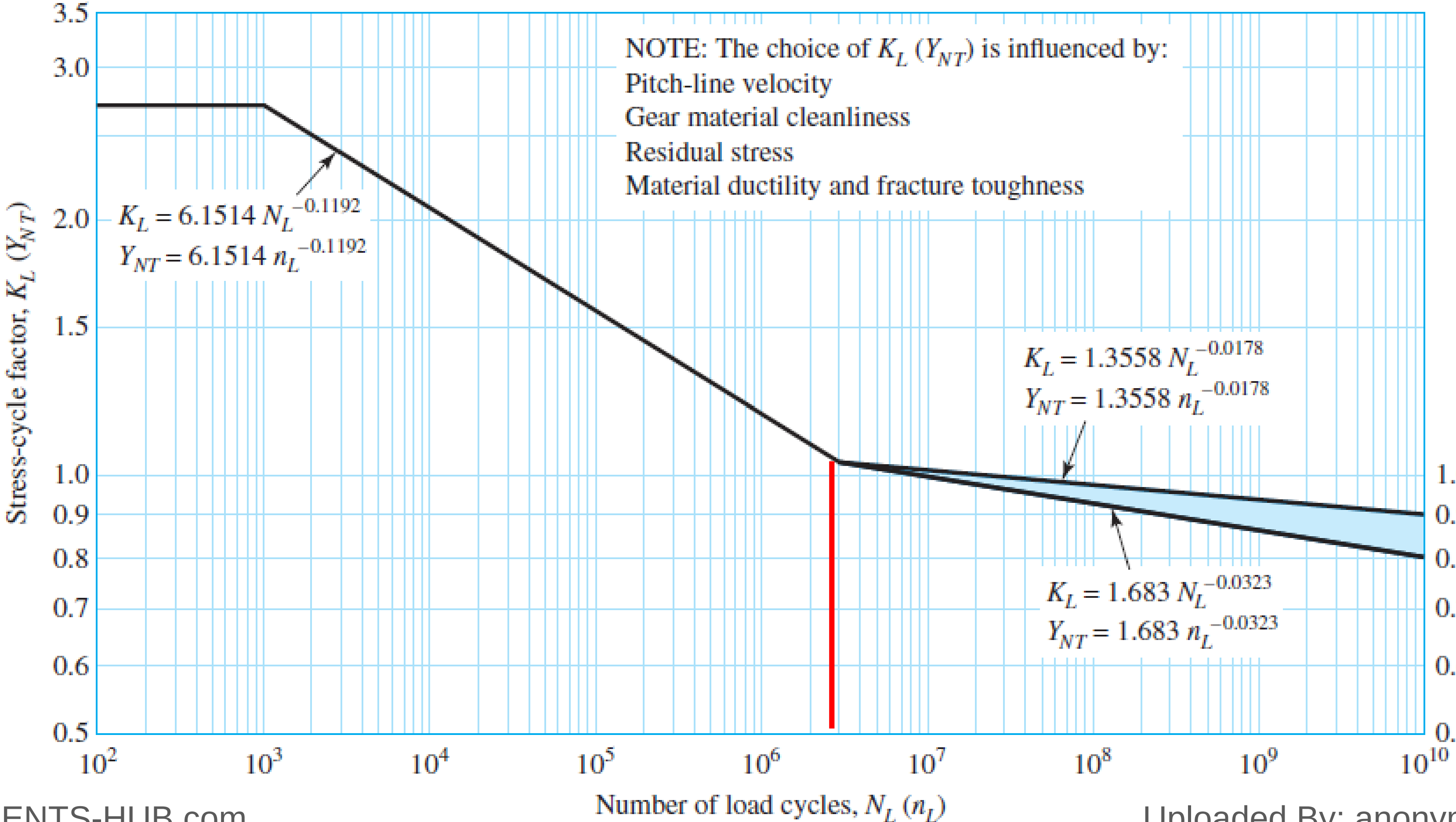
Bending Stress

$$s_t = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J}$$

Permissible Bending Stress Equation

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R}$$



15.4 Straight Bevel Gear Analysis – Problem 15.10

Bending Stress

$$s_t = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J}$$

Permissible Bending Stress Equation

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R}$$

Reliability Factors C_R (Z_Z) and K_R (Y_Z)

Table 15–3 displays the reliability factors. Note that $C_R = \sqrt{K_R}$ and $Z_Z = \sqrt{Y_Z}$. Logarithmic interpolation equations are

$$Y_Z = K_R = \begin{cases} 0.50 - 0.25\log(1-R) & 0.99 \le R \le 0.999 \quad (15\text{–}19) \\ 0.70 - 0.15\log(1-R) & 0.90 \le R < 0.99 \quad (15\text{–}20) \end{cases}$$

Table 15–3

Reliability Factors

Source: ANSI/AGMA 2003-B97.

Requirements of Application	Reliability Factors for Steel* C_R (Z_Z)	K_R (Y_Z)†
Fewer than one failure in 10 000	1.22	1.50
Fewer than one failure in 1000	1.12	1.25
Fewer than one failure in 100	1.00	1.00
Fewer than one failure in 10	0.92	0.85‡
Fewer than one failure in 2	0.84	0.70§

15.4 Straight Bevel Gear Analysis – Problem 15.10

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Bending Stress

$$s_t = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J}$$

Permissible Bending Stress Equation

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R}$$

Temperature Factor K_T (K_θ)

$$K_T = \begin{cases} 1 & 32°\text{F} \le t \le 250°\text{F} \\ (460 + t)/710 & t > 250°\text{F} \end{cases}$$

$$K_\theta = \begin{cases} 1 & 0°\text{C} \le \theta \le 120°\text{C} \\ (273 + \theta)/393 & \theta > 120°\text{C} \end{cases}$$

15.4 Straight Bevel Gear Analysis – Problem 15.10

Bending Stress

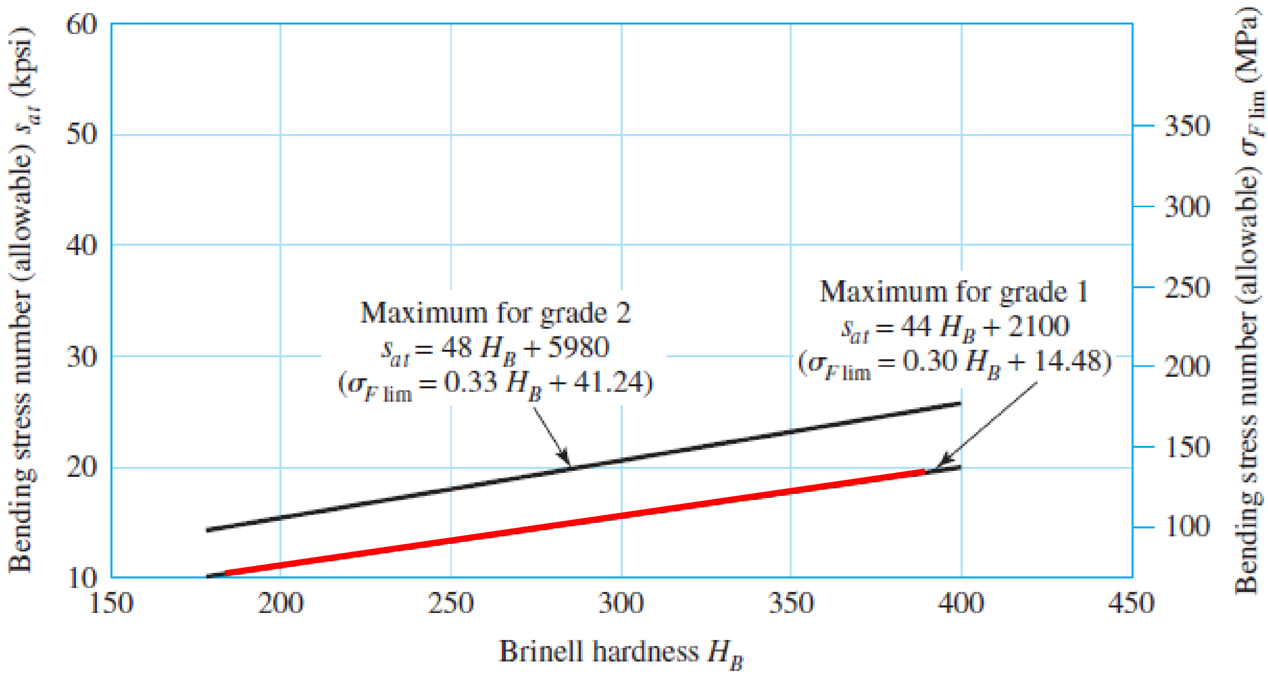
$$s_t = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J}$$

Permissible Bending Stress Equation

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R}$$

Figure 15–13

Allowable bending stress number for through-hardened steel gears, $s_{at}(\sigma_{F\,\lim})$. *(Source: ANSI/AGMA 2003-B97.)*



15.4 Straight Bevel Gear Analysis – Problem 15.10

Bending Stress

$$s_t = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J}$$

Permissible Bending Stress Equation

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R}$$

Allowable Bending Stress Numbers

Tables 15–6 and 15–7 provide s_{at} ($\sigma_{F\,\text{lim}}$) for steel gears and for iron gears, respectively. Figure 15–13 shows graphically allowable bending stress s_{at} ($\sigma_{H\,\text{lim}}$) for through-hardened steels. The equations are

$$\begin{aligned} s_{at} &= 44H_B + 2100 \text{ psi} && \text{grade 1} \\ \sigma_{F\,\text{lim}} &= 0.30H_B + 14.48 \text{ MPa} && \text{grade 1} \\ s_{at} &= 48H_B + 5980 \text{ psi} && \text{grade 2} \\ \sigma_{H\,\text{lim}} &= 0.33H_B + 41.24 \text{ MPa} && \text{grade 2} \end{aligned} \quad (15\text{–}23)$$

15.4 Straight Bevel Gear Analysis – Problem 15.10

Bending Stress

$$s_t = \frac{W^t}{F} P_d K_o K_v \frac{K_s K_m}{K_x J}$$

Permissible Bending Stress Equation

$$s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R}$$

13.1 Types of Gears

Figure 13–1

Spur gears are used to transmit rotary motion between parallel shafts.

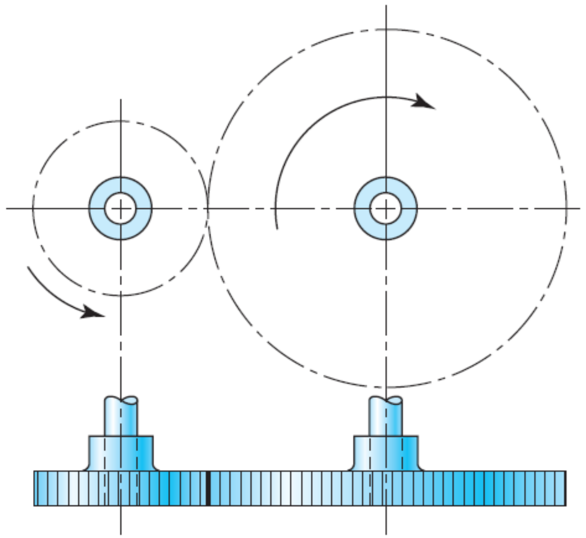
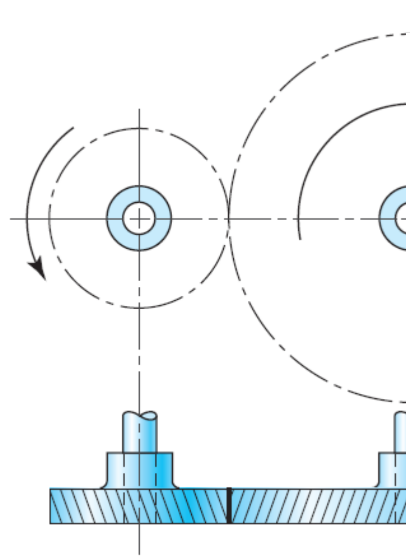


Figure 13–2

Helical gears are used to transmit motion between parallel or nonparallel shafts.



