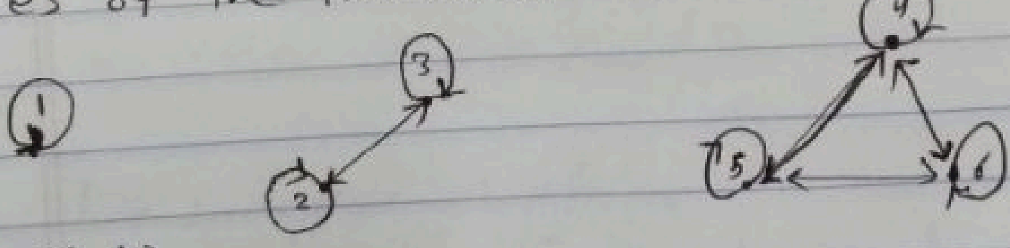
Q.50 let $S = \{1, 2, 3, 4, 5\}$ and let $\mathcal{A} = \{\{1,2\}, \{3\}, \{4,5\}\}$
list all ~~members~~ ordered pairs of the equivalence relation promised by Theorem 4.5 :-

$R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (1,2), (2,1), (4,5), (5,4)\}$

Q.51 The accompanying digraph represents an equivalance relation on $S = \{1, 2, 3, 4, 5, 6\}$. list the equivalence classes of the indicated relation :



$A_1 = \{(1,1)\}$
$A_2 = \{(2,2), (2,3), (3,2), (3,3)$

$A_3 = \{ (4,4), (5,5), (6,6), (4,5), (5,4), (4,6), (6,4)$
$(5,6), (6,5) \}$

* $[1] = \{1\}$
* $[2] = \{2,3\} = [3]$
* $[4] = [5] = [6] = \{4,5,6\}$

$R = \{ \{1\}, \{2,3\}, \{4,5,6\} \}$

$\boxed{Q.54}$ a) let $X = R \times R$. for each $(a,b) \in X$, let
$D_{(a,b)} = \{(x,y) \in X : x = a\} \cup \{(x,y) \in X : x^2 + y^2 = a^2 + b^2\}$
Is $\{ D_{(a,b)} : (a,b) \in X \}$ a partion of X?

Exercises 4.5 :

Q.69

$a^2 \equiv b^2 (\text{mod } n)$

$25 \equiv 9 (\text{mod } 4)$

$25 = 4k + 9$

$25 - 9 = 4k \Rightarrow \boxed{k = 4}$

$a^2 = 25$
$b^2 = 9$
$n = 4$
$a = \pm 5$
$b = \pm 3$
$n = 4$

Thats mean $25 \equiv 9 (\text{mod } 4)$ is Correct statment.

$\Rightarrow 5 \equiv 3 (\text{mod } 4)$

$5 = 4k + 3$

$2 = 4k$

$k = \frac{1}{2}$; k must be integer

so $5 \not\equiv 3 \text{ mod}(4)$

Q.70 $a \equiv b \text{ (mod}(n)) \Rightarrow \exists K \in \mathbb{Z} : a - b = Kn$ — ①

Exercises 4.6 :

Q.90 let $R = \{(1,2),(3,5),(2,2)\}$ and let $S = \{(2,1),(5,3),(5,1)\}$. Find each of the following:

a) → $R^{-1} = \{(2,1),(5,3),(2,2)\}$

→ $S^{-1} = \{(1,2),(3,5),(1,5)\}$

b) $S \circ R = \{(1,1),(3,3),(2,1),(3,1)\}$

c) $R \circ S = \{(2,2),(5,5),(5,2)\}$

d) $R^{-1} \circ S^{-1} = \{(1,1),(3,3),(1,3),(1,2)\}$

e) $S^{-1} \circ R^{-1} = \{(2,2),(5,5),(2,5)\}$

Q.91 For each of the following statments, Provide a proof or a counterexample :

a) if R and S are relations that are reflexive on a set X, then $R \circ S$ is reflexive on X :- False.

