

10.9

22]

$$f(x) = \frac{2}{(1-x)^3}$$

$$f'(x) = 6(1-x)^{-4}$$

$$f''(x) = 24(1-x)^{-5}$$

$$f'''(x) = 120(1-x)^{-6}$$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = 2 + 6x + \frac{24}{2!} x^2 + \frac{120}{3!} x^3 + \dots$$

$$= 2 + 6x + 12x^2 + 20x^3 + \dots$$

$$= \sum_{n=0}^{\infty} (n+2)(n+1) x^n$$

$$\frac{2}{(1-x)^3} = \frac{d^2}{dx^2} \left[\frac{1}{1-x} \right] = \frac{d^2}{dx^2} \sum_{n=0}^{\infty} x^n$$

$$= \sum_{n=2}^{\infty} n(n-1) x^{n-2}$$

40]

$$\sqrt{1+x} = 1 + \frac{x}{2}$$

$$|x| < 0.01$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \dots$$

$$|e| < \left| -\frac{x^2}{8} \right| < \frac{(0.01)^2}{8} = 1.25 \times 10^{-5}$$

10]

$$\frac{1}{2-x}$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$\frac{1}{2} \left[\frac{1}{1-\frac{x}{2}} \right] = \frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{1}{2} x \right)^n$$

$$= \sum_{n=0}^{\infty} \frac{x^n}{2^{n+1}}$$

Haytham
Shehadeh

12]

$$x^2 \sin x$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$x^2 \sin x = x^2 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{(2n+1)!}$$

18] $\sin^2 x$

$$\sin^2 x = \frac{1}{2} [1 - \cos 2x]$$

$$= \frac{1}{2} - \frac{1}{2} \cos 2x = \frac{1}{2} - \frac{1}{2} \left[1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \dots \right]$$

$$= \frac{(2x)^2}{2 \cdot 2!} - \frac{(2x)^4}{2 \cdot 4!} + \frac{(2x)^6}{2 \cdot 6!} - \dots$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (2x)^{2n}}{2 \cdot (2n)!}$$

28] $\ln(1+x) - \ln(1-x)$

$$\int \frac{1}{1+x} - \int \frac{1}{1-x}$$

$$= \int 1 + x + x^2 + x^3 + \dots - \int 1 - x + x^2 - x^3 + \dots$$

$$= \cancel{x} + \cancel{\frac{x^2}{2}} + \cancel{\frac{x^3}{3}} + \cancel{\frac{x^4}{4}} + \dots - \cancel{x} + \cancel{\frac{x^2}{2}} - \cancel{\frac{x^3}{3}} + \cancel{\frac{x^4}{4}} - \dots$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots$$

$$= 2x + \frac{2x^3}{3} + \frac{2x^5}{5} + \dots$$

$$= \sum_{n=0}^{\infty} \frac{2x^{2n+1}}{(2n+1)}$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n = 1 - x + x^2 - x^3 + \dots$$

$$-\ln(1-x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots$$

35] $P_3(x) = x - \frac{x^3}{6}$

$\sin x$ at $x=0.1$

$f(x) = \sin x = P_3(x) + R_3(x)$

$f(x) = \sin x = x - \frac{x^3}{3!} + R_3(x)$

$|E| < \frac{(0.1)^4}{4!}$

$\approx 4.167 \times 10^{-6}$

Error = $|R_3(x)|$

$R_3(x) = \frac{f^{(4)}(c)}{4!} x^4$

37] $\sin x = x - \frac{x^3}{6}$

$E < 5 \times 10^{-4}$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$\frac{|x|^5}{5!} < 5 \times 10^{-4}$

$\sin x = P(x) + R(x)$

$|x| < \sqrt[5]{0.06} \approx 0.5698$

$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

41] $e^x = 1 + x + \frac{x^2}{2}$

$|x| < 0.1$

$f(x) = P_2(x) + R_2(x)$

$|R_2(x)| < \frac{e^c |x|^3}{3!}$

$f^{(3)}(x) = f^3(x)$

$< \frac{e^{0.1} (0.1)^3}{3!} = 1.87 \times 10^{-4}$

$f^3(x) < e^x \leq e^{0.1} < 3^{0.1}$