13.1 Types of Gears

Figure 13–3

Bevel gears are used to transmit rotary motion between intersecting shafts.

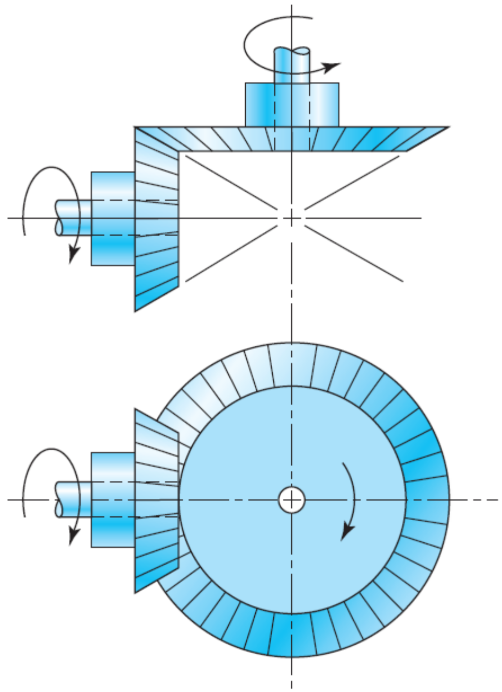
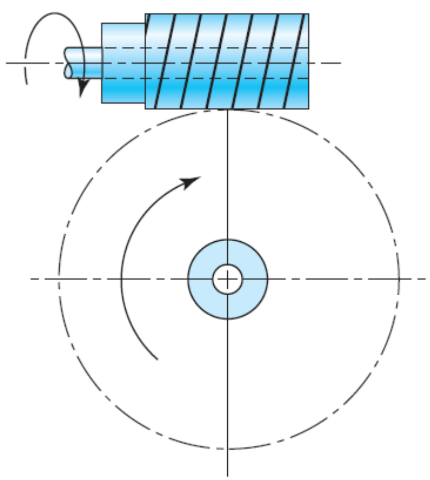
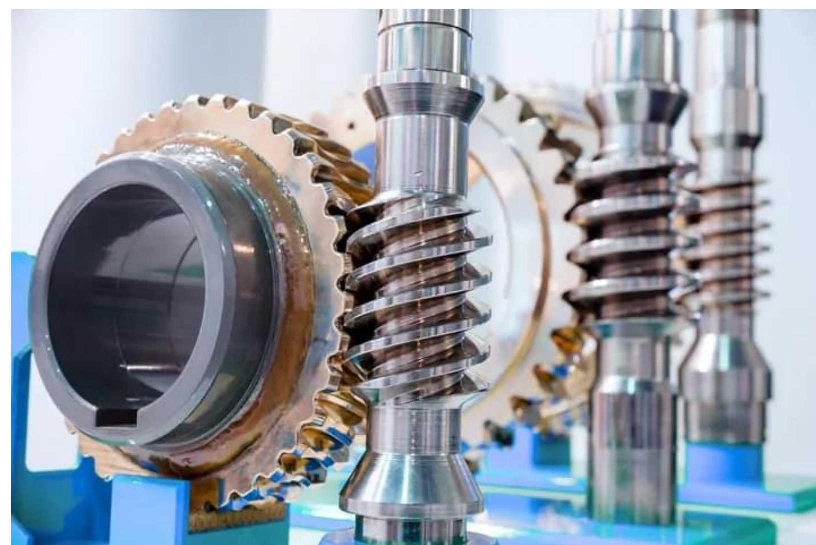


Figure 13–4

Worm gearsets are used to transmit rotary motion between nonparallel and nonintersecting shafts.





13.11 Worm Gears

Table 8–5

Coefficients of Friction f for Threaded Pairs

Source: H. A. Rothbart and T. H. Brown, Jr., *Mechanical Design Handbook,* 2nd ed., McGraw-Hill, New York, 2006.

Screw Material	Nut Material: Steel	Bronze	Brass	Cast Iron
Steel, dry	0.15–0.25	0.15–0.23	0.15–0.19	0.15–0.25
Steel, machine oil	0.11–0.17	0.10–0.16	0.10–0.15	0.11–0.17
Bronze	0.08–0.12	0.04–0.06	—	0.06–0.09

Table 8–6

Thrust-Collar Friction Coefficients

Source: H. A. Rothbart and T. H. Brown, Jr., *Mechanical Design Handbook,* 2nd ed., McGraw-Hill, New York, 2006.

Combination	Running	Starting
Soft steel on cast iron	0.12	0.17
Hard steel on cast iron	0.09	0.15
Soft steel on bronze	0.08	0.10
Hard steel on bronze	0.06	0.08

13.11 Worm Gears

13.11 Worm Gears

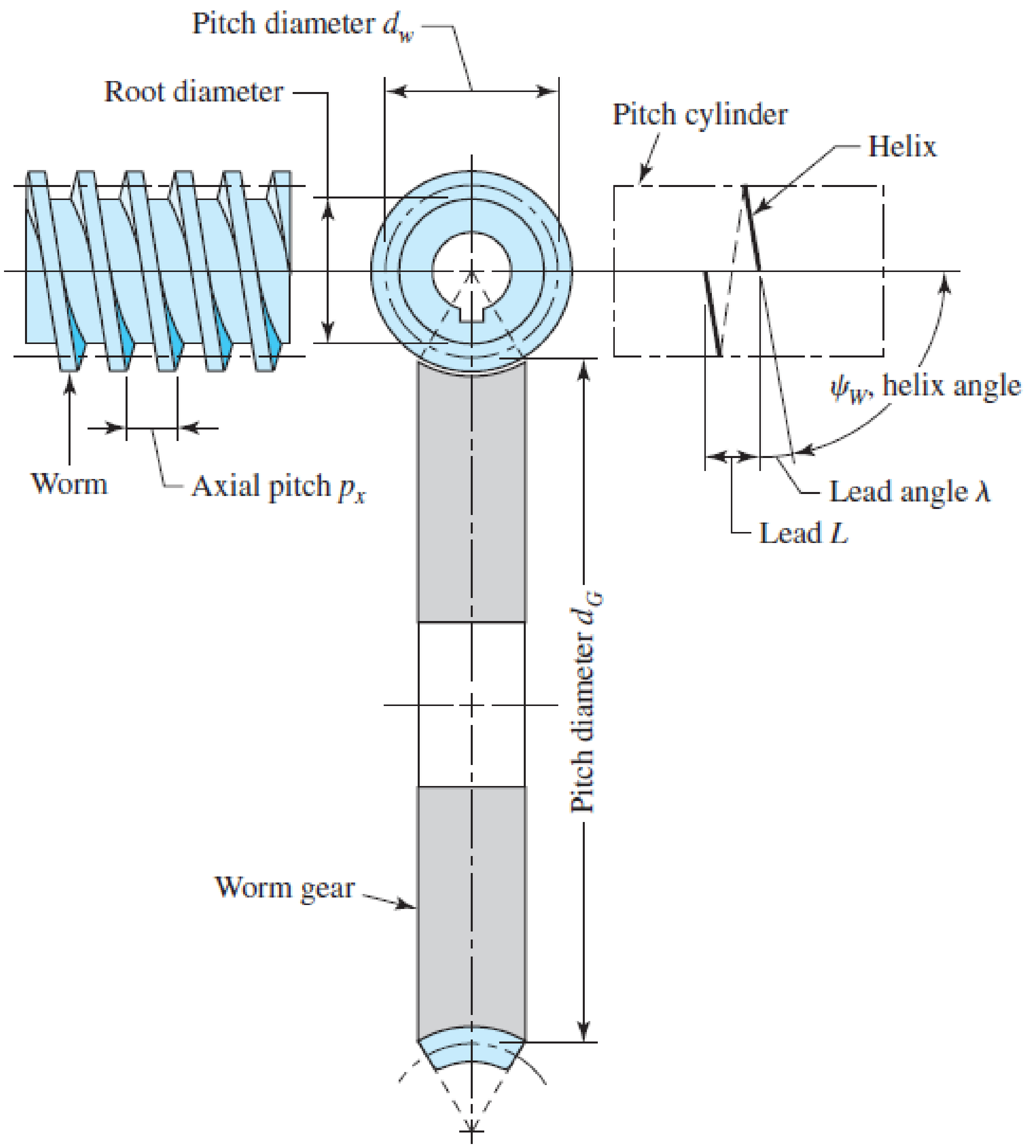
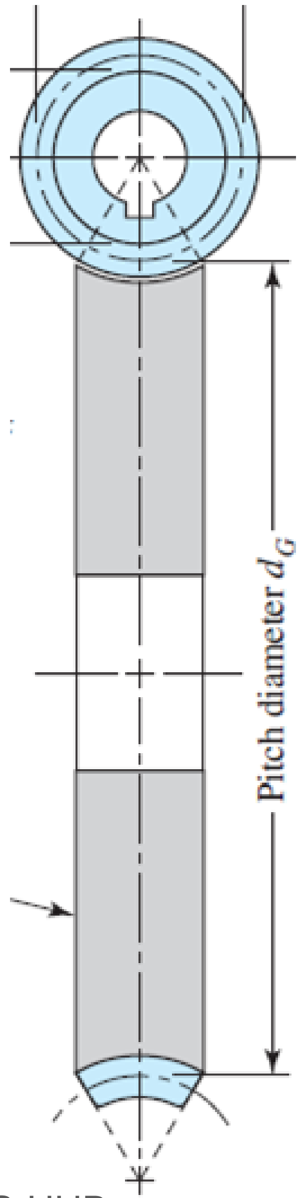


Figure 13–24

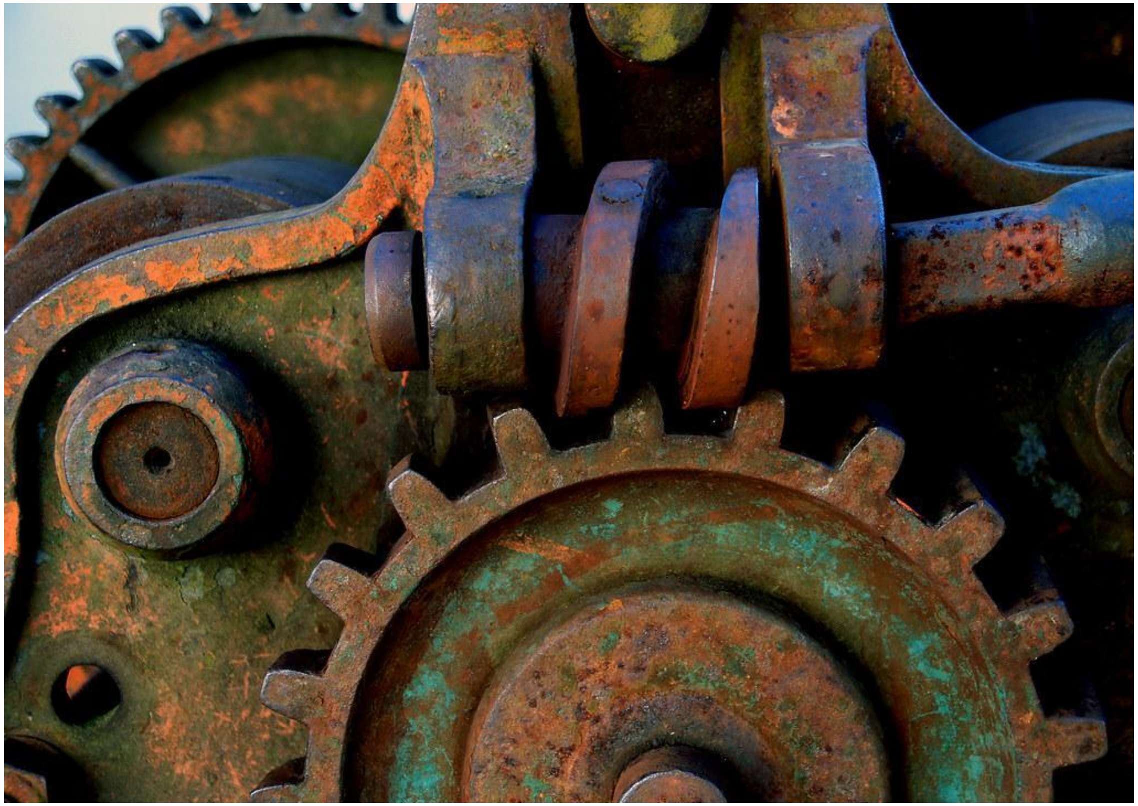
Nomenclature of a single-enveloping worm gearset.