let $a \in X$ ⇒ since R is reflexive $(a,a) \in R$

⇒ since S is reflexive $(a,a) \in S$

$\Rightarrow$ The symmetric difference $R \oplus S$ contains all elements in R or S, but not these that are in both sets

$(a,a) \notin R \oplus S$

$\Rightarrow$ $R \oplus S$ is irreflexive (as we know that $(a,a) \in R \cap S$ for every element $a \in X$)

b) if R and S are symmetric relation's, so is $R \oplus S$:-

let $R = \{(1,1)\}$ and $S = \{(1,2), (2,1), (3,3)\}$ $\Rightarrow$ symmetric

$\Rightarrow R \oplus S = \{(2,1)\}$ $\Rightarrow$ not symmetric.

c) if R and S are transitive relations so is $R \oplus S$:-

let $R = \{(1,2), (2,1), (1,1)\}$ and $S = \{(1,1), (1,2)\}$ $\Rightarrow$ transitive

$\Rightarrow R \oplus S = \{(1,2), (1,1)\}$ $\Rightarrow$ not transitive

d) if $R \circ S = S \circ R$, then $S = R$:-

let $R = \{(1,4), (4,5), (6,6)\}$ and $S = \{(1,4), (4,5)\}$

$\Rightarrow R \circ S = \{(1,5)\}$, $S \circ R = \{(1,5)\}$

$R \circ S = S \circ R$ but $R \neq S$

e) if R, S and T are relations from A to A, then:
$R \circ (S \cup T) = (R \circ S) \cup (R \circ T)$:-

Suppose $(a,b) \in R \circ (S \cup T) \Leftrightarrow (a,c) \in (S \cup T)$ and $(c,b) \in R$
$\Leftrightarrow (a,c) \in S$ or $(a,c) \in T$ and $(c,b) \in R$
$\Leftrightarrow (a,c) \in S$ and $(c,b) \in R$ or $(a,c) \in T$ and $(c,b) \in R$
$\Leftrightarrow (a,b) \in (R \circ S)$ or $(a,b) \in (R \circ T)$
$\Leftrightarrow (a,b) \in (R \circ S) \cup (R \circ T)$ ✓

f) if R, S and T are relations from A to A then
$R \circ (S \cap T) = (R \circ S) \cap (R \circ T)$:-

Suppose $(a,b) \in R \circ (S \cap T) \Leftrightarrow (a,c) \in (S \cap T)$ and $(c,b) \in R$
$\Leftrightarrow (a,c) \in S$ and $(a,c) \in T$ and $(c,b) \in R$
$\Leftrightarrow (a,c) \in S$ and $(c,b) \in R$ and $(a,c) \in T$ and $(c,b) \in R$
$\Leftrightarrow (a,b) \in (R \circ S)$ and $(a,b) \in (R \circ T)$
$\Leftrightarrow (a,b) \in (R \circ S) \cap (R \circ T)$ ✓

g) if R and S are symmetric relations, then $(R \circ S)^{-1} = R \circ S$

let $R = \{(1,1)\}$ and $S = \{(1,2), (2,1), (3,3)\} \Rightarrow$ symmetric

$\Rightarrow R \circ S = \{(2,1)\}$
$\Rightarrow (R \circ S)^{-1} = \{(1,2)\}$ $\Rightarrow R \circ S \neq (R \circ S)^{-1}$

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natural numbers ... each pair cong...

