Exercises :

Q5: let X have the p.d.f $f(x) = \begin{cases} 4x^3 & , 0<x<1 \\ 0 & , \text{elsewhere} \end{cases}$

show that $Y = -2\ln X^{4}$ is $\chi^2(2)$.

→ Find the dist. of Y same Q1.76

- $G(y) = Pr(Y \le y) = Pr(-8\ln x \le y)$

$$= Pr\left(\ln x \ge \frac{-y}{8}\right)$$

$$= Pr\left(x \ge e^{-\frac{y}{8}}\right)$$

$$= \int_{e^{-\frac{y}{8}}}^{\infty} f(x)\,dx = \int_{e^{-\frac{y}{8}}}^{1} 4x^3\,dx = 1-e^{-\frac{y}{2}} , \quad y>0$$

$$\Rightarrow G(y) = \begin{cases} 0 & , y<0 \\ 1-e^{-\frac{y}{2}} & , y>0 \end{cases}$$

- Now find the $g(y) = \bar{G}(y)$ since X r.v continuous

$$g(y) = \begin{cases} \frac{1}{2} e^{-\frac{y}{2}} & , y>0 \\ 0 & , \text{else} \end{cases}$$

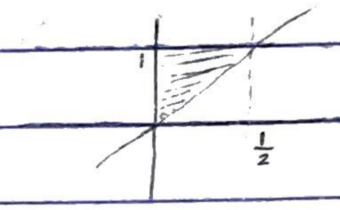
$\Rightarrow \beta = 2 , \alpha = 1 , r = 2\alpha = 2$.

So $Y \sim \chi^2(2)$.

Q7: let X_1, X_2 be a Random sample from the distribution having p.d.f $f(x) = \begin{cases} 2x & , 0<x<1 \\ 0 & , \text{elsewhere} \end{cases}$

Find $Pr\left(\frac{X_1}{X_2} \le \frac{1}{2}\right)$ and $Pr\left(X_1 X_2 \ge \frac{1}{4}\right)$.

$$f(x_1, x_2) = f(x_1) f(x_2) = \begin{cases} 4x_1 x_2 & , 0<x_1<1 , 0<x_2<1 \\ 0 & , \text{elsewhere} \end{cases}$$



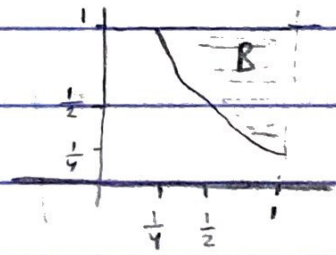
$$\rightarrow Pr\left(\frac{X_1}{X_2} \le \frac{1}{2}\right) = \iint_A f(x_1, x_2)\, dx_2\, dx_1$$

$$= \int_0^{\frac{1}{2}} \int_{2x_1}^{1} 4x_1 x_2 \, dx_2\, dx_1$$

$$= \int_0^{\frac{1}{2}} \left[2x_1 x_2^2 \Big|_{2x_1}^{1} \right] dx_1$$

$$= \int_0^{\frac{1}{2}} 2x_1 - 8x_1^3 \, dx_1$$

$$= x_1^2 - 2x_1^4 \Big|_0^{\frac{1}{2}} = \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$$



$$\rightarrow Pr\left(X_1 X_2 \ge \frac{1}{4}\right) = \iint_B f(x_1, x_2)\, dx_2\, dx_1$$

$$= \int_{\frac{1}{4}}^{1} \int_{\frac{1}{4x_1}}^{1} 4x_1 x_2 \, dx_2\, dx_1$$

$$= \frac{15}{16} - \frac{1}{4} \ln 2 .$$

check it

Q11: let X_1 and X_2 denote two i.i.d random variables, each from distribution that is $N(0,1)$. Find the p.d.f of $Y = X_1^2 + X_2^2$.

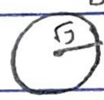
$$G(y) = Pr(Y \le y) = Pr(X_1^2 + X_2^2 \le y)$$

$$= \begin{cases} \iint_B f(x_1, x_2)\, dx_1 dx_2 & , y \ge 0 \\ 0 & , y < 0 \end{cases}$$

$$\Rightarrow f(x_1, x_2) = f(x_1) f(x_2) \quad \rightarrow N(0,1)$$

$$= \frac{1}{2\pi} e^{-\frac{x_1^2 + x_2^2}{2}}$$

B



use polar coordinates : let $r^2 = x_1^2 + x_2^2$

$$dx_1 dx_2 = r\, dr\, d\theta$$

$$\Rightarrow G(y) = \begin{cases} \int_0^{\sqrt{y}} \int_0^{2\pi} \frac{1}{2\pi} e^{-\frac{r^2}{2}} r\, d\theta\, dr & , y \ge 0 \\ 0 & , y < 0 \end{cases}$$

$$= \begin{cases} \int_0^{\sqrt{y}} r e^{-\frac{r^2}{2}} dr & , y \ge 0 \\ 0 & , y < 0 \end{cases}$$

$$= \begin{cases} \end{cases}$$