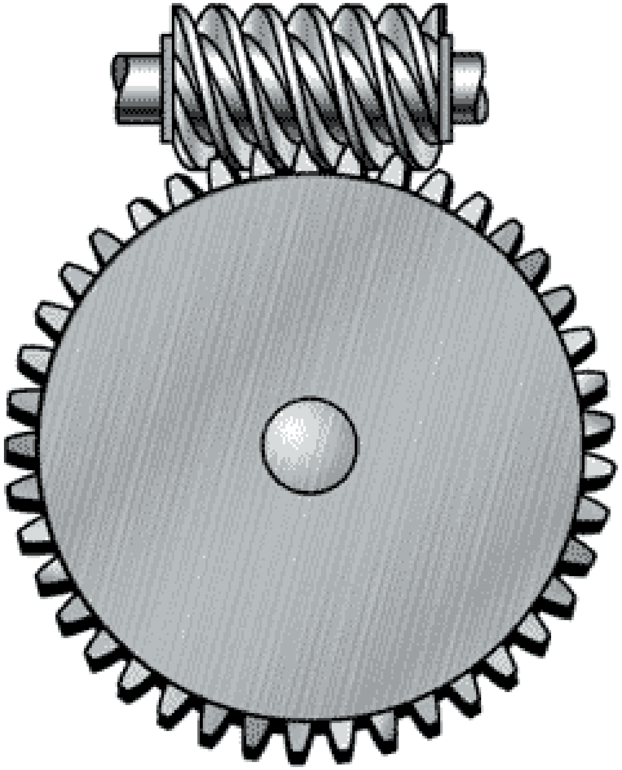
$$d_G = \frac{N_G p_t}{\pi}$$

13.11 Worm Gears



13.11 Worm Gears

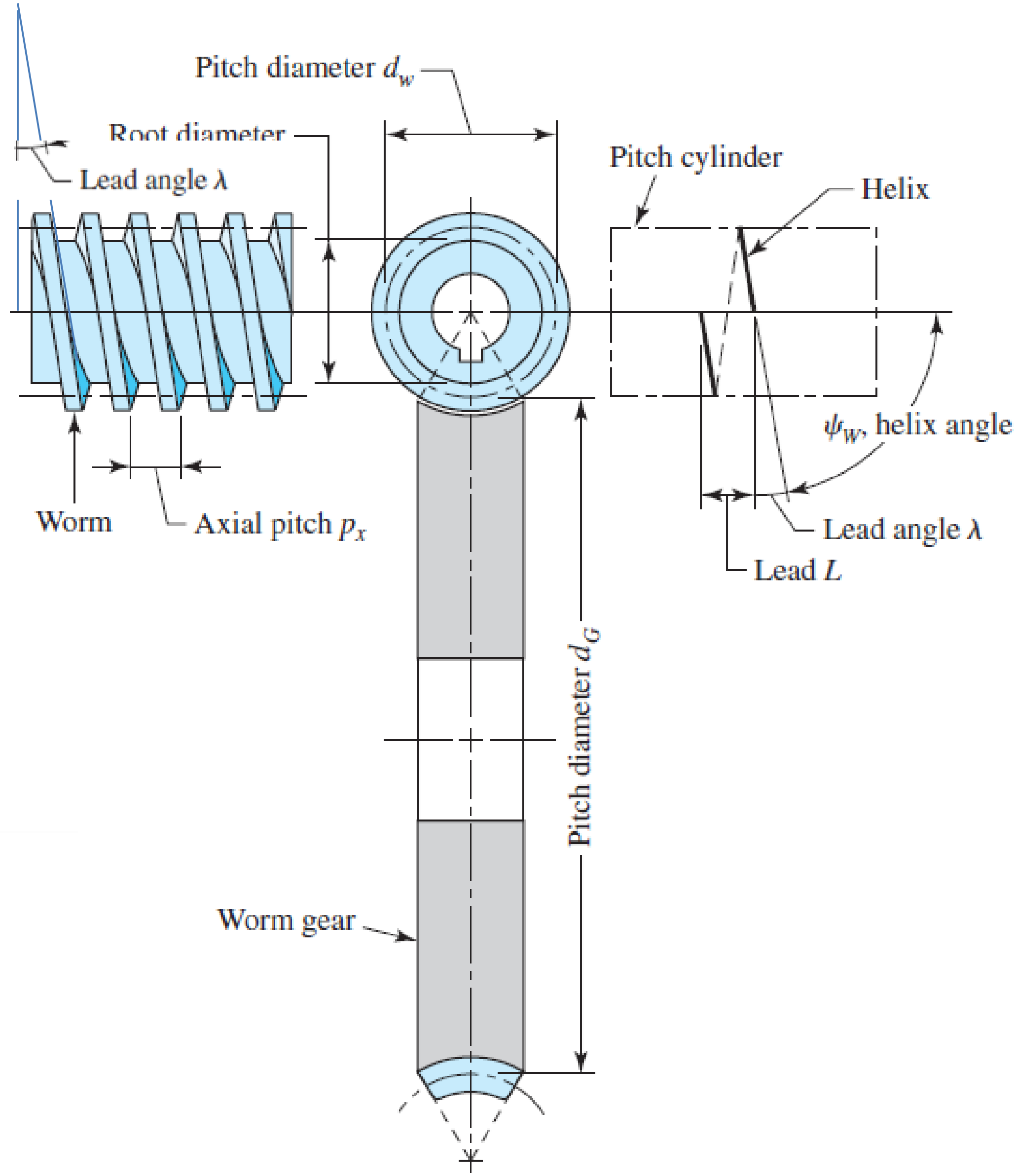




13.11 Worm Gears

Figure 13–24

Nomenclature of a single-enveloping worm gearset.

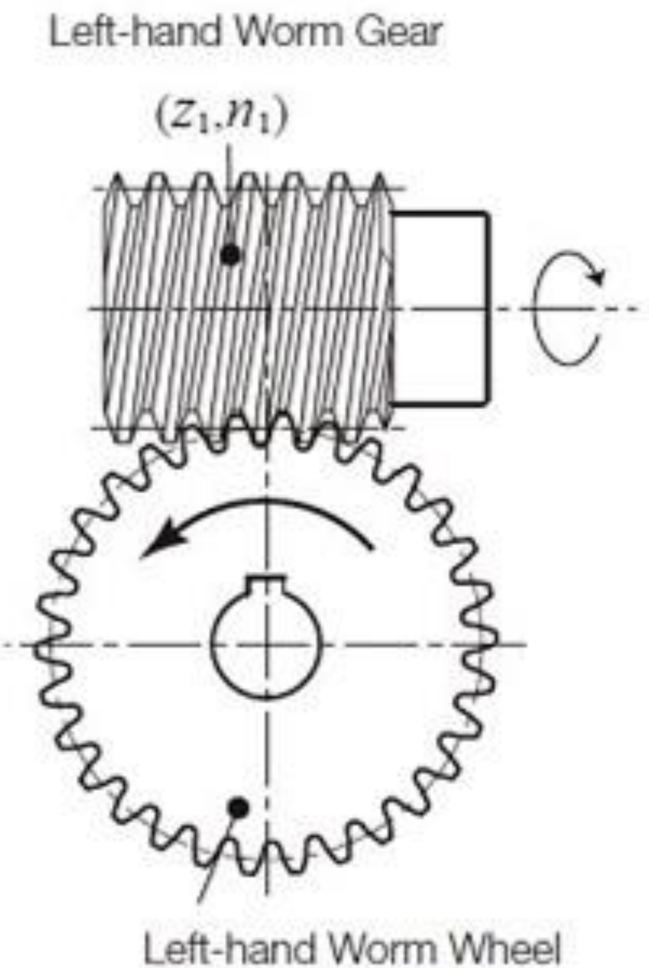


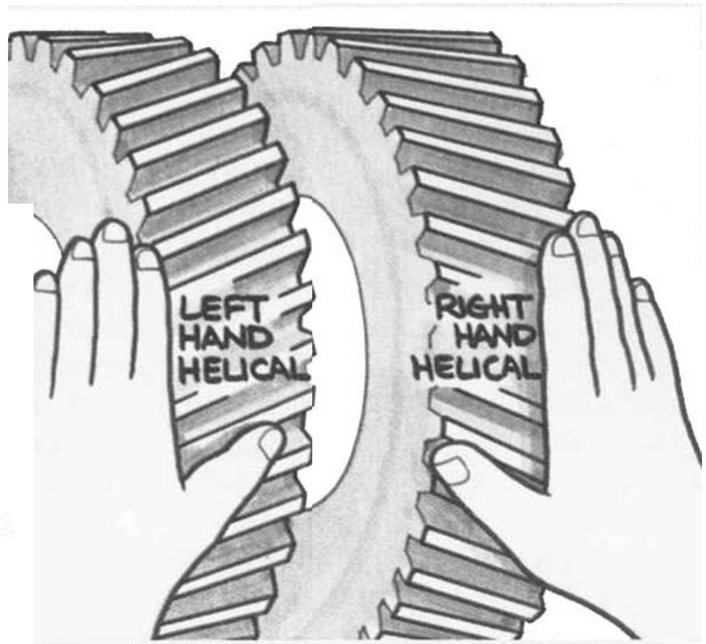
$$\frac{C^{0.875}}{3.0} \leq d_W \leq \frac{C^{0.875}}{1.7}$$

