Chapter 3:

Resistors in Series and Parallel

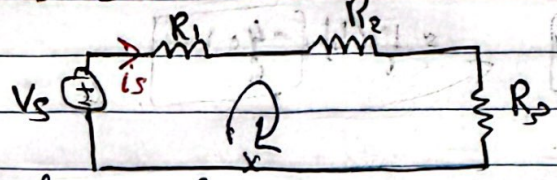
VDR

CDR

Δ − y

y − Δ

* Resistors in Series:



series elements have the same current.

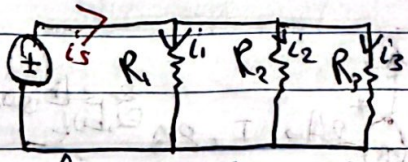
↳ KVL → $-V_s + R_1 i_s + R_2 i_s + R_3 i_s = 0$

$(R_1 + R_2 + R_3) i_s = V_s$

R_{eq}

$$R_{eq} = \sum_{i=1}^{N} R_i = R_1 + R_2 + R_3 + \cdots + R_N$$

* Resistors in Parallel:



Parallel elements have the same voltage.

$i_1 = \frac{V_s}{R_1}$, $i_2 = \frac{V_s}{R_2}$, $i_3 = \frac{V_s}{R_3}$

KCL → $I_s = i_1 + i_2 + i_3$

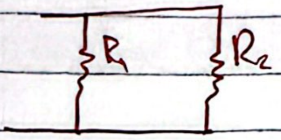
$= \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3}$

$I_s = V_s \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$

$\frac{1}{R_{eq}}$

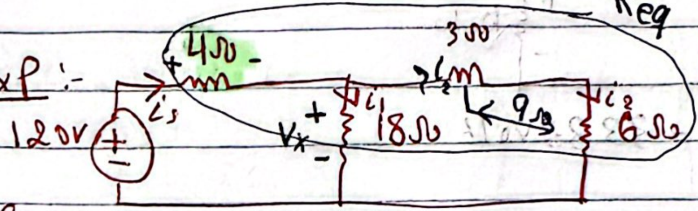
$$\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i}$$

→

* 

$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

* Exp :-



find i_s , i_1 , i_2 :

$R_{eq} = \{(6+3) // 18\} + 4$ (6+3: series; //18: Parallel; +4: series) , $6+3=9$ توالي

$9//18 = \frac{9 \cdot 18}{9+18} = 6$ توازي

$6+4 = \boxed{10}$ توالي

$= 10\,\Omega$

$\therefore V_s = R_{eq} \cdot I_s$

$120 = 10\, I_s \rightarrow \boxed{I_s = 12\,A}$

$\hookrightarrow V_x = 120 - 4(12)$

$= 120 - 48 = 72\,Volt$

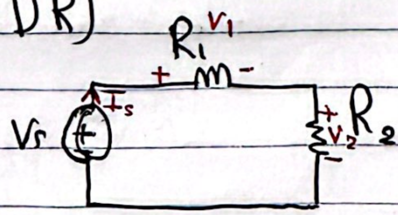
or $V_x = i_s \times (18 // 9)$

$= 12 \times 6 = 72\,Volt$

$\therefore i_1 = \frac{V_x}{18} = \frac{72}{18} = \boxed{4A}$

$i_2 = \frac{V_x}{9} = \frac{72}{9} = \boxed{8A}$

* Voltage Divider Rule: (VDR)



$V_1 \rightarrow V_s$

$V_2 \rightarrow V_s$

① $R_{eq} = R_1 + R_2$

هاي الطريقة بس للدارة البسيطة، لو في current source بتربطش

② $I_s = \frac{V_s}{R_{eq}}$ ($R_{eq} = R_1 + R_2$)

③ $V_1 = I_s R_1 \hookrightarrow \frac{R_1}{R_1+R_2} V_s \rightarrow \boxed{V_1 = V_s \cdot \frac{R_1}{R_1+R_2}}$

$V_2 = I_s R_2 \hookrightarrow \boxed{V_2 = V_s \cdot \frac{R_2}{R_1+R_2}}$ → VDR