Q.92 Give an example to show that there are relations R and S such that $(RoS)^{-1} \neq R^{-1} o S^{-1}$

let $R = \{(2,3), (4,6), (3,3)\}$, and $S = \{(3,2), (6,4), (6,3)\}$

$\Rightarrow R^{-1} = \{(3,2), (6,4), (3,3)\}$, $S^{-1} = \{(2,3), (4,6), (2,6)\}$

$\Rightarrow (RoS) = \{(6,6), (6,3), (3,3)\}$

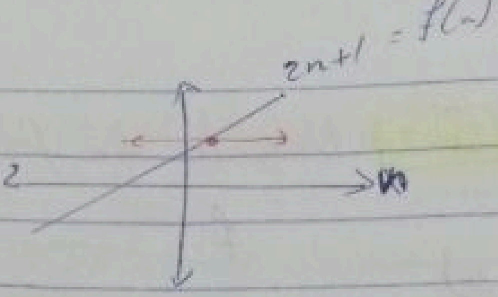
$\Rightarrow (RoS)^{-1} = \{(6,6), (3,6), (3,3)\}$

$\Rightarrow R^{-1} o S^{-1} = \{(2,2), (2,3), (2,4), (4,4)\}$

$\Rightarrow (RoS)^{-1} \neq R^{-1} o S^{-1}$ ✓

Q.93 let $R = \{(n,p) \in \mathbb{N} \times \mathbb{N} : n < p\}$ and let $S = \{(n,p) \in \mathbb{N} \times \mathbb{N} : n \text{ divides } p\}$ show that

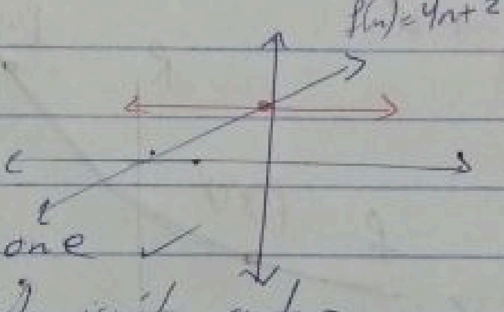
Exercises 5.1 :



Q.1 a) $f(n) = 2n+1$

$\Rightarrow f(n)$ is one to one

$\Rightarrow \{n, f(n)\} \in \mathbb{Z} \times \mathbb{Z} \longrightarrow \mathbb{Z} \neq \mathbb{R}$ so isn't onto

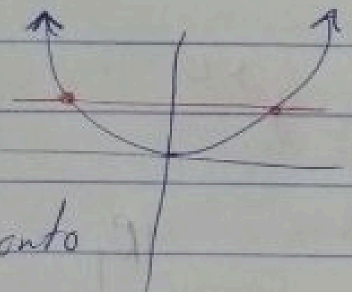


b) $f(n) = 4n + 2$

$f(n) : \mathbb{Z} \to \mathbb{Z} \rightarrow$ one to one ✓

Range : $\mathbb{R} \neq \mathbb{Z} \Rightarrow f(n)$ isn't onto .

