

Discussion 7.5

16 $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3} \quad \left(\frac{0}{0}\right)$

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{3x^2} = \lim_{x \rightarrow 0} \frac{-\sin x}{6x} = \lim_{x \rightarrow 0} \frac{-\cos x}{6} = -\frac{1}{6}$$

56 $\lim_{x \rightarrow \infty} x^{\frac{1}{\ln x}} \quad \left(\frac{0}{\infty}\right)$

$$\lim_{x \rightarrow \infty} e^{\ln x^{\frac{1}{\ln x}}} = \lim_{x \rightarrow \infty} e^{\frac{\ln x}{\ln x}} = e^1 = e$$

68 $\lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{\sqrt{\sin x}} \quad \left(\frac{0}{0}\right)$

$$\lim_{x \rightarrow 0^+} \frac{\frac{1}{2\sqrt{x}}}{\frac{\cos x}{2\sqrt{\sin x}}} = \lim_{x \rightarrow 0^+} \frac{1}{\cos x} \sqrt{\frac{\sin x}{x}} = \sqrt{\lim_{x \rightarrow 0^+} \frac{\sin x}{x}} = 1$$

OR $\lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{\sqrt{\sin x}} = \lim_{x \rightarrow 0^+} \sqrt{\frac{x}{\sin x}}$

$$= \lim_{x \rightarrow 0^+} \sqrt{\frac{1}{\frac{\sin x}{x}}} = \frac{1}{\sqrt{\lim_{x \rightarrow 0^+} \frac{\sin x}{x}}} = 1$$

$$(27) \lim_{\theta \rightarrow 0} \frac{\sin \theta}{3 - 1} \quad \left(\frac{0}{0}\right)$$

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{3} \cdot \frac{\ln 3}{1} \cdot \frac{\cos \theta}{1} = (1)(\ln 3)(1) = \ln 3$$

$$(60) \lim_{x \rightarrow 0^+} \left(1 + \frac{1}{x}\right)^x \quad (\infty^0)$$

$$\lim_{x \rightarrow 0^+} e^{\ln \left(1 + \frac{1}{x}\right)^x} = e^{\lim_{x \rightarrow 0^+} x \ln \left(1 + \frac{1}{x}\right)} \quad (0 \cdot \infty)$$

$$= e^{\lim_{x \rightarrow 0^+} \frac{\ln \left(1 + \frac{1}{x}\right)}{\frac{1}{x}}} \quad \left(\frac{\infty}{\infty}\right)$$

$$= e^{\lim_{x \rightarrow 0^+} \frac{-\frac{1}{x^2}}{1 + \frac{1}{x}}} = e^{\lim_{x \rightarrow 0^+} \frac{1}{1 + \frac{1}{x}}} = e^{\lim_{x \rightarrow 0^+} \frac{x}{x+1}} = e^0 = e^1 = e$$

$$(72) \lim_{x \rightarrow -\infty} \frac{2^x + 4^x}{5^x - 2^x} \quad \text{Divid by } 2^x$$

$$= \lim_{x \rightarrow -\infty} \frac{1 + \left(\frac{4}{2}\right)^x}{\left(\frac{5}{2}\right)^x - 1} = \frac{1 + 0}{0 - 1} = -1$$

