

Problem

Let $A = \{1, 2, 3, 4, 5\}$ and define a function

$F: P(A) \rightarrow \mathbf{Z}$ as follows: For all sets X in $P(A)$,

$$F(X) = \begin{cases} 0 & \text{if } X \text{ has an even} \\ & \text{number of elements.} \\ 1 & \text{if } X \text{ has an odd} \\ & \text{number of elements.} \end{cases}$$

Find the following:

a. $F(\{1, 3, 4\})$

b. $F(\emptyset)$

c. $F(\{2, 3\})$

d. $F(\{2, 3, 4, 5\})$

Step-by-step solution

Step 1 of 4

Consider the set,

$$A = \{1, 2, 3, 4, 5\}.$$

Define a function, $F(X) = \begin{cases} 0, & \text{if } X \text{ has an even number of elements} \\ 1, & \text{if } X \text{ has an odd number of elements} \end{cases}$.

(a)

The objective is to determine the value of $F(\{1, 3, 4\})$.

Since, $X = \{1, 3, 4\}$ have odd number of elements.

By the definition of $F(X)$,

$$\begin{aligned} F(\{X\}) &= F(\{1, 3, 4\}) \\ &= \boxed{1} \end{aligned}$$

Step 2 of 4

(b)

The objective is to determine the value of $F(\emptyset)$.

Since, $X = \{\emptyset\}$ have even number of elements.

By the definition of $F(X)$,

$$\begin{aligned} F(\{X\}) &= F(\emptyset) \\ &= \boxed{0} \end{aligned}$$

Since, the empty set has 0 elements (its cardinality is 0), and 0 is even number.

Step 3 of 4

(c)

The objective is to determine the value of $F(\{2,3\})$.

Since, $X = \{2,3\}$ have even number of elements.

By the definition of $F(X)$,

$$\begin{aligned} F(\{X\}) &= F(\{2,3\}) \\ &= \boxed{0} \end{aligned}$$

Step 4 of 4

(d)

The objective is to determine the value of $F(\{2,3,4,5\})$.

Since, $X = \{2,3,4,5\}$ have even number of elements.

By the definition of $F(X)$,

$$\begin{aligned} F(\{X\}) &= F(\{2,3,4,5\}) \\ &= \boxed{0} \end{aligned}$$