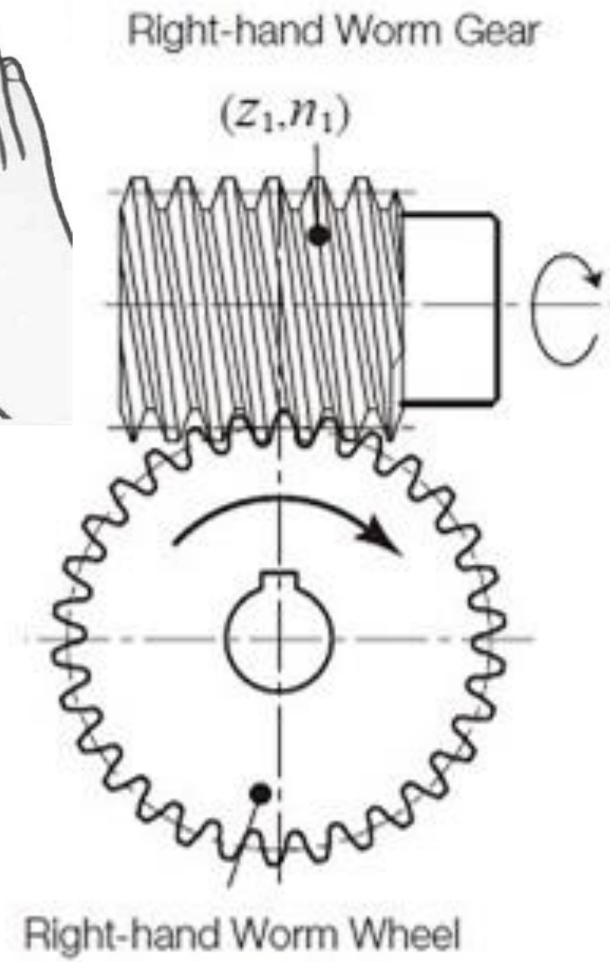
$$L = p_x N_W$$

$$\tan \lambda = \frac{L}{\pi d_W}$$

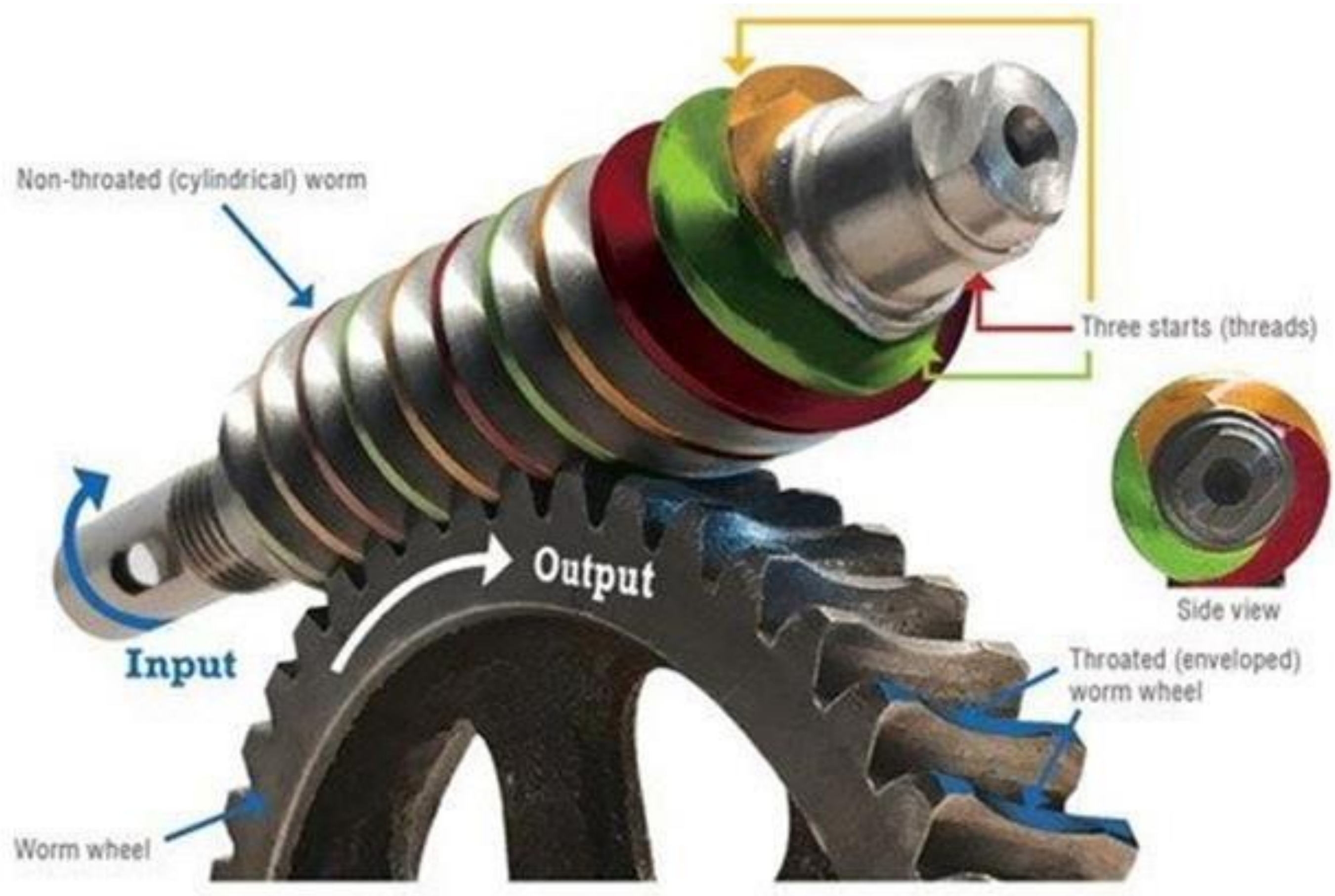
13.11 Worm Gears







13.11 Worm Gears



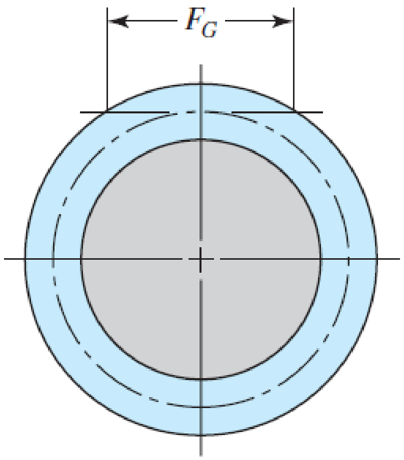
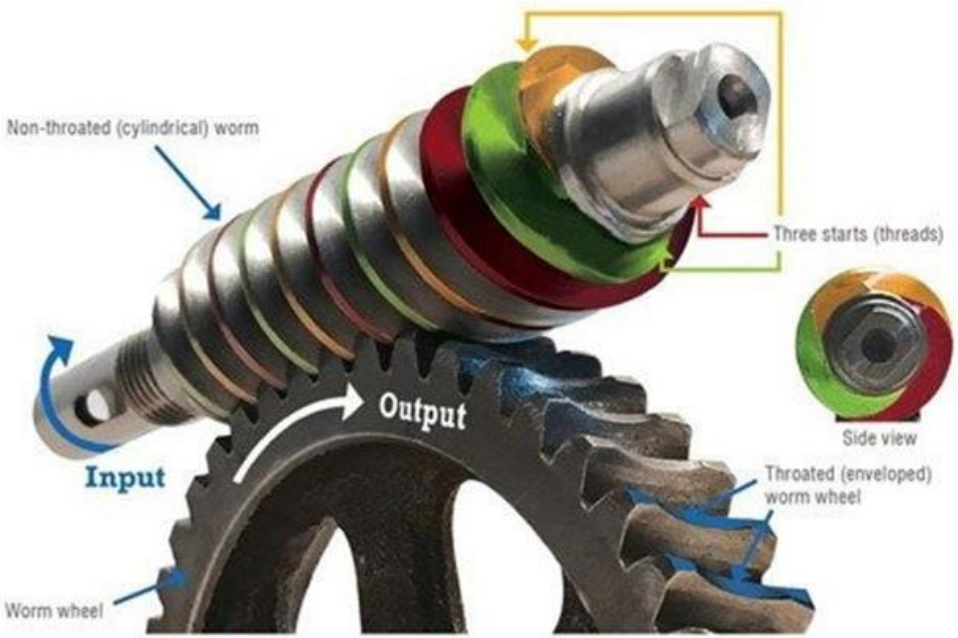


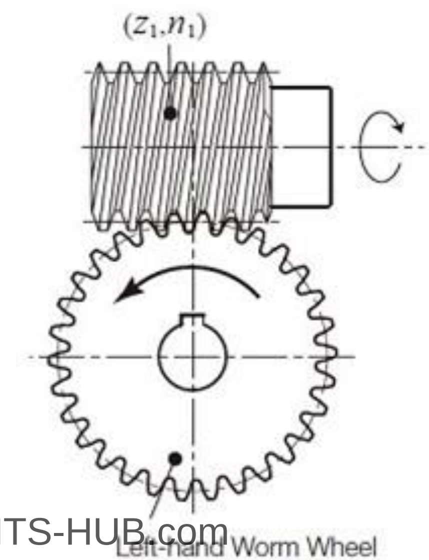
Figure 13–25

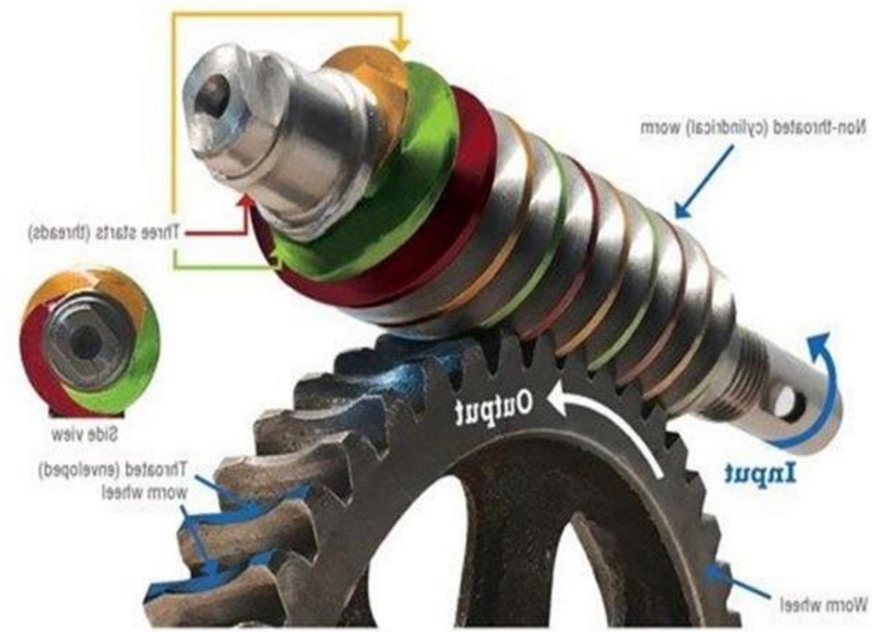
A graphical depiction of the face width of the worm of a worm gearset.

13.11 Worm Gears

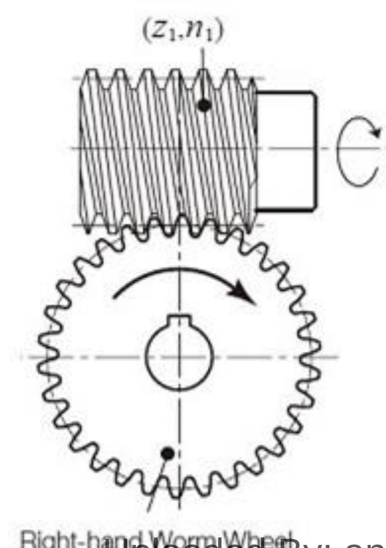


Left-hand Worm Gear



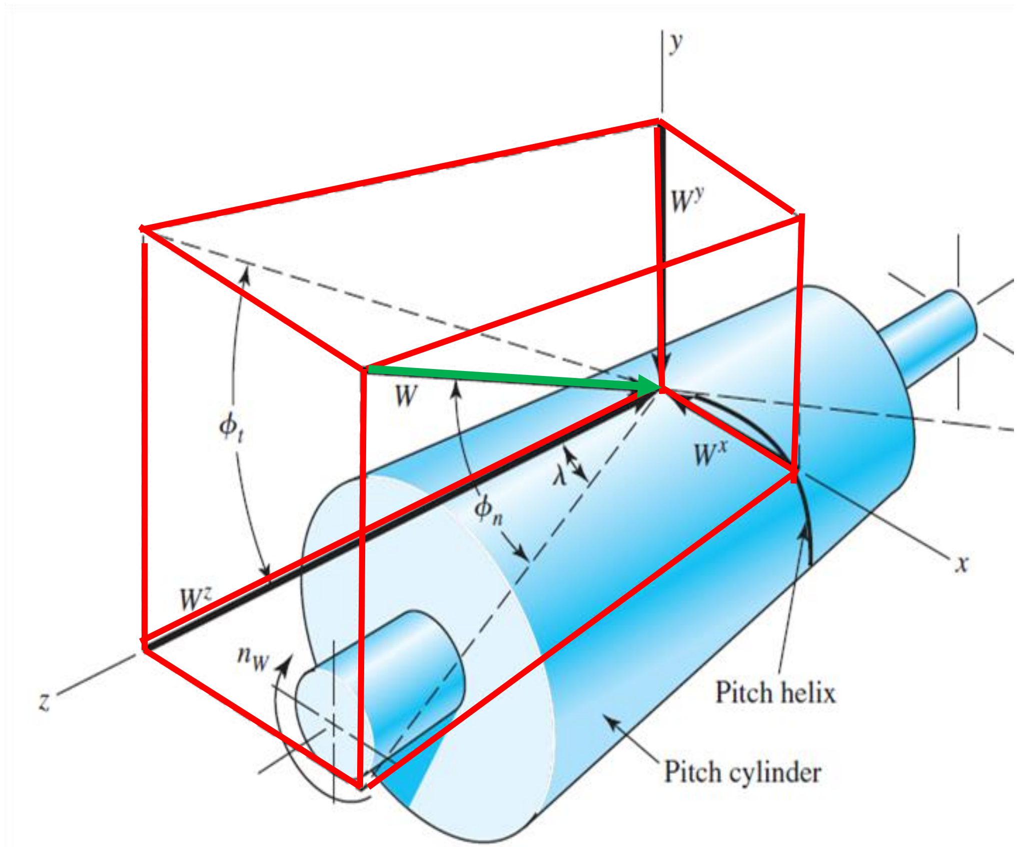


Right-hand Worm Gear



13.17 Force Analysis - Worm Gearing

Figure 13.40: Drawing of the pitch cylinder of a worm, showing the forces exerted upon it by the worm gear.



