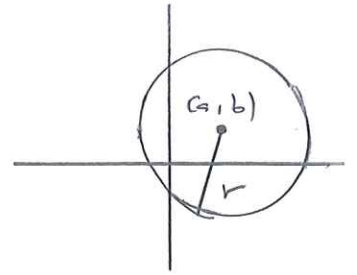


Circle, ellipse and parabolas, Hyperbola in the plane.

Circle : $(x-a)^2 + (y-b)^2 = r^2$

center (a, b) with radius $= r$



Ellipse with the horizontal major axis:

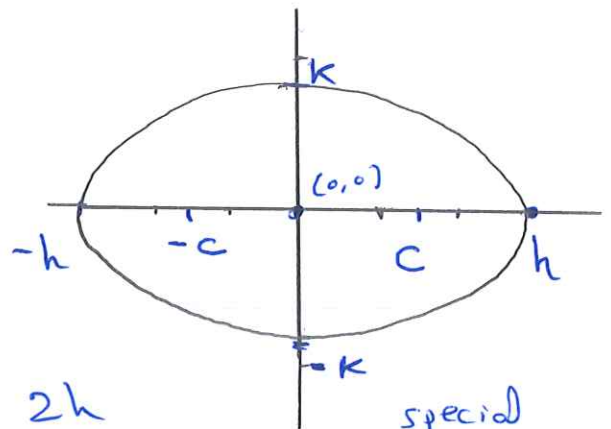
$$\frac{(x-a)^2}{h^2} + \frac{(y-b)^2}{k^2} = 1$$

1) Center (a, b)

2) length of the major axis is $2h$

3) length of the minor axis is $2k$

4) Distance between the center and either focus is C



special case

$(a, b) = \text{origin}$

with $C^2 = h^2 - k^2$, $h > k > 0$

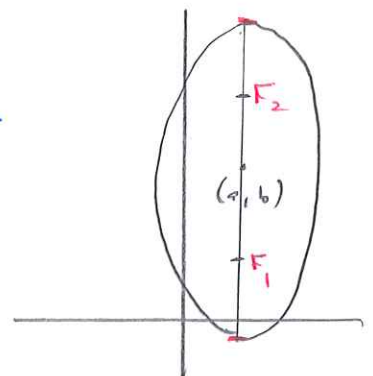
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Focus : $(a \pm c, b)$

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Ellipse with the vertical major axis:

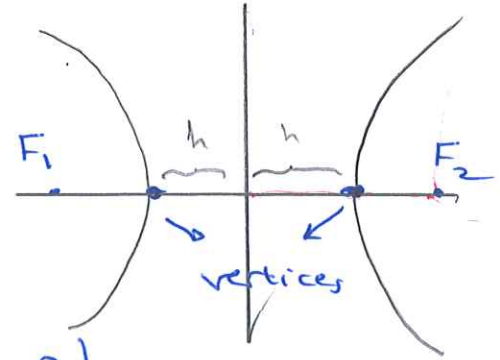
$$\frac{(x-a)^2}{k^2} + \frac{(y-b)^2}{h^2} = 1$$



(1), (2), (3) and (4) the same as above. Focus $(a, b \pm c)$

Hyperbola with the horizontal transverse axis:

$$\frac{(x-a)^2}{h^2} - \frac{(y-b)^2}{k^2} = 1$$



1) Center (a, b)

2) Distance between the vertices is $2h$

3) Distance between the foci is $2k$

4) $c^2 = h^2 + k^2$, Focus $(a \pm c, b)$

Hyperbola with the vertical transverse axis

$$\frac{(x-a)^2}{k^2} - \frac{(y-b)^2}{h^2} = 1$$

parabola with the Horizontal axis:

$$(y-b)^2 = 4p(x-a), p \neq 0$$

vertex (a, b) , Focus $(a+p, b)$

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parabola with vertical axis

$$(x-a)^2 = 4p(y-b), p \neq 0$$

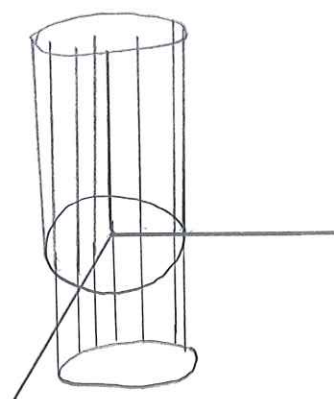
12.6 Cylinders and Quadric Surfaces:

Cylinders is a surface that is generated by moving a straight line along a given planar

Curve while holding the line parallel to a given fixed line.
generating curve

Example: $x^2 + y^2 = 1, z$.

We have a circular cylinder where the generating curves are circles.

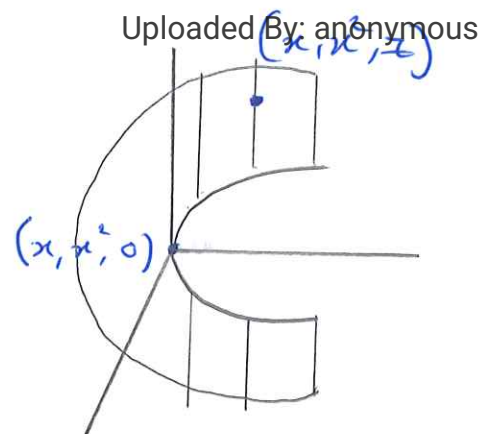


Example Find an equation for the cylinder made by lines parallel to the z -axis that pass through the parabola

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$$y = x^2, z = 0$$

— This is called the cylinder $y = x^2$.



Quadric Surfaces

"إسطوانات"

A quadric surface is the graph in space of a second-degree equation in x , y , and z .

We focus on $Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz = E$

where A, B, C, D and E are constants.

spheres are special cases "كروية"

Example ① The ellipsoid. $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

If $x=0$, $\frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, ellipse in yz -plane

If $y=0$, $\frac{x^2}{a^2} + \frac{z^2}{c^2} = 1$, ellipse in xz -plane

If $z=0$, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, ellipse in xy -plane

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If $a=b=c \iff x^2 + y^2 + z^2 = a^2$ (sphere)

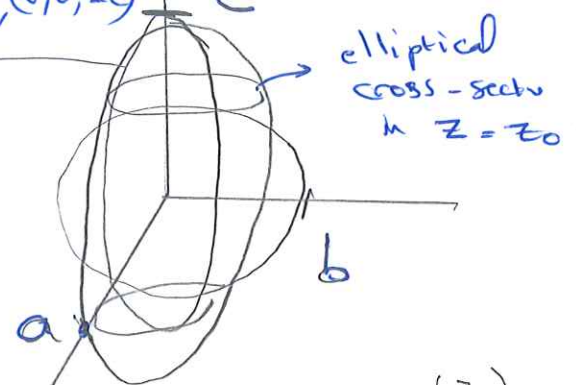
Coordinate axes at $(\pm a, 0, 0)$, $(0, \pm b, 0)$, $(0, 0, \pm c)$

بالنسبة لمحاور

المحاور الثلاثة

American football.

$$\frac{x^2}{a^2} + \frac{z^2}{c^2} = 1$$



(2)