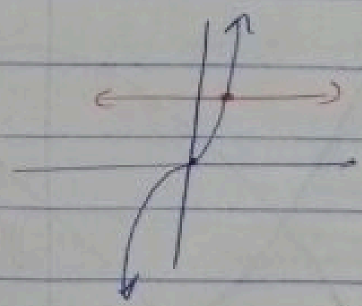
c) $f(n) = n^2$



$\Rightarrow f(n)$ isn't one to one

$\Rightarrow$ Range : $[0, \infty) \neq \mathbb{Z} \Rightarrow$ isn't onto

d) $f(n) = n^3 - n^2 = n^2(n-1)$

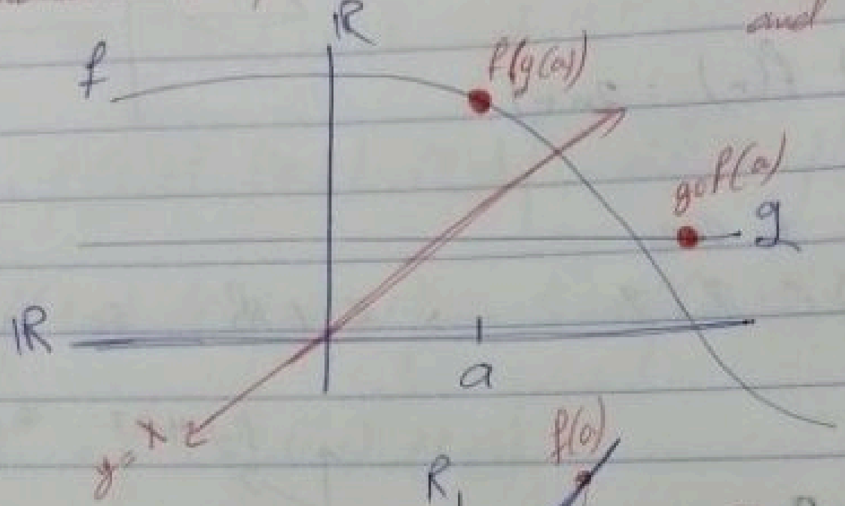


$\Rightarrow f(n)$ is one to one

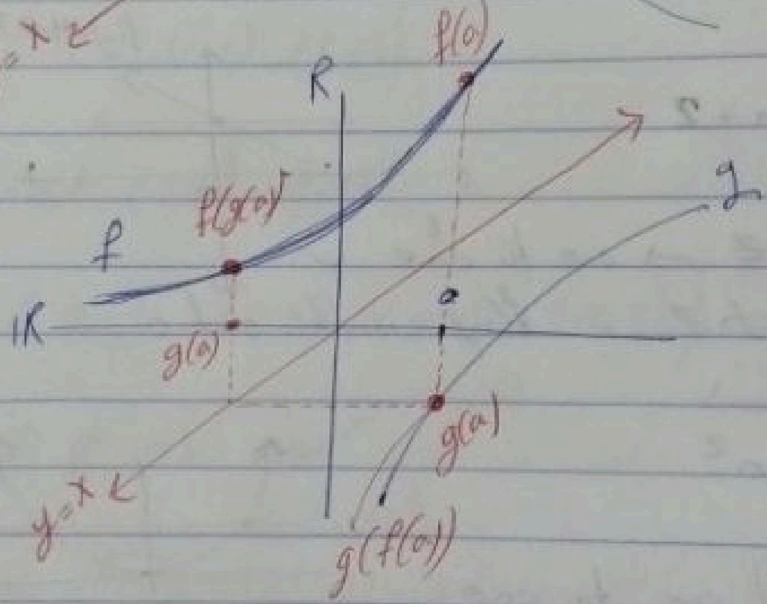
$\Rightarrow f(n)$ isn't onto since $\mathbb{R} \neq \mathbb{Z}$

Q.2 Indicate the position of $(a, f \circ g(a))$ on each one and $(a, g \circ f(a))$?

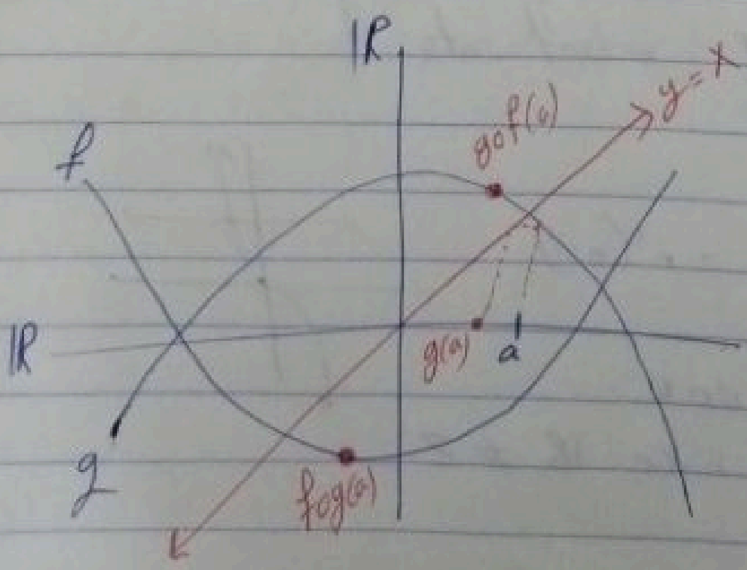
a)



b)



c)