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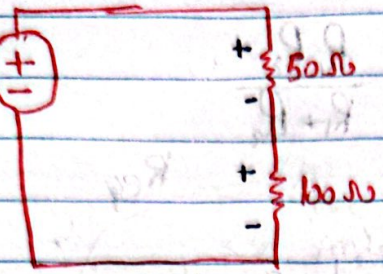
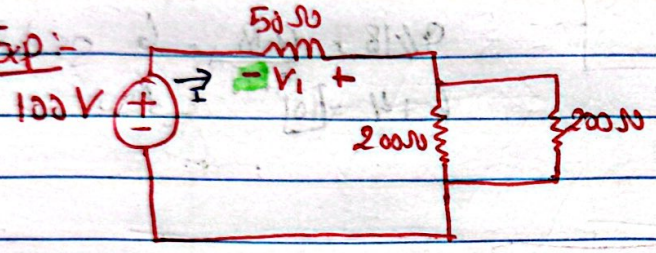
Exp :- 100V



$$V_{50\Omega} = \left(\frac{50}{50+100}\right) 100 = 33.33 \text{ Volt}$$

$$V_{100\Omega} = \left(\frac{100}{100+50}\right) .100 = 66.66 \text{ Volt}$$

Exp :-

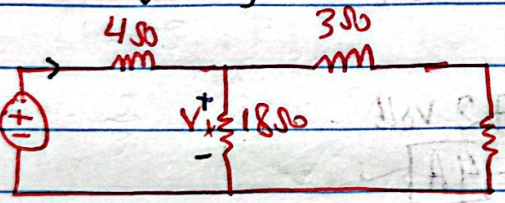


$V_1 = ?$

لإيجاد V_1 نستخدم قانون التجزئة

$$V_1 = \frac{50}{50 + (200 // 200)} * 100$$

Exp :-



$V_x = ?$

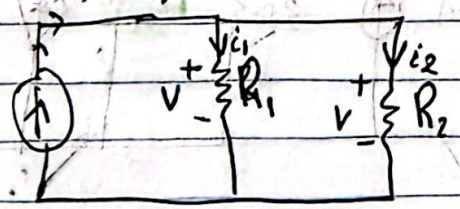
$$V_x = \frac{(18 // 9)}{(18 // 9) + 4} * 120 = \frac{6}{6+4} * 120 = \boxed{7.2 \text{ Volt}}$$

* Current Divider Rule: (CDR)

$V = R_1 i_1$, $V_2 = R_2 i_2$

$V = i_s \cdot R_{eq}$

$V = i_s \cdot \frac{R_1 R_2}{R_1 + R_2}$



the same voltage.

KCL

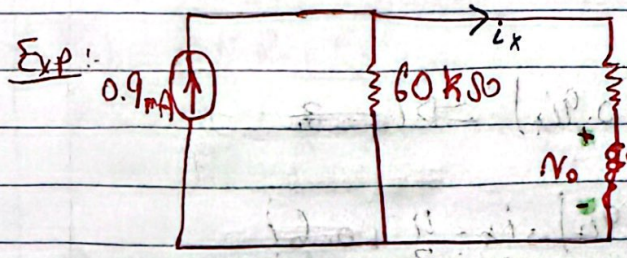
$i_s = i_1 + i_2$

$R_1 i_1 = \frac{R_1 R_2}{R_1 + R_2} i_s$

$$i_1 = \frac{R_2}{R_2 + R_1} * i_s$$

$$i_2 = \frac{R_1}{R_1 + R_2} * i_s$$

Exp:



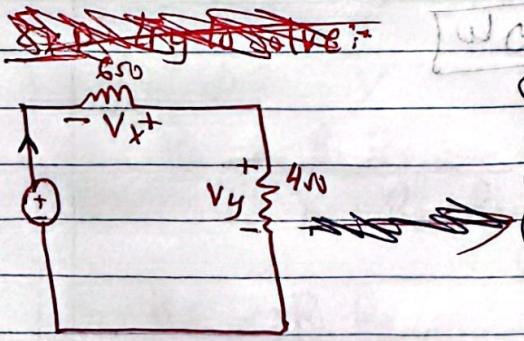
find V_o ?

