

SUN.



MON.



TUES.



WED.



THUR.



FRI.



SAT.



NO.

DATE

/ /

L2: Statics of particles

→ equilibrium → $\sum F = 0$

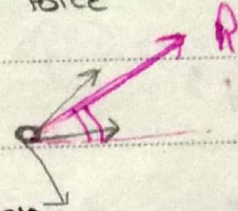
F = Vector → magnitude [N], direction

المقدار magnitude

الاتجاه direction

Resultant Force

القوة المحصلة



graphical

Triangle rule + Trapezoid ⇒ graphical.

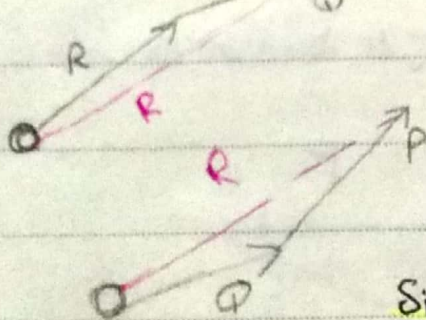
Law of cosines :-

$$R^2 = P^2 + Q^2 - 2PQ \cos B$$

الزاوية بين P و Q

head to tail

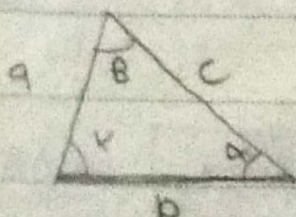
الرأس مع الذيل



Law of sines :-

$$\frac{\sin A}{Q} = \frac{\sin B}{P} = \frac{\sin C}{R}$$

$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}, \quad \vec{P} - \vec{Q} = \vec{Q} - \vec{P}$$



$$\Rightarrow \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

NO.

SUN.

MON.

TUES.

WED.

THUR.

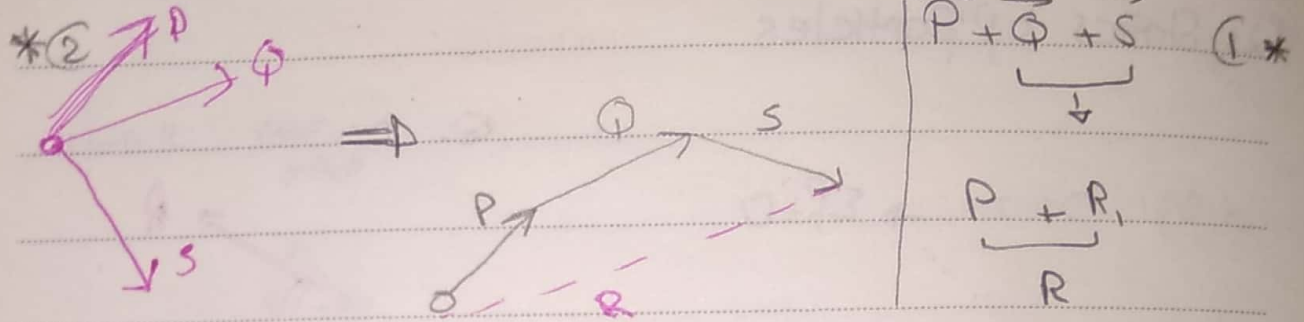
FRI.

SAT.

DATE

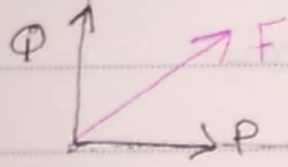
/

/

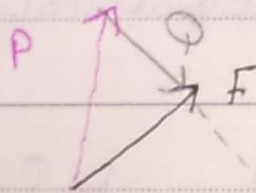


*Concurrent forces → Point of application.

