Leen Sabri

Q8 $\sum\limits_{n=1} (-1)^{n+1} \frac{10^n}{(n+1)!}$, $U_n = \frac{10^n}{(n+1)!}$ $\Rightarrow$ applay Def (Conv Abs) (conv Abs Test)

$(n+2)! = (n+2)(n+1)!$

$\sum |a_n| = \sum U_n = \sum\limits_{n=1} \frac{10^n}{(n+1)!} \xrightarrow{\text{Ratio Test}} \lim\limits_{n\to\infty} \frac{10^{n+1}}{(n+2)!} \cdot \frac{(n+1)!}{10^n} = \lim\limits_{n\to\infty} \frac{\cancel{10^n}\cdot 10}{(n+2)\cancel{(n+1)!}} \cdot \frac{\cancel{(n+1)!}}{\cancel{10^n}} = \lim\limits_{n\to\infty} \frac{10}{(n+2)} = 0 < 1$ $\therefore$ converge by (RT)

Hence $\sum\limits_{n=1} (-1)^{n+1} \frac{10^n}{(n+1)!}$ conv Abs by conv Abs Test

Q13 $\sum\limits_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}+1}{n+1}$ $\Rightarrow U_n = \frac{\sqrt{n}+1}{n+1}$

1) $U_n > 0$

2) $f(x) = \frac{\sqrt{x}+1}{x+1} \Rightarrow f'(x) = \frac{(x+1)\left(\frac{1}{2\sqrt{x}}\right) - (\sqrt{x}+1)(1)}{(x+1)^2} \Rightarrow \frac{x+1}{2\sqrt{x}} - (\sqrt{x}-1) = 0 \Rightarrow \frac{x+1}{2\sqrt{x}} = (\sqrt{x}-1) \Rightarrow 2\sqrt{x}(\sqrt{x}-1) = (x+1) \Rightarrow 2x - 2\sqrt{x} = x+1 \Rightarrow -2\sqrt{x} = x+1-2x \Rightarrow -2\sqrt{x} = -x+1 \Rightarrow 4x = x^2 - 2x + 1 \Rightarrow x^2 - 6x + 1 = 0$

$\Rightarrow$ للوهم بنذكل دبتشالع $f(x) < 0$ متناقصة

3) $\lim\limits_{n\to\infty} U_n = \lim\limits_{n\to\infty} \frac{\sqrt{n}+1}{n+1} = \frac{\infty}{\infty} \Rightarrow \lim\limits_{n\to\infty} \frac{\frac{1}{2\sqrt{n}}}{1} = \lim\limits_{n\to\infty} \frac{1}{2\sqrt{n}} = 0$ او مقارنة درجة البسط < درجة المقام

$\therefore \sum\limits_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}+1}{n+1}$ conv by Alternating series Test (AST)

$\lim \frac{x^n}{n!} = 0 \therefore \lim \frac{n!}{x^n} = \infty$

Q20 $\sum\limits_{n=1} (-1)^{n+1} \frac{n!}{2^n} \Rightarrow \sum |a_n| = \sum U_n = \sum \frac{n!}{2^n} \Rightarrow \lim\limits_{n\to\infty} \frac{n!}{2^n} = \infty$ $\therefore$ diverge by n^{th} term test $\therefore \sum\limits_{n=1} (-1)^{n+1} \frac{n!}{2^n}$ div

Q25 $\sum\limits_{n=1}^{\infty} (-1)^{n+1} \frac{1+n}{n^2} = \sum\limits_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{n^2} + \frac{1}{n}\right) \Rightarrow \sum |a_n| = \sum U_n = \sum\left(\frac{1}{n^2} + \frac{1}{n}\right) \Rightarrow \lim\limits_{n\to\infty} \underbrace{\left(\frac{1}{n^2} + \frac{1}{n}\right)}_{\text{p-test}} = 1 + \infty = \infty$ $\therefore$ diverg by n^{th} term test