Q.7 a) Give an example of a one to one function $f: \mathbb{N} \to \mathbb{N}$ that does not map $\mathbb{N}$ onto $\mathbb{N}$:-

$f: \mathbb{N} \to \mathbb{N}$ • f is 1-1 but not onto

$n \to n+1$ since $1 \to 2$, $2 \to 3$

Range $= \{2, 3, 4, \dots\} \neq \mathbb{N}$

b) $f: \mathbb{N} \to \mathbb{N}$ • f onto but not 1-1

$n \to n-1$, $n \neq 1$

since $1 \to 1$, $2 \to 1$, $3 \to 2$, $4 \to 3$

c) $f: \mathbb{N} \to \mathbb{N}$ • f is not onto not 1-1

$n \to \begin{cases} 4 & \text{if } n = 1 \\ n^2 & \text{if } n \neq 1 \end{cases}$

$1 \to 4$, $2 \to 4$, $3 \to 9$, $4 \to 16$

d) let $X = \{1, 2, 3, 4\}$ Does there exist one to one function $f: X \to X$ that doesn't onto ? or f onto but not 1-1

No , No , since it finite any 1-1 must be onto and any onto must be 1-1

Q.11 let f and g be functions. Either prove or give a counterexample to the conjecture that $f \cup g$ is a function:

$\Rightarrow f \cup g$ isn't always a function.

$f = \{(2,2)\}$, $g = \{(2,3)\}$

$f \circ g = \{\}$

$f(2) = 2$

$g(2) = 3$

$\Rightarrow (f \cup g)(1) = ??$ not defined, so it isn't a function.

Q.12 a) let A be a set and let f and g be function from A to A. Prove that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

let $(a,c) \in (f \circ g)^{-1} \Leftrightarrow (c,a) \in f \circ g$

$\Leftrightarrow \exists b \in A, (c,b) \in g$ and $(b,a) \in f$

$\Leftrightarrow \exists b \in A, (b,c) \in g^{-1}$ and $(a,b) \in f^{-1}$

$\Leftrightarrow (a,c) \in g^{-1} \circ f^{-1}$ ✓

b) give an example to show that there is a set A and functions f and g from A to A such that $(f \circ g)^{-1} \neq f^{-1} \circ g^{-1}$:

let $A = \mathbb{N}$ and $f(x) = \frac{1}{2}x$ and $g(x) = x - 1$

$f^{-1}(x) = 2x$, $g^{-1}(x) = x+1$

$\Rightarrow (f \circ g)(x) = f(g(x)) = f(x-1) = \frac{x-1}{2}$

$\Rightarrow (f \circ g)^{-1}(x) = 2x+1$

$\Rightarrow (f^{-1} \circ g^{-1})(x) = f^{-1}(g^{-1}(x)) = f^{-1}(x+1) = 2x+2$

so $(f \circ g)^{-1}(0) = 1 \neq (f^{-1} \circ g^{-1})(0) = 2$

Q.6 $f(n)$ be the number of digits in n

exp. $f(437) = 3$, $f(19) = 2$

* $f(n)$ is not 1-1

$f(19) = f(15) = 2$

$x_1 \neq x_2 \rightarrow f(x_1) = f(x_2)$

Hence , $f(n)$ is onto on $\mathbb{N}$

$\boxed{Q.4}$ $f = \{(a,b) \in \mathbb{R} \times \mathbb{R} \mid \text{if } a<b \text{ then } f(a) < f(b)\}$

* let A and B sets of numbers and let $f: A \to B$ be a strictly increasing function. let a and b be distinct elements of $A = \mathbb{R}$, we need to show that $f(a) \neq f(b)$

Since $a \neq b$, there's two cases

- Case 1: $a<b$, since f is strictly increasing, this implies that $f(a) < f(b)$ so $f(a) \neq f(b)$

- Case 2: $b<a$. Since f is strictly increasing, this implies that $f(b) < f(a)$, so $f(a) \neq f(b)$

In either case, we have that $f(a) \neq f(b)$