Example ②: The Hyperbolic paraboloid
قسطر هائبيكولويدي

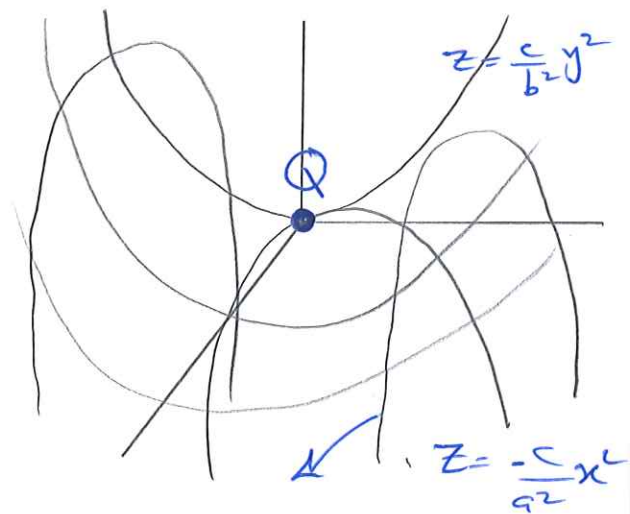
$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z}{c}, \quad c > 0$$

If $x=0$: $\frac{y^2}{b^2} = \frac{z}{c} \Rightarrow z = \frac{c}{b^2} y^2$
(parabola in yz -plane)

If $y=0$: $z = -\frac{c}{a^2} x^2$ (parabola in xz -plane)

If $z = z_0 > 0$ (z is constant)
we have hyperbola

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z_0}{c}$$



$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1, \quad z_0 = c$$

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The point Q is called

saddle point

النقطة مثل سرج الفرس

قطع مكافئ بيضاوي الشكل
 ③ Elliptical Paraboloid.

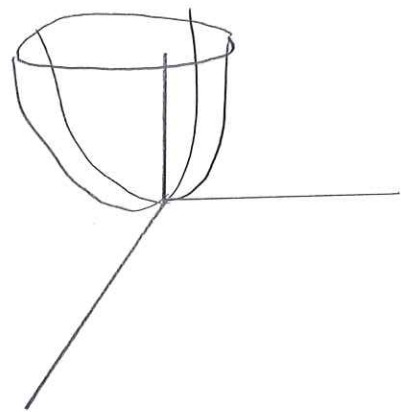
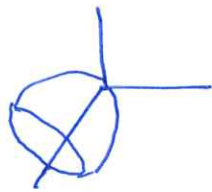
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$$

If $x=0 \Rightarrow z = \frac{c}{b^2} y^2$ (parabola) in yz -plane

If $y=0 \Rightarrow z = \frac{c}{a^2} x^2$ (parabola) in xz -plane

If $z=c \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (ellipse in xy -plane)

Example: $\frac{y^2}{a^2} + \frac{z^2}{b^2} = \frac{x}{c}$



④ Elliptical Cone
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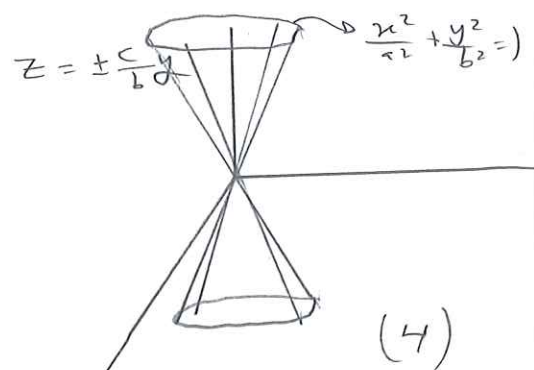
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$$

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If $x=0 \Rightarrow z^2 = \frac{c^2}{b^2} y^2 \Rightarrow z = \pm \frac{c}{b} y$ (Line)

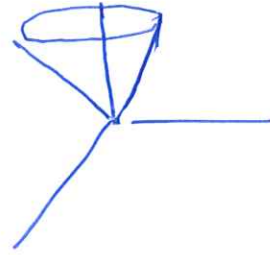
If $y=0 \Rightarrow z = \pm \frac{c}{a} x$ (lines)

If $z=c \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (ellipse)