$\therefore$ The series diverg by n^{th} term test

Q30 $\sum\limits_{n=1}^{\infty} (-1)^n \frac{\ln n}{n - \ln n} \Rightarrow f(x) = \frac{\ln x}{x - \ln x} \Rightarrow f'(x) = \frac{(x - \ln x)\frac{1}{x} - \ln x\left(1 - \frac{1}{x}\right)}{(x - \ln x)^2} = \frac{1 - \cancel{\left(\frac{\ln x}{x}\right)} - \ln x + \cancel{\left(\frac{\ln x}{x}\right)}}{(x - \ln x)^2} = \frac{1 - \ln x}{(x - \ln x)} \Rightarrow 1 - \ln x = 0 \Rightarrow \ln x = 1 \Rightarrow x = e$

$+ + + + + + \mid - - - - -$ (e)

$\therefore f(x) \overset{\text{متناقص}}{<} 0 \quad \forall x > e$ يعني بقدر أحكي انه متناقص لا n تكون كبيرة

إذا تحقق أول شرطين بدي أشوف الـ lim لـ U_n

$\lim\limits_{n\to\infty} \frac{\ln n}{n - \ln n} = \frac{\infty}{\infty} \Rightarrow \frac{\frac{1}{n}}{1 - \frac{1}{n}} = \frac{0}{1} = 0$ $\therefore$ converg by AST

ما بدي أشوف اذا conv Abs من خلال الـ lim لـ $\sum U_n$

$\lim \sum \frac{\ln n}{n - \ln n} \Rightarrow \sum \frac{\ln n}{n + \ln n} > \sum \frac{1}{n}$ and $\sum \frac{1}{n}$ div by P-series test $\therefore \sum \frac{\ln n}{n - \ln}$ div by DCT

Q39 $\sum\limits_{n=1}^{\infty} (-1)^n \frac{(2n)!}{2^n n! n} \Rightarrow$ Ratio Test $\Rightarrow \lim \left| \frac{\overset{(2n+2)!}{(2(n+1))!}}{2^{n+1}(n+1)!(n+1)} \cdot \frac{2^n n! (n+1)}{2n!} \right| = \lim \left| \frac{(2n+2)(2n+1)}{\cancel{2^n} 2 (n+1) \cancel{n!} \cancel{(n+1)}} \cdot \frac{\cancel{2^n} \cancel{n!} \cancel{(n+1)}}{2\, n!} \right| =$

Q42 $\sum\limits_{n=1}^{\infty} (-1)^n \left(\sqrt{n^2+n} - n\right)$

$\lim\left(\sqrt{n^2+n} - n\right) \cdot \frac{(\sqrt{n^2+n} + n)}{(\sqrt{n^2+n} + n)} = \lim \frac{n}{\sqrt{n^2+n} + n} = \lim \frac{n}{\left(\sqrt{n^2\left(1+\frac{1}{n}\right)}\right) + n} = \lim \frac{n}{n\sqrt{1+\frac{1}{n}} + n} = \lim \frac{n}{n\left(\sqrt{1+\frac{1}{n}} + 1\right)} = \lim \frac{1}{\sqrt{1+\frac{1}{n}} + 1} = \frac{1}{\sqrt{1+0} + 1} = \frac{1}{2} \neq 0$ $\therefore$ div by n^{th} term Test

Q50 $\sum\limits_{n=1}^{\infty} (-1)^{n+1} \frac{1}{10^n} \Rightarrow \frac{1}{10} - \frac{1}{10^2} + \frac{1}{10^3} - \frac{1}{10^4} + \frac{1}{10^5} + \dots$

$|\text{error}| < \left|\frac{1}{10^5}\right| = 0.00001$

Q54 $|\text{error}| < 0.001 \Rightarrow U_{n+1} < 0.001 \Rightarrow \frac{n+1}{(n^2+1)+1} < 0.001 \Rightarrow \frac{(n+1)^2+1}{n+1} > 1000 \Rightarrow (n+1)^2+1 > 1000(n+1) \Rightarrow n^2+2n+1+1 > 1000n + 1000$

$\Rightarrow n > \frac{998 + \sqrt{998^2 + 4(998)}}{2} \approx 998.9999 \Rightarrow n \geq 999$