12.3 Integrals Involving Exponential and Logarithmic Functions

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Integrals for Exponential function

$$\boxed{\int \underset{\uparrow}{e^{f(x)}}\, f'(x)\, dx = e^{f(x)} + c}$$

Exp ① $y = e^{x} \Rightarrow \int y' = \int e^{x}$

$y = \int e^{x}\, dx$

$\int e^{(x)}(1)\, dx = y = e^{x} + c$

$\int e^{x}\, dx = e^{x} + c$

$\int e^{y}\, dy = e^{y} + c$

② $\int \underline{2x\, e^{x^2}}\, dx = e^{x^2} + c$ ✓

تشتق

$y = x^{2}$

$y' = 2x$

$\frac{dy}{dx} = 2x$

$dy = 2x\, dx$

$\int e^{x^2}\, \boxed{2x\, dx}$

✓ $\int e^{y}\, dy = e^{y} + c$

$= e^{x^2} + c$

③ $\int \frac{\overset{f}{e^{\sqrt{x}}}}{2\sqrt{x}}\, dx = e^{\sqrt{x}} + c$ ✓

f'

$y = \sqrt{x} = x^{\frac{1}{2}}$

$2\sqrt{}$

f'

$$\int e^{\sqrt{x}} \frac{dx}{2\sqrt{x}}$$

$y=\sqrt{x} = x^{\frac{1}{2}}$

$y' = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2} \frac{1}{\sqrt{x}}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$dy = \frac{dx}{2\sqrt{x}}$

$$\int e^{y}\, dy = e^{y} + c$$

$$= e^{\sqrt{x}} + c$$

⑦ $\int \frac{x}{e^{x^2}}\, dx = \frac{1}{-2} \int \overset{f'}{-2x}\, e^{\overset{f}{-x^2}}\, dx$

$$= -\frac{1}{2} e^{-x^2} + c$$ ✓

$\overset{!}{=} \int x e^{-x^2} dx$

$y = -x^2$

$y' = -2x$

$\frac{dy}{dx} = -2x$

$dy = -2x\, dx$

$\frac{dy}{-2x} = dx$

$$\int \cancel{x}\, e^{-x^2} \frac{dy}{-2\cancel{x}}$$

$$= -\frac{1}{2} \int e^{y} dy$$

$e^{y} + c$

$$\frac{dy}{-2x} = dx$$

$$= -\frac{1}{2} e^{y} + C$$

$$= -\frac{1}{2} e^{-x^{2}} + C$$

⑤ $\int e^{\overset{\to f}{2x-1}} \, dx = \frac{1}{2} \int \overset{f'}{2} \, e^{\overset{f}{2x-1}} \, dx$

$$= \frac{1}{2} e^{2x-1} + C$$

or $\int e^{2x-1} \, dx$

$y = 2x-1$

$y' = 2$

$\frac{dy}{dx} = 2 \Rightarrow dy = 2dx$

$\frac{dy}{2} = dx$

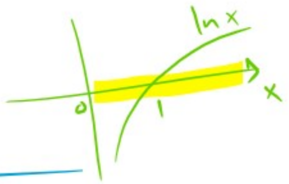
$\int e^{y} \frac{dy}{2}$

$\frac{1}{2} \int e^{y} dy = \frac{1}{2} e^{y} + c$

$= \frac{1}{2} e^{2x-1} + C$

Integrals for lograithmic Functions

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C$$



Exp ① $$\int \frac{\overset{f'}{1}}{\underset{f}{x}}\, dx = \ln|x| + c$$

② $$\int \frac{\overset{f'}{2}}{\underset{f}{2x+5}}\, dx = \ln|2x+5| + c$$

③ $$\frac{1}{2}\int \frac{2\overset{f'}{(x-1)}}{\underset{f}{x^2-2x+5}}\, dx = \frac{1}{2}\ln\left|x^2-2x+5\right| + c$$

$f' = 2x-2$

$= 2(x-1)$

or

$$\int \frac{x-1}{x^2-2x+5}\, dx$$

$y = x^2-2x+5$

$y' = 2x-2$

$\frac{dy}{dx} = 2x-2$

$$\int \frac{\cancel{x-1}}{} \quad \frac{dy}{}$$

$$\int \frac{\cancel{x-1}}{y} \quad \frac{dy}{2(\cancel{x-1})}$$

$$\frac{\quad}{dx} = 2x - 2$$

$$dy = (2x-2)\,dx$$

$$\frac{dy}{2(x-1)} = dx$$

$$\frac{1}{2}\int \frac{1}{y}\,dy = \frac{1}{2}\ln|y| + c$$

$$= \frac{1}{2}\ln|x^2 - 2x + 5| + c$$

④ $$\int \frac{\overset{f'}{3x^2 - 2x}}{\underset{f}{x^3 - x^2 + 1}}\,dx = \ln|f(x)| + c$$

$$= \ln|x^3 - x^2 + 1| + c$$

Exp $\overline{MR} = R'(x) = 6\,e^{0.01x}$

x : # of units produced

① Find Total Revenue

② Find Revenue if 100 units are produced ..

① $R(x) = \int \overline{MR}\,dx = \int 6\,e^{0.01x}\,dx$

$y = \boxed{0.01x}$ ←

$y' = 0.01$

$\frac{dy}{dx} = 0.01$

$dy = 0.01\, dx$

$\frac{dy}{0.01} = \boxed{dx}$ ←

$= \int 6 e^{y} \frac{dy}{0.01}$

$= \frac{6}{\frac{1}{100}} \int e^{y}\, dy$

$= 600 e^{y} + c$

$$\boxed{R(x) = 600 e^{0.01x} + c}$$ ✓

$R(x) = px$

$R(0) = p(0) = 0$

$\boxed{R(0) = 0}$

$R(0) = 600 e^{0.01(0)} + c$

$0 = 600 e^{0} + c$

$0 = 600(1) + c$

$\boxed{-600 = c}$

$$\boxed{R(x) = 600 e^{0.01x} - 600}$$

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$\int 6 e^{0.01x} dx = \frac{6}{0.01} \int \overset{f'}{0.01}\, e^{\overset{f}{0.01x}} dx$

$= \frac{6}{0.01} e^{0.01x} + c$

$= 600 e^{0.01x} + c$ ✓

$R(100) = 600 e^{\cancel{0.01}(\cancel{100})} - 600$

$= 600\, e \quad + c$

② $R(100) = 600\, e^{\cancel{0.01}(\cancel{100})} - 600$

$= 600\, e^{1} - 600$

$= 600(e - 1)$ $\qquad e \approx 2.718$

$= 600(2.718 - 1)$

$= 600(1.718) \approx \$\ 1030.8$

∫ exp. $\Rightarrow$ $\int e^{f(x)} f'(x)\, dx = e^{f(x)} + c$

$\int e^{y} dy = e^{y} + c$

∫ log. $\Rightarrow$ $\int \frac{f'(x)}{f(x)}\, dx = \ln|f(x)| + c$

$\int \frac{1}{y}\, dy = \ln|y| + c$