$$W_{Wt} = -W_{Ga} = W^x$$

$$W_{Wr} = -W_{Gr} = W^y$$

$$W_{Wa} = -W_{Gt} = W^z$$

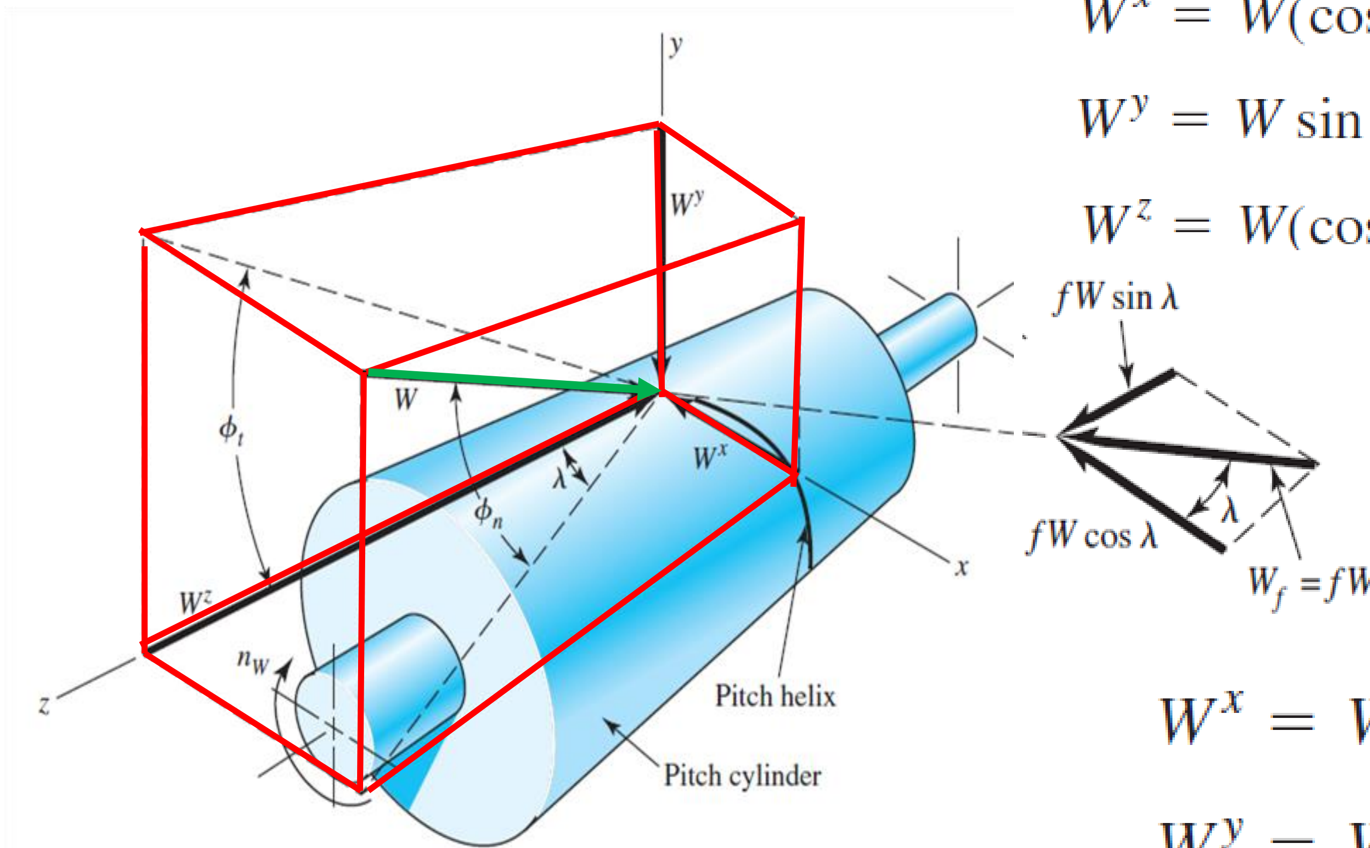
$$W^x = W \cos \phi_n \sin \lambda$$

$$W^y = W \sin \phi_n$$

$$W^z = W \cos \phi_n \cos \lambda$$

13.17 Force Analysis - Worm Gearing

Figure 13.40: Drawing of the pitch cylinder of a worm, showing the forces exerted upon it by the worm gear.



$$W^x = W(\cos\phi_n \sin\lambda + f\cos\lambda)$$

$$W^y = W\sin\phi_n$$

$$W^z = W(\cos\phi_n \cos\lambda - f\sin\lambda)$$

$$W^x = W\cos\phi_n \sin\lambda$$

$$W^y = W\sin\phi_n$$

$$W^z = W\cos\phi_n \cos\lambda$$

13.17 Force Analysis - Worm Gearing

$$W_f = fW = \frac{fW_{Gt}}{f\sin\lambda - \cos\phi_n\cos\lambda}$$

$$W_{Wt} = W_{Gt}\frac{\cos\phi_n\sin\lambda + f\cos\lambda}{f\sin\lambda - \cos\phi_n\cos\lambda}$$

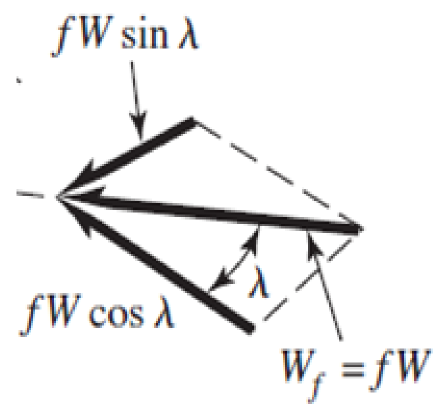
$$\eta = \frac{W_{Wt}\,(\text{without friction})}{W_{Wt}\,(\text{with friction})}$$

$$\eta = \frac{\cos\phi_n - f\tan\lambda}{\cos\phi_n + f\cot\lambda}$$

$$W^x = W(\cos\phi_n\sin\lambda + f\cos\lambda)$$

$$W^y = W\sin\phi_n$$

$$W^z = W(\cos\phi_n\cos\lambda - f\sin\lambda)$$



$$W^x = W\cos\phi_n\sin\lambda$$

$$W^y = W\sin\phi_n$$

$$W^z = W\cos\phi_n\cos\lambda$$

13.17 Force Analysis - Worm Gearing

Table 13–5

Recommended Pressure Angles and Tooth Depths for Worm Gearing

Lead Angle λ, deg	Pressure Angle ϕ_n, deg	Addendum a	Dedendum b_G
0–15	$14\frac{1}{2}$	$0.3683p_x$	$0.3683p_x$
15–30	20	$0.3683p_x$	$0.3683p_x$
30–35	25	$0.2865p_x$	$0.3314p_x$
35–40	25	$0.2546p_x$	$0.2947p_x$
40–45	30	$0.2228p_x$	$0.2578p_x$

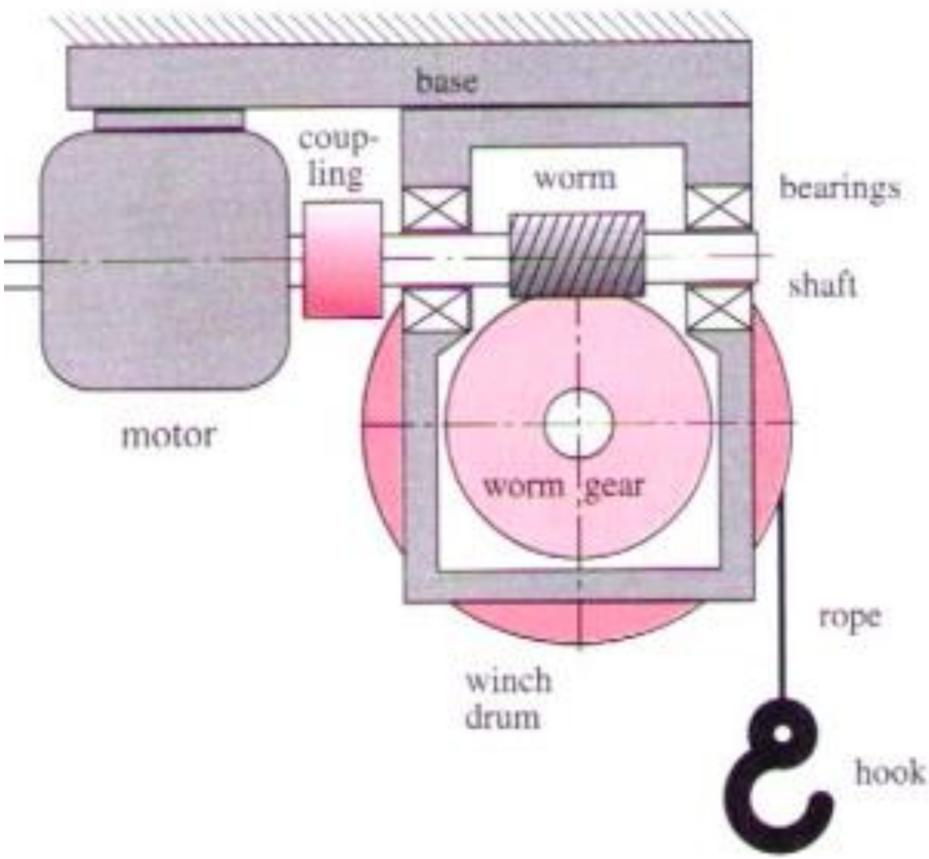
Table 13–6

Efficiency of Worm Gearsets for $f = 0.05$

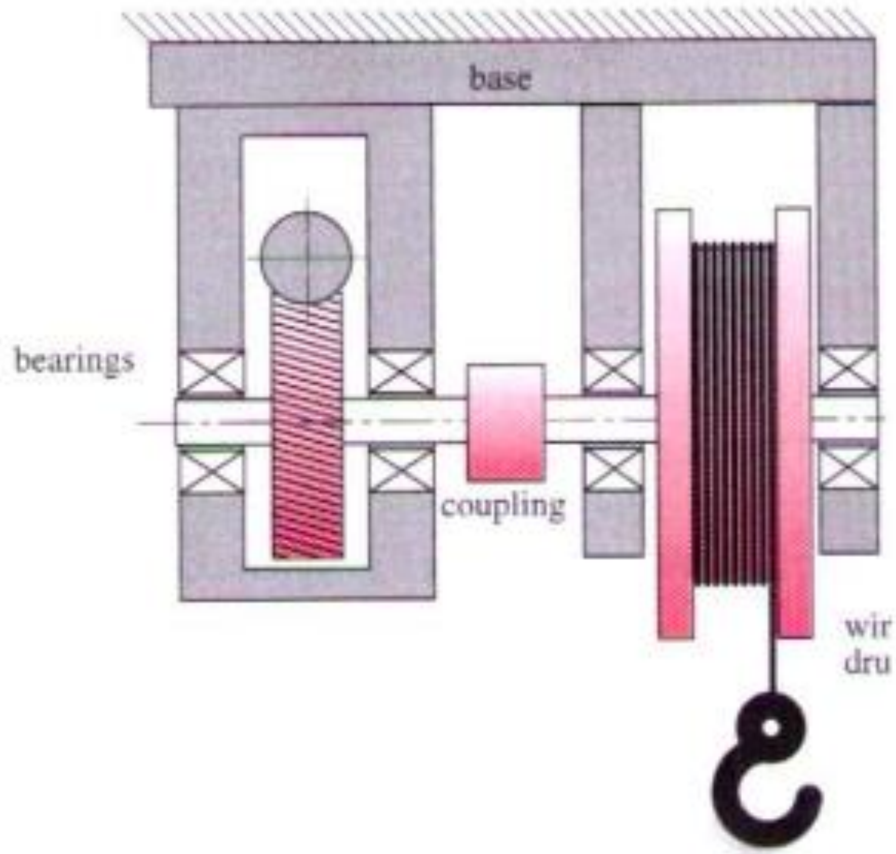
Lead Angle λ, deg	Efficiency η, %
1.0	25.2
2.5	45.7
5.0	62.6
7.5	71.3
10.0	76.6
15.0	82.7
20.0	85.6
30.0	88.7

