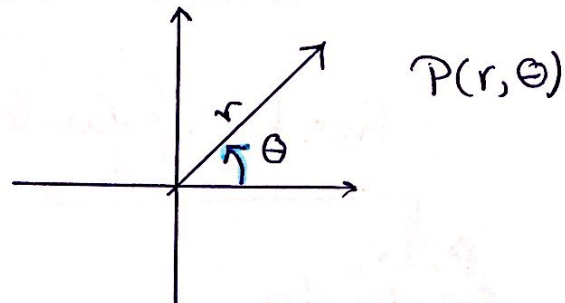


Polar Coordinates

- r : directed distance from origin
- θ : directed angle with positive x -axis



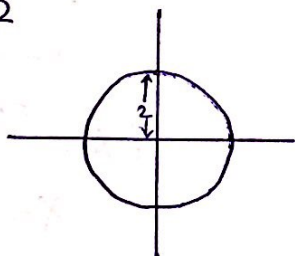
Polar equations & Graphs

معطيات a - دائرة نصف القطر a
 a - $r = a$

$$\theta = \theta_0$$

Circle of radius $|a|$ centered at origin.
line through origin making an angle θ_0 with the initial ray.

Ex:- $r = 2$



⚠ Note:
if you have :-

$$P_1 = P_2$$

$$(r_1, \theta_1) = (r_2, \theta_2)$$

That does not necessarily mean that $r_1 = r_2$ and $\theta_1 = \theta_2$

Relating Polar and Cartesian Coordinates

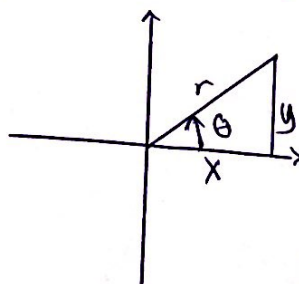
↳ Equations Relating Polar and Cartesian Coordinates

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$r^2 = x^2 + y^2$$

$$\tan \theta = \frac{y}{x}$$



Alaa Ebrahimi

How to solve Problems

First : finding Coordinates :

Polar Coordinates

- If the Question gives you a Cartesian coordinates and wants the Polar coordinates Then use:-

$$r = \sqrt{x^2 + y^2}$$

$$x = r \cos \theta \quad / \quad y = r \sin \theta$$

Ex: Point (1,1)

$$x=1, \quad y=1$$

$$r = \sqrt{2}$$

$$\begin{cases} x = \sqrt{2} \cos \theta \Rightarrow \frac{1}{\sqrt{2}} = \cos \theta \\ y = \sqrt{2} \sin \theta \Rightarrow \frac{1}{\sqrt{2}} = \sin \theta \end{cases}$$

$$\theta = \frac{\pi}{4}$$

$$\text{So } P(\sqrt{2}, \frac{\pi}{4})$$

Cartesian Coordinates

- If the Question gives you a Polar coordinates and wants the Cartesian coordinates Then you use:-

$$x = r \cos \theta$$

$$y = r \sin \theta$$

Ex: $P(-\sqrt{2}, \frac{\pi}{4})$

$$x = r \cos \theta = -\sqrt{2} \left(\frac{1}{\sqrt{2}} \right) = -1$$

$$y = r \sin \theta = (-\sqrt{2}) \left(\frac{1}{\sqrt{2}} \right) = -1$$

So the point is (-1, -1)

Hana Elbawi

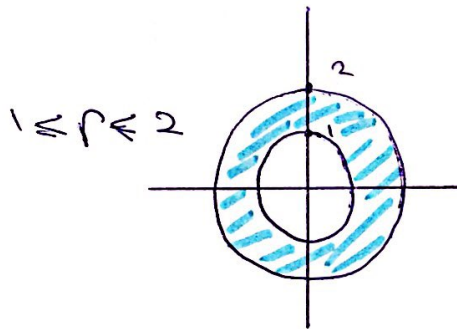
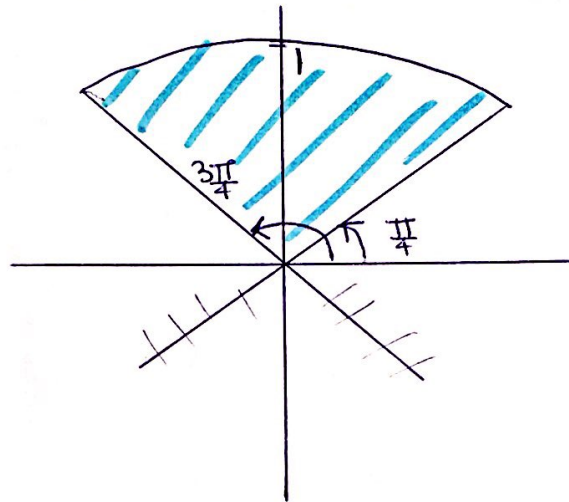
How to Graph in Polar Coordinates

Graph :-

$$\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$$

$$0 \leq r \leq 1$$

↑ Positive



Equations

Pol $\rightarrow$ Cart

Polar $\leftrightarrow$ Cart

Cart $\rightarrow$ Pol

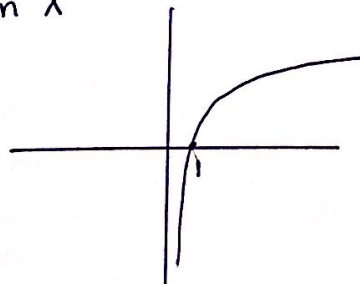
Ex: $r \sin \theta = \ln r + \ln \cos \theta$

$$r \sin \theta = \ln r \cos \theta$$

$$e^{r \sin \theta} = r \cos \theta$$

$$e^y = x$$

$$y = \ln x$$



Ex: $x^2 + xy + y^2 = 1$

$$r^2 \cos^2 \theta + r^2 \cos \theta \sin \theta + r^2 \sin^2 \theta = 1$$

$$r^2 + r^2 \cos \theta \sin \theta = 1$$

$$r^2 (1 + \cos \theta \sin \theta) = 1$$

Alaa Elbawi