Vector Force components

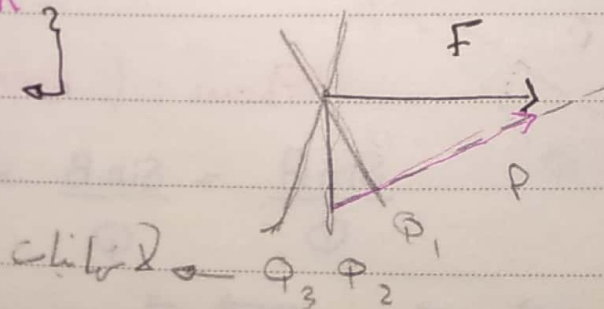


إذا عرف F و عرف واحدة من المركبات بطلع المركبة التامة

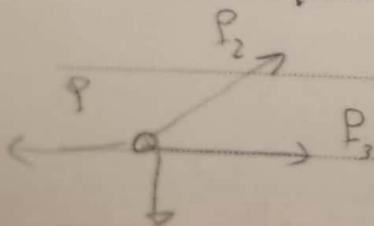


إذا عرف واحد من direction للمركبات و F

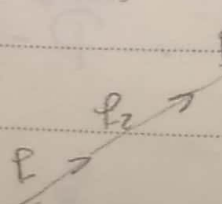
*minimum
 $Q \perp P$



Concurrent Forces

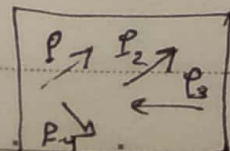


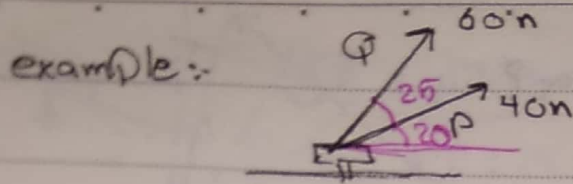
Collinear Force



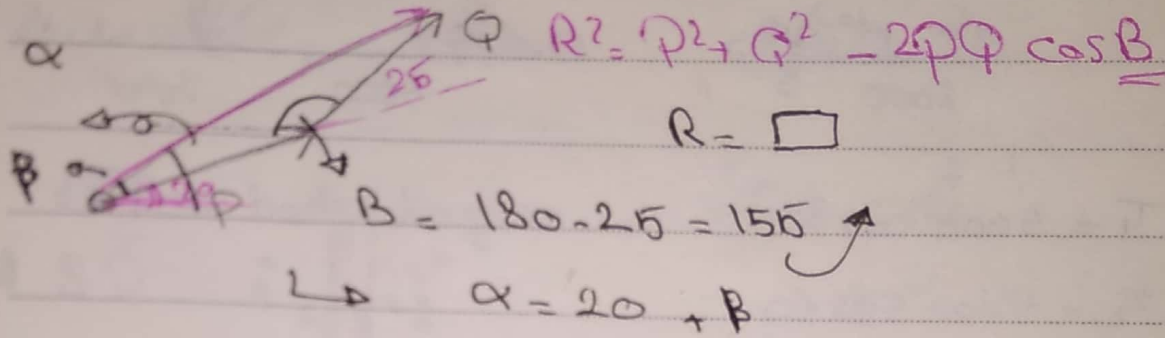
Line of action نفس الـ بطلع

Coplanar Forces





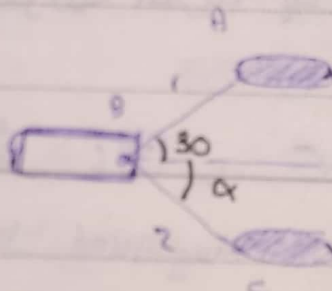
determine the R



$$\frac{60}{\sin B} = \frac{R}{\sin 155} = B = \square$$

L3:

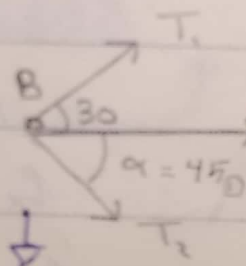
example:



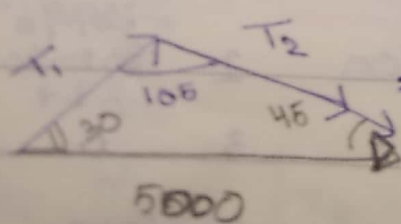
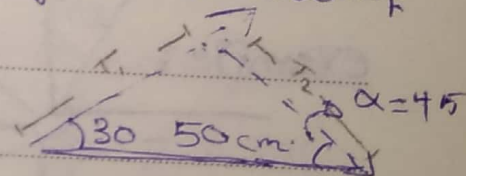
$$R = 5000 \text{ lbf}$$

$$DT_1, T_2: \alpha = 45^\circ$$

2) $\alpha?$: T_2 minimum



5000 kg approximately $\Delta 1 \text{ cm} = 1000 \text{ lbf}$



$$R^2 = T_1^2 + T_2^2 - 2T_1 T_2 \cos B$$

$$\frac{5000}{\sin 105} = \frac{T_1}{\sin 45} = \frac{T_2}{\sin 30} \Rightarrow T_1 = 3660.21 \text{ lb}$$

$$\Rightarrow T_2 = 2588.11 \text{ lb}$$

NO.

DATE / /

SUN.

MON.

TUES.

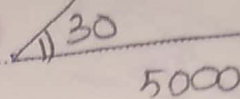
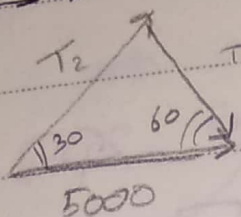
WED.

THUR.

FRI.

SAT.

2) T_2 ? $\alpha = \text{min}$ T_2 minimum : α ?

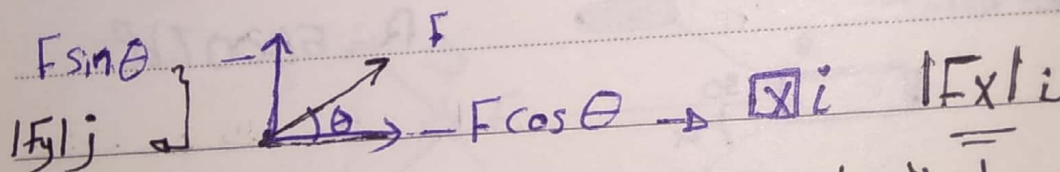


$$T_1 = 5000 \cos 30$$

$$T_2 = 5000 \cos 60$$

3 Rectangle component of force.

* ينحلل القوة إلى x_{com} و y_{com} وبما أن هذه مركبة القوة
بالتحليل : الحركة القريبة يتأخذ $\cos$ والبعدة يتأخذ $\sin$