$$\eta = \frac{\cos\phi_n - f\tan\lambda}{\cos\phi_n + f\cot\lambda}$$

13.17 Force Analysis - Worm Gearing



Front view



Side view

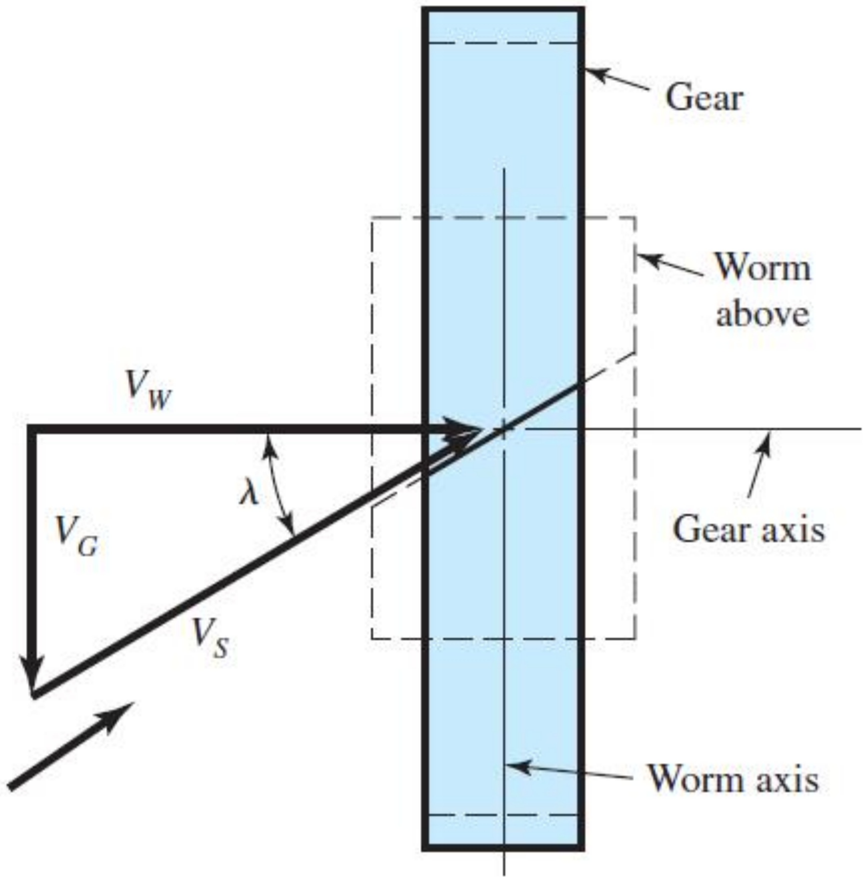
13.17 Force Analysis - Worm Gearing

Figure 13–41

Velocity components in worm gearing.

$$\mathbf{V}_W = \mathbf{V}_G + \mathbf{V}_S;$$

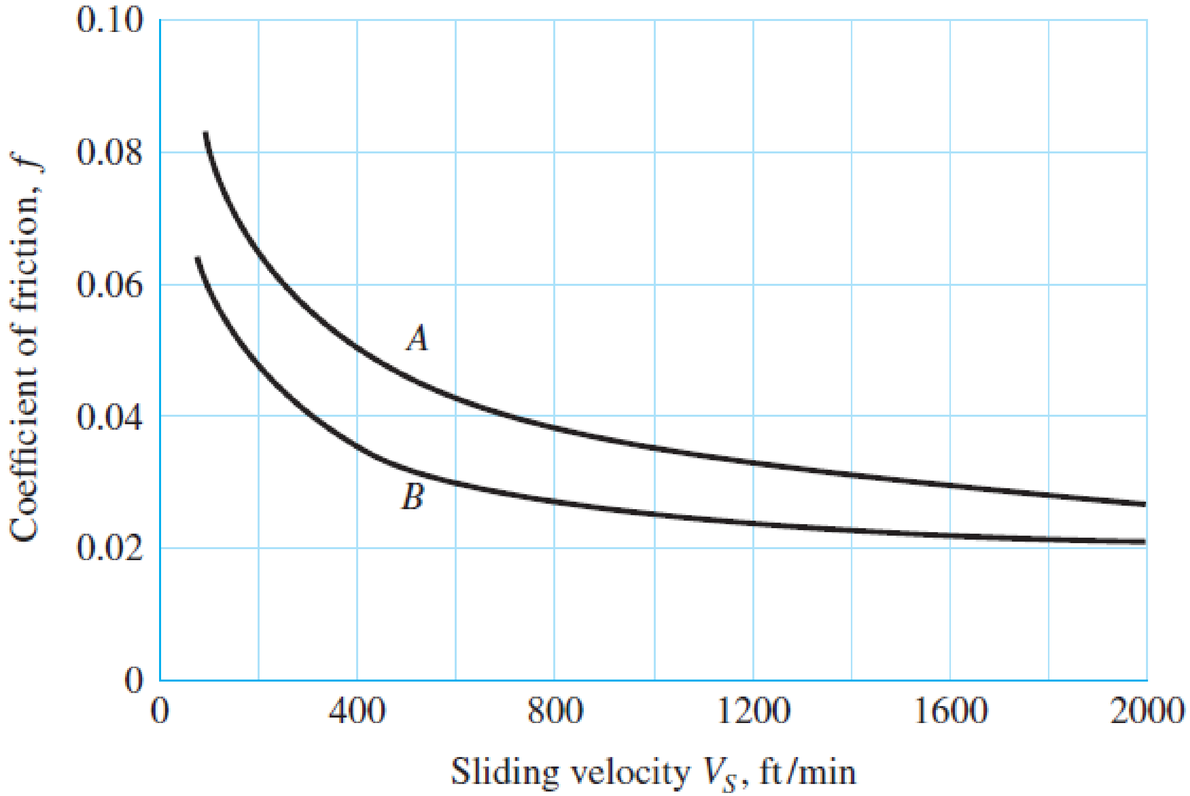
$$V_S = \frac{V_W}{\cos\lambda}$$



13.17 Force Analysis - Worm Gearing

Figure 13–42

Representative values of the coefficient of friction for worm gearing. These values are based on good lubrication. Use curve *B* for high-quality materials, such as a case-hardened steel worm mating with a phosphor-bronze gear. Use curve *A* when more friction is expected, as with a cast-iron worm mating with a cast-iron worm gear.



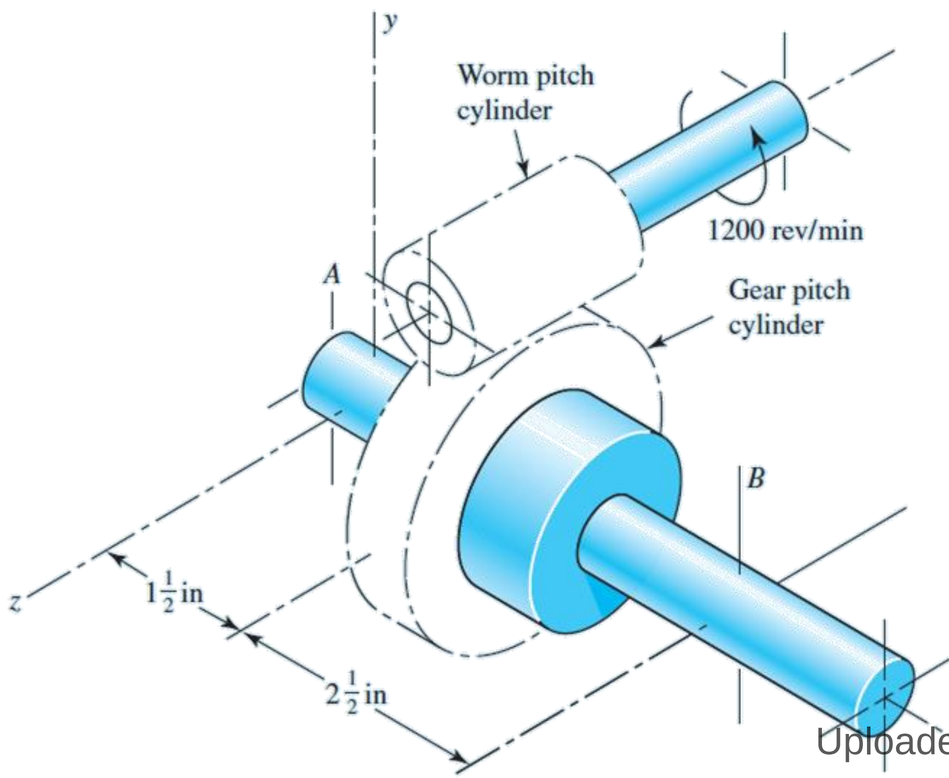
13.17 Force Analysis - Worm Gearing – Example 13.10

EXAMPLE 13–10

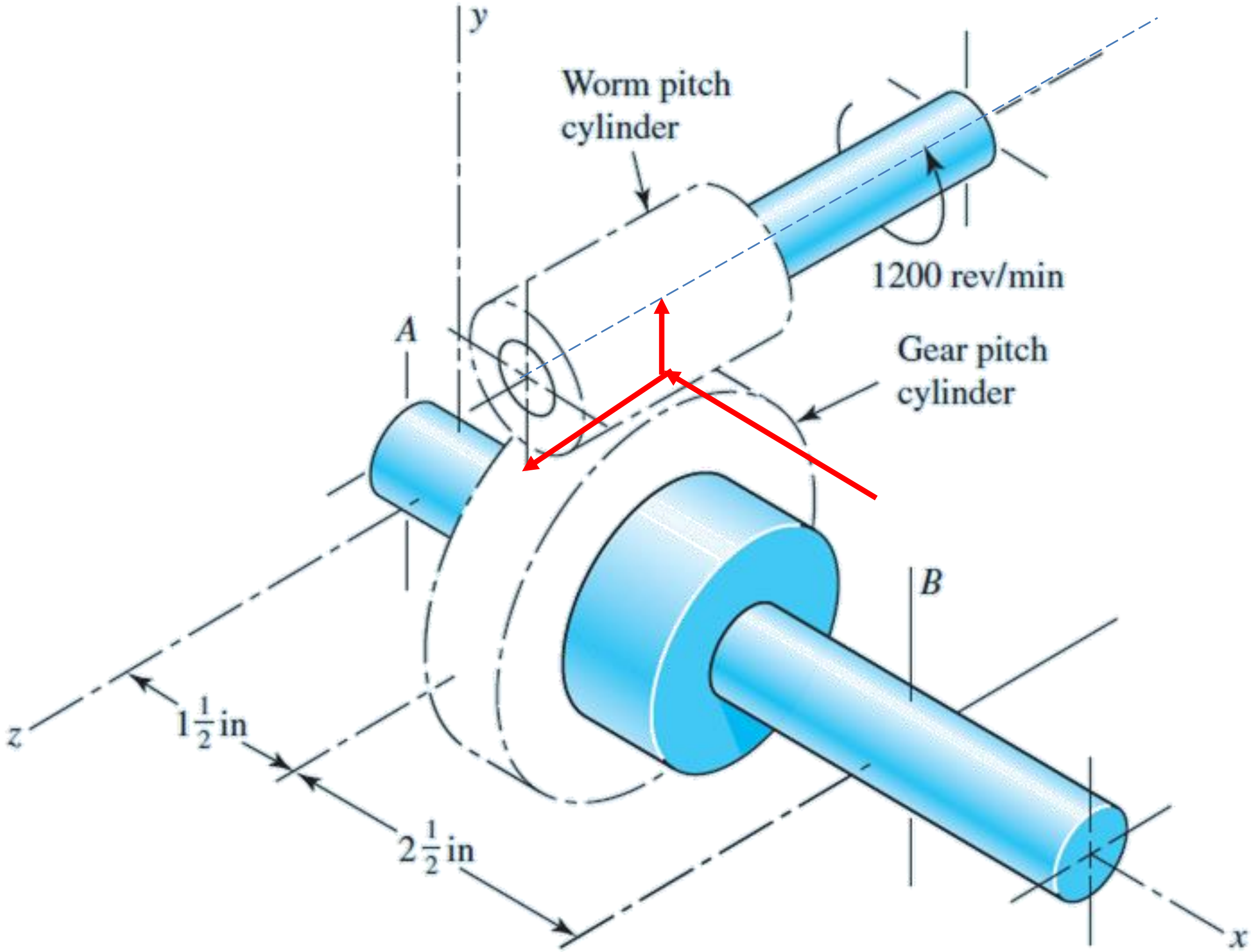
A 2-tooth right-hand worm transmits 1 hp at 1200 rev/min to a 30-tooth worm gear. The gear has a transverse diametral pitch of 6 teeth/in and a face width of 1 in. The worm has a pitch diameter of 2 in and a face width of $2\frac{1}{2}$ in. The normal pressure angle is $14\frac{1}{2}^{\circ}$. The materials and quality of the gearing to be used are such that curve *B* of Fig. 13–42 should be used to obtain the coefficient of friction.

(*a*) Find the axial pitch, the center distance, the lead, and the lead angle.

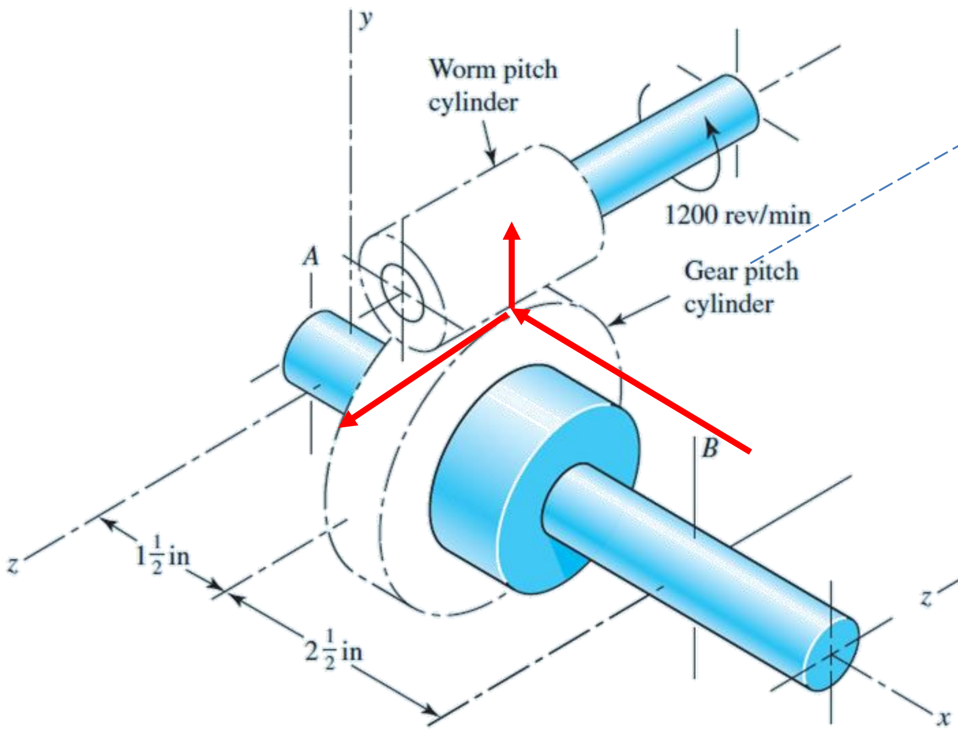
(*b*) Figure 13–43 is a drawing of the worm gear oriented with respect to the coordinate system described earlier in this section; the gear is supported by bearings *A* and *B*. Find the forces exerted by the bearings against the worm-gear shaft, and the output torque.

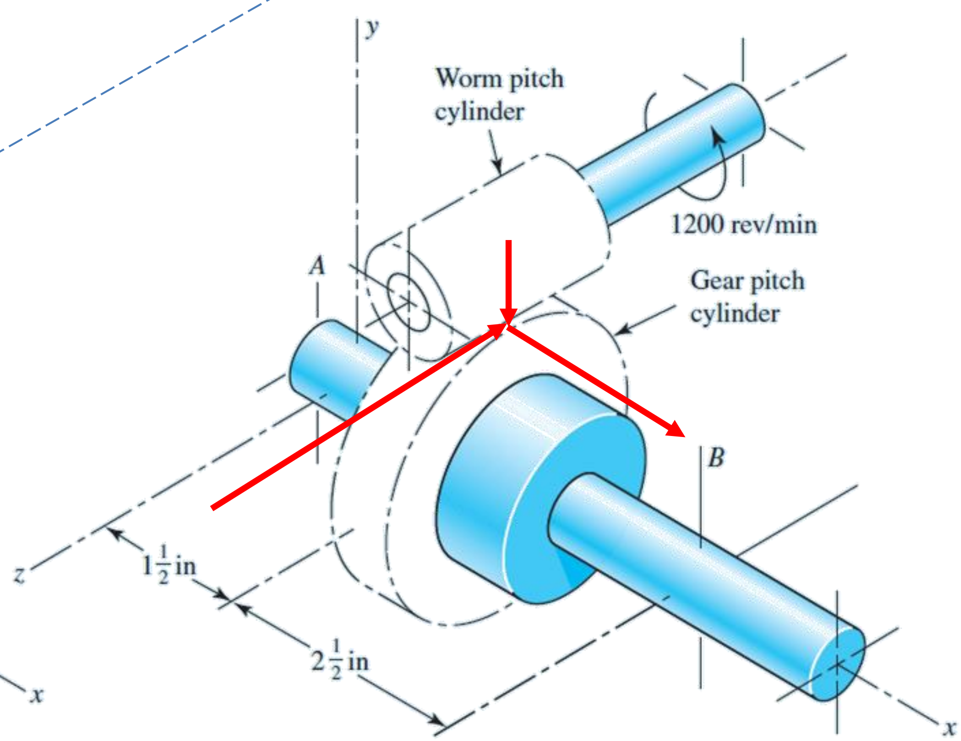


13.17 Force Analysis - Worm Gearing – Example 13.10

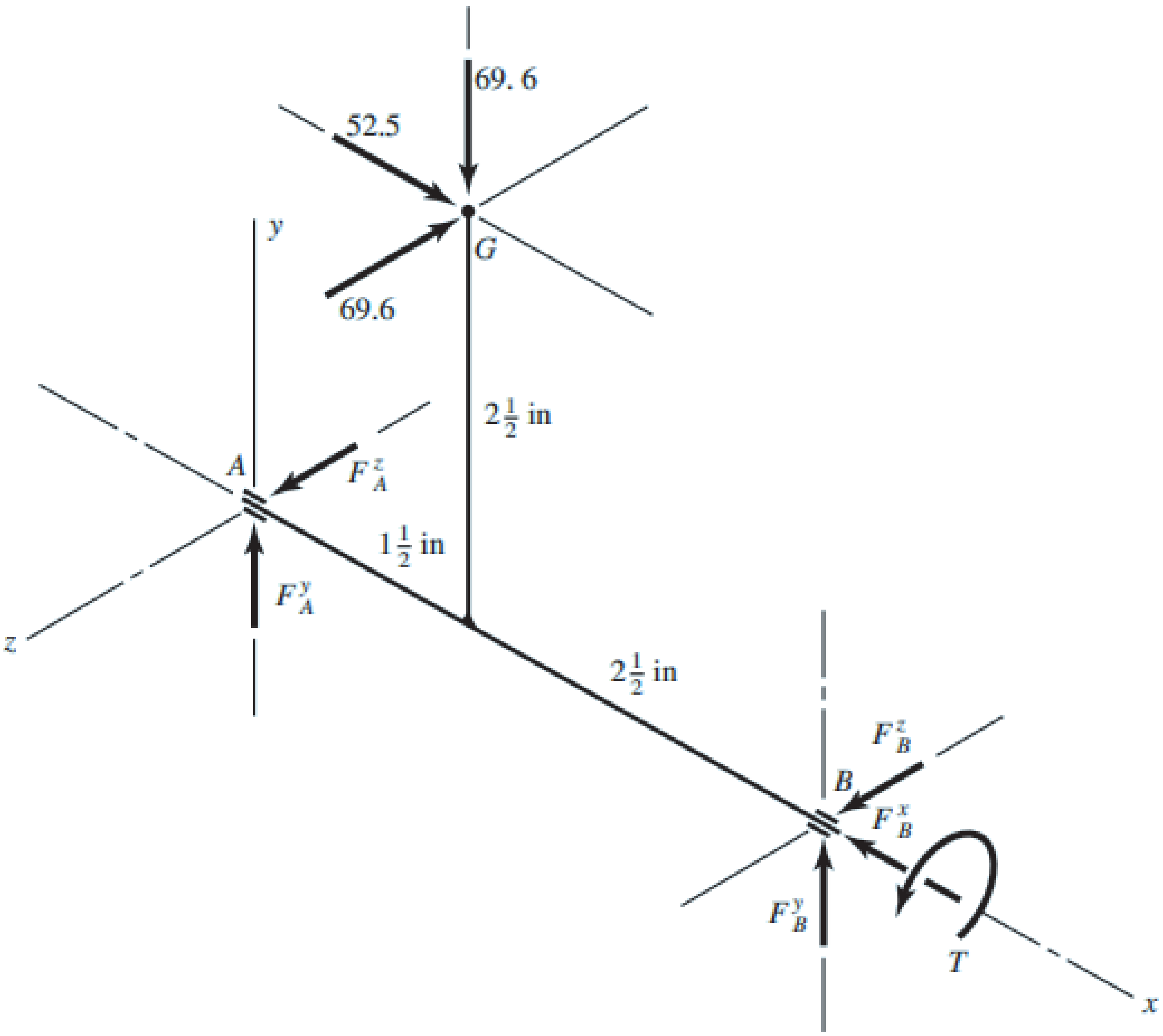


13.17 Force Analysis - Worm Gearing – Example 13.10





13.17 Force Analysis - Worm Gearing – Example 13.10



15.6 Worm Gearing – AGMA Equation

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Contact Stress

$$(W^t)_{\text{all}} = C_s D_m^{0.8} F_e C_m C_v$$

Bending Stress

$$\sigma_a = \frac{W_G^t}{p_n F_e y}$$

Wear Stress

$$(W_G^t)_{\text{all}} = K_w d_G F_e$$

15.6 Worm Gearing – AGMA Equation

BIRZEIT UNIVERSITY

Contact Stress

$$(W^t)_{\text{all}} = C_s D_m^{0.8} F_e C_m C_v$$

where C_s = materials factor

D_m = mean gear diameter, in

F_e = effective face width of the gear (actual face width, but not to exceed $0.67d_m$, the mean worm diameter), in

C_m = ratio correction factor

C_v = velocity factor

15.6 Worm Gearing – AGMA Equation

Contact Stress

$$(W^t)_{\text{all}} = C_s D_m^{0.8} F_e C_m C_v$$

$$C_s = 720 + 10.37C^3 \qquad C \le 3 \text{ in} \qquad (15\text{–}32)$$

For sand-cast gears,

$$C_s = \begin{cases} 1000 & C > 3 \quad D_m \le 2.5 \text{ in} \\ 1190 - 477 \log D_m & C > 3 \quad D_m > 2.5 \text{ in} \end{cases} \qquad (15\text{–}33)$$

For chilled-cast gears,

$$C_s = \begin{cases} 1000 & C > 3 \quad D_m \le 8 \text{ in} \\ 1412 - 456 \log D_m & C > 3 \quad D_m > 8 \text{ in} \end{cases} \qquad (15\text{–}34)$$

For centrifugally cast gears,

$$C_s = \begin{cases} 1000 & C > 3 \quad D_m \le 25 \text{ in} \\ 1251 - 180 \log D_m & C > 3 \quad D_m > 25 \text{ in} \end{cases} \qquad (15\text{–}35)$$

15.6 Worm Gearing – AGMA Equation

BIRZEIT UNIVERSITY

Contact Stress

$$(W^t)_{\text{all}} = C_s D_m^{0.8} F_e C_m C_v$$

The ratio correction factor C_m for gear ratio m_G is given by

$$C_m = \begin{cases} 0.02\sqrt{-m_G^2 + 40m_G - 76} + 0.46 & 3 < m_G \leq 20 \\ 0.0107\sqrt{-m_G^2 + 56m_G + 5145} & 20 < m_G \leq 76 \\ 1.1483 - 0.006\,58m_G & m_G > 76 \end{cases} \qquad (15\text{–}36)$$

The velocity factor C_v is given by

$$C_v = \begin{cases} 0.659\exp(-0.0011V_s) & V_s < 700 \text{ ft/min} \\ 13.31\,V_s^{-0.571} & 700 \leq V_s < 3000 \text{ ft/min} \\ 65.52\,V_s^{-0.774} & V_s > 3000 \text{ ft/min} \end{cases} \qquad (15\text{–}37)$$

15.6 Worm Gearing – AGMA Equation

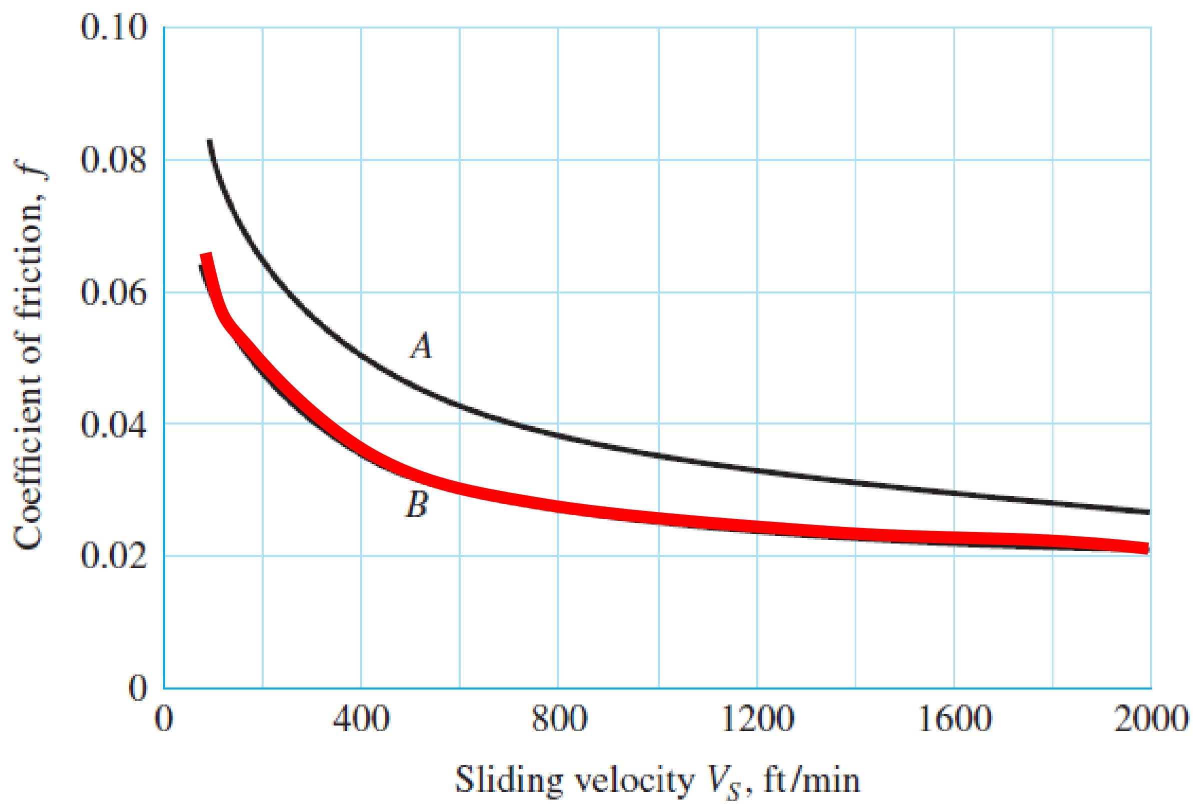
BIRZEIT UNIVERSITY

AGMA reports the coefficient of friction f as

$$f = \begin{cases} 0.15 & V_s = 0 \\ 0.124 \exp(-0.074 V_s^{0.645}) & 0 < V_s \le 10 \text{ ft/min} \\ 0.103 \exp(-0.110 V_s^{0.450}) + 0.012 & V_s > 10 \text{ ft/min} \end{cases} \quad (15\text{–}38)$$

Figure 13–42

Representative values of the coefficient of friction for worm gearing. These values are based on good lubrication. Use curve *B* for high-quality materials, such as a case-hardened steel worm mating with a phosphor-bronze gear. Use curve *A* when more friction is expected, as with a cast-iron worm mating with a cast-iron worm gear.



15.6 Worm Gearing – AGMA Equation

Now we examine some worm-gear mesh geometry. The addendum a and dedendum b are

$$a = \frac{p_x}{\pi} = 0.3183 p_x \qquad (15\text{–}39)$$

$$b = \frac{1.157 p_x}{\pi} = 0.3683 p_x \qquad (15\text{–}40)$$

The full depth h_t is

$$h_t = \begin{cases} \dfrac{2.157 p_x}{\pi} = 0.6866 p_x & p_x \geq 0.16 \text{ in} \\[2ex] \dfrac{2.200 p_x}{\pi} + 0.002 = 0.7003 p_x + 0.002 & p_x < 0.16 \text{ in} \end{cases} \qquad (15\text{–}41)$$

Table 15–8

Cylindrical Worm Dimensions Common to Both Worm and Gear*

		ϕ_n		
		14.5°	20°	25°
Quantity	Symbol	$N_W \leq 2$	$N_W \leq 2$	$N_W > 2$
Addendum	a	$0.3183p_x$	$0.3183p_x$	$0.286p_x$
Dedendum	b	$0.3683p_x$	$0.3683p_x$	$0.349p_x$
Whole depth	h_t	$0.6866p_x$	$0.6866p_x$	$0.635p_x$

*The table entries are for a tangential diametral pitch of the gear of $P_t = 1$.

15.6 Worm Gearing – AGMA Equation

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The worm outside diameter d_o is

$$d_o = d + 2a$$

The worm root diameter d_r is

$$d_r = d - 2b$$

The worm-gear throat diameter D_t is

$$D_t = D + 2a$$

The worm-gear root diameter D_r is

$$D_r = D - 2b$$

where D is the worm-gear pitch diameter.

The clearance c is

$$c = b - a \qquad (15\text{–}46)$$

The worm face width (maximum) $(F_W)_{\max}$ is

$$(F_W)_{\max} = 2\sqrt{\left(\frac{D_t}{2}\right)^2 - \left(\frac{D}{2} - a\right)^2} = 2\sqrt{2Da} \qquad (15\text{–}47)$$

which was simplified using Eq. (15–44). The worm-gear face width F_G is

$$F_G = \begin{cases} 2d_m/3 & p_x > 0.16 \text{ in} \\ 1.125\sqrt{(d_o + 2c)^2 - (d_o - 4a)^2} & p_x \le 0.16 \text{ in} \end{cases} \qquad (15\text{–}48)$$

15.6 Worm Gearing – AGMA Equation

Bending Stress

$$\sigma_a = \frac{W_G^t}{p_n F_e y}$$

where $p_n = p_x \cos \lambda$ and y is the Lewis form factor related to circular pitch. For $\phi_n = 14.5°$, $y = 0.100$; $\phi_n = 20°$, $y = 0.125$; $\phi_n = 25°$, $y = 0.150$; $\phi_n = 30°$, $y = 0.175$.

15.6 Worm Gearing – AGMA Equation

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Wear Stress – Buckingham Stress

$$(W_G^t)_{all} = K_w d_G F_e$$

where K_w = worm-gear load factor

d_G = gear-pitch diameter

F_e = worm-gear effective face width

Table 15–11

Wear Factor K_w for Worm Gearing

Source: Earle Buckingham, *Design of Worm and Spiral Gears*, Industrial Press, New York, 1981.

Material		Thread Angle ϕ_n			
Worm	**Gear**	**$14\frac{1}{2}°$**	**20°**	**25°**	**30°**
Hardened steel*	Chilled bronze	90	125	150	180
Hardened steel*	Bronze	60	80	100	120
Steel, 250 BHN (min.)	Bronze	36	50	60	72
High-test cast iron	Bronze	80	115	140	165
Gray iron†	Aluminum	10	12	15	18
High-test cast iron	Gray iron	90	125	150	180
High-test cast iron	Cast steel	22	31	37	45
High-test cast iron	High-test cast iron	135	185	225	270
Steel 250 BHN (min.)	Laminated phenolic	47	64	80	95
Gray iron	Laminated phenolic	70	96	120	140

*Over 500 BHN surface.

†For steel worms, multiply given values by 0.6.

15.6 Worm Gear Analysis

To reduce cooling load, use multiple-thread worms. Also keep the worm pitch diameter as small as possible.

Multiple-thread worms can remove the self-locking feature of many worm-gear drives. When the worm drives the gearset, the mechanical efficiency e_W is given by

$$e_W = \frac{\cos \phi_n - f \tan \lambda}{\cos \phi_n + f \cot \lambda} \tag{15–54}$$

With the gear driving the gearset, the mechanical efficiency e_G is given by

$$e_G = \frac{\cos \phi_n - f \cot \lambda}{\cos \phi_n + f \tan \lambda} \tag{15–55}$$

To ensure that the worm gear will drive the worm,

$$f_{\text{stat}} < \cos \phi_n \tan \lambda \tag{15–56}$$

$$W_W^t = W_G^t \frac{\cos \phi_n \sin \lambda + f \cos \lambda}{\cos \phi_n \cos \lambda - f \sin \lambda} \tag{15–57}$$

15.6 Worm Gear Analysis

The mechanical efficiency of most gearing is very high, which allows power in and power out to be used almost interchangeably. Worm gearsets have such poor efficiencies that we work with, and speak of, output power. The magnitude of the gear transmitted force W_G^t can be related to the output horsepower H_0, the application factor K_a, the efficiency e, and design factor n_d by

$$W_G^t = \frac{33\,000 n_d H_0 K_a}{V_G e} \tag{15–58}$$

We use Eq. (15–57) to obtain the corresponding worm force W_W^t. It follows that the worm and gear transmitted powers in hp are

$$H_W = \frac{W_W^t V_W}{33\,000} = \frac{\pi d_W n_W W_W^t}{12(33\,000)} \tag{15–59}$$

$$H_G = \frac{W_G^t V_G}{33\,000} = \frac{\pi d_G n_G W_G^t}{12(33\,000)} \tag{15–60}$$

13.17 Force Analysis - Worm Gearing – Example 13.10

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Try to design a worm-gear mesh with one of FOS (contact, wear, or bending) to connect an induction motor to a centrifugal pump. The motor speed is 1125 rev/min, and the velocity ratio is to be 11:1. The input power requirement is 15 hp. For this service K_a 1.25 is appropriate. Additionally, a design factor n_d of 1.1 is to be included to address other unquantifiable risks. Materials are High test cast iron for worm and cast bronze for the worm gear. Worm axial pitch = 1.75 in.