$V_o = (80k)\, i_x$

$= (80k)\left(\frac{60k}{60k + (80+40)k} * 0.9m\right)$

$= 24$ Volt

Solve:

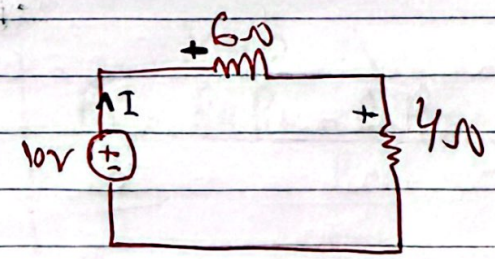


KVL → $-V_x + V_y - 10 = 0$

$-10 - (-6I) + 4I = 0$

$-10 + 6I + 4I = 0$

$I = 1A$

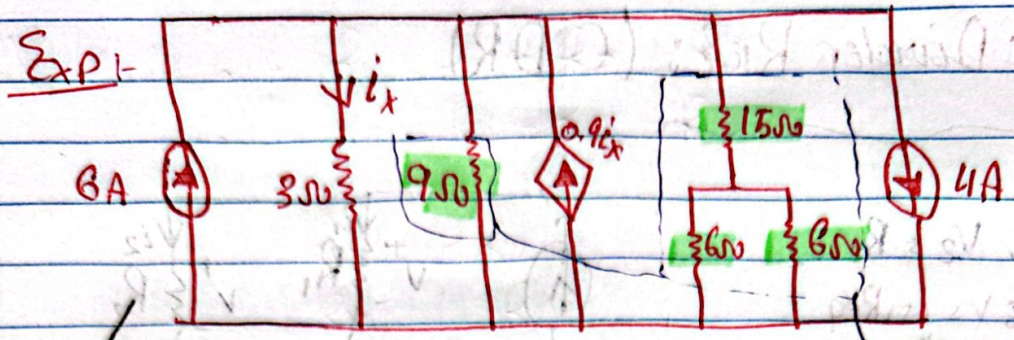


KVL

$-10 + 6I + 4I = 0$

$I = 1A$

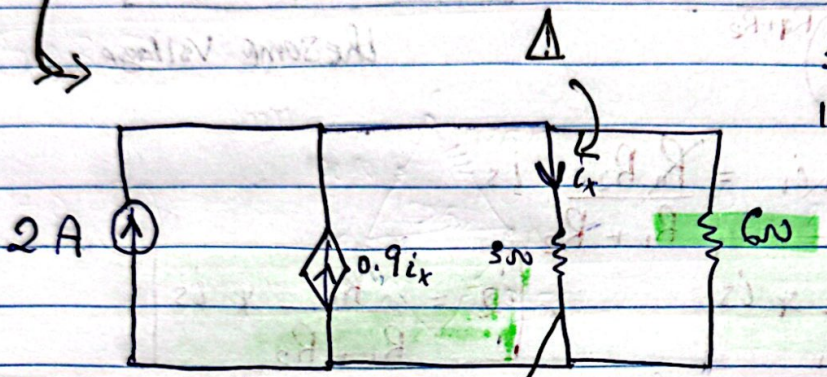
Ex1-



$6 // 6 = \frac{6 \times 6}{12} = 3$

$3 + 15 = 18$

$18 // 9 = \frac{18 \times 9}{18+9} = \boxed{6\Omega}$



i_x هنوع نوضحها توازي عادي مانوع

$$i_x = \frac{6}{6+3} * (2 + 0.9i_x) \Rightarrow i_x = 3$$

$$i_x = \frac{2}{3}(2 + 0.9i_x) \rightarrow i_x = \frac{4}{3} + 0.6i_x$$

$$-0.6i_x \quad -0.6i_x$$

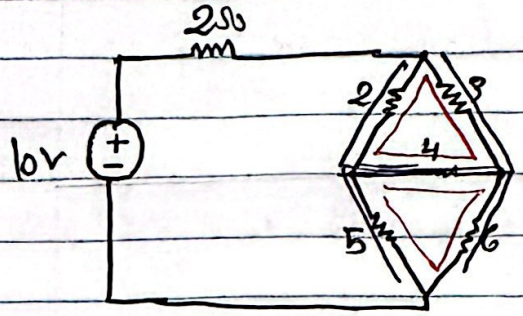
$$0.4i_x = \frac{4}{3} \rightarrow \boxed{i_x = 3.33A}$$