Exempl: (a) $x^2 + y^2 = z^2 \Rightarrow$ Circular Cone 

(b) $z = \sqrt{x^2 + y^2} \geq 0$

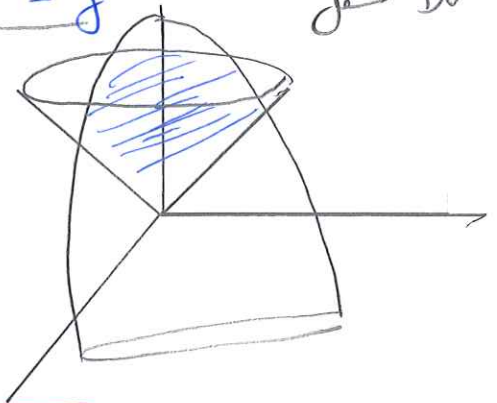


(c) $x^2 + y^2 = -z^2 \Leftrightarrow x^2 + y^2 + z^2 = 0$ (Point $(0,0,0)$)

(d) sketch the common region:

$z = \sqrt{x^2 + y^2}$
elliptical cone.

$z = 9 - x^2 - y^2$
elliptical paraboloid
down



(5) Hyperboloid of one Sheet

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

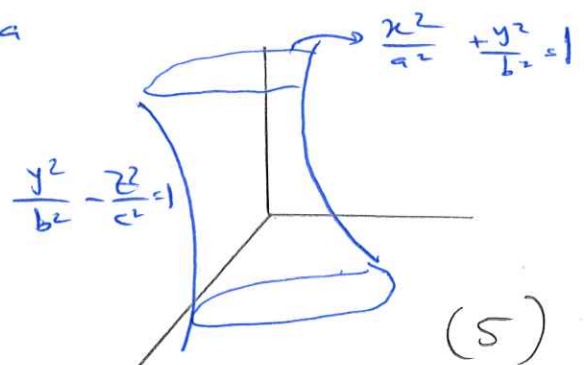
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If $x=0$: $\frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ (part of hyperbola in yz -plane)

If $y=0$: $\frac{x^2}{a^2} - \frac{z^2}{c^2} = 1$ (hyperbola)

If $z=0$: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (ellipse)

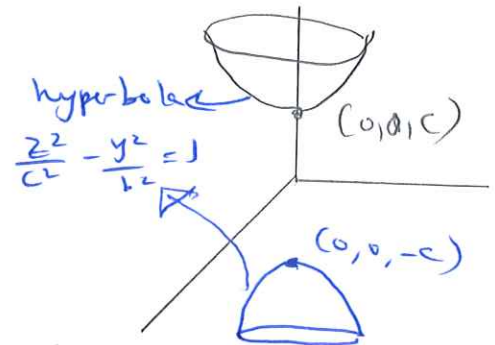


⑥ Hyperboloid of Two sheets

$$\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

If $z = c\sqrt{2} \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (ellipse)

If $x = 0 \Rightarrow \frac{z^2}{c^2} - \frac{y^2}{b^2} = 1$
(hyperbola in yz -plane)



If $y = 0 \Rightarrow \frac{z^2}{c^2} - \frac{x^2}{a^2} = 1$ (hyperbola in xz plane)

Q1) Discussion:

1) $x^2 + y^2 + 4z^2 = 10$ (d) (ellipsoid)

2) $z^2 + 4y^2 - 4x^2 = 4$ (i) (hyperboloid of one sheet)

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3) $9y^2 + z^2 = 16$ (a) (cylinder)

4) $y^2 + z^2 = x^2$ (g) (circular cone)

~~10~~

5) $x = y^2 - z^2$ (l) (hyperbolic paraboloid)

6) $x = -y^2 - z^2$ (e) (elliptical paraboloid)

7) $x^2 + 2z^2 = 8$ (b) (cylinder)

8) $z^2 + x^2 - y^2 = 1$ (j) (hyperboloid of one sheet)

9) $x = z^2 - y^2$ (k) (hyperbolic paraboloid)

10) $z = -4x^2 - y^2$ (f) (elliptical paraboloid)

11) $x^2 + 4z^2 = y^2$ (h) (cone)

12) $9x^2 + 4y^2 + 2z^2 = 36$ (c) (ellipsoid)