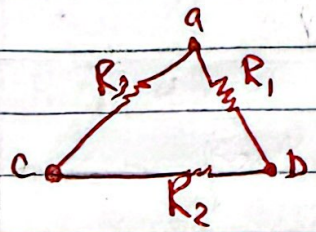
$$V = i_x R = \boxed{10V}$$

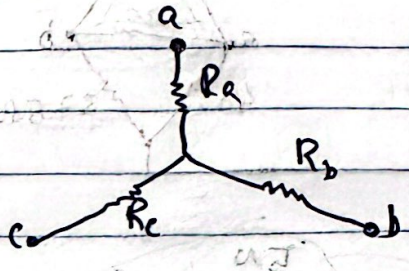
$$P_x = -V(0.9i_x) = -10(0.9 * 3.33)$$

$$= \boxed{30W}$$

Δ → Y
Y → Δ
→ transformation





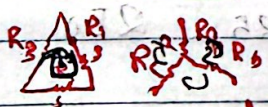


$$R_{ab} = R_1 // (R_2 + R_3)$$

$$R_{ab} = \frac{R_1 (R_2 + R_3)}{R_1 + R_2 + R_3} = R_a + R_b \quad \text{---} (1)$$

$$R_{bc} = \frac{R_2 (R_1 + R_3)}{R_1 + R_2 + R_3} = R_b + R_c \quad \text{---} (2)$$

$$R_{ca} = \frac{R_3 (R_1 + R_2)}{R_1 + R_2 + R_3} = R_c + R_a \quad \text{---} (3)$$



Δ → Y

$$R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

Y → Δ

$$R_1 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

$$R_2 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$

$$R_3 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

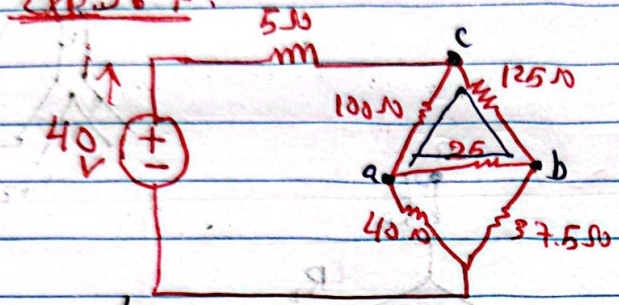
Note :

- if $R_1 = R_2 = R_3 = R_\Delta$
- if $R_a = R_b = R_c = R_Y$

$\therefore R_\Delta = 3R_Y$

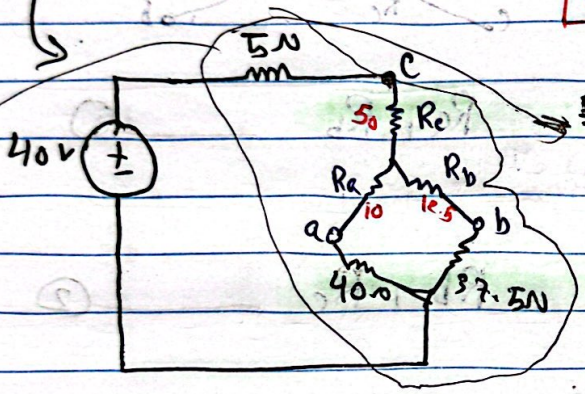
$R_Y = \frac{1}{3} R_\Delta$

Ex 3.7 :



P_{40V} ??

$\Delta \rightarrow Y$



~~5 + 50 = 55~~

~~55 + (10+40)~~

$R_a = \frac{25 \times 100}{100 + 25 + 125} = \frac{2500}{250} = 10\Omega$

$R_b = \frac{25 \times 125}{250} = 12.5\Omega$

$R_c = \frac{100 \times 125}{250} = 50\Omega$

$\frac{50 \times 50}{100} = 25$

$R_{eq} = \frac{5 + 50}{55} + \frac{(10+40) \,//\, (12.5+37.5)}{25} = 80\Omega$

$P_{80\Omega} = \frac{V^2}{R_{eq}} = \frac{40^2}{80}$ (abs.) , $P_{40V} = -\frac{40^2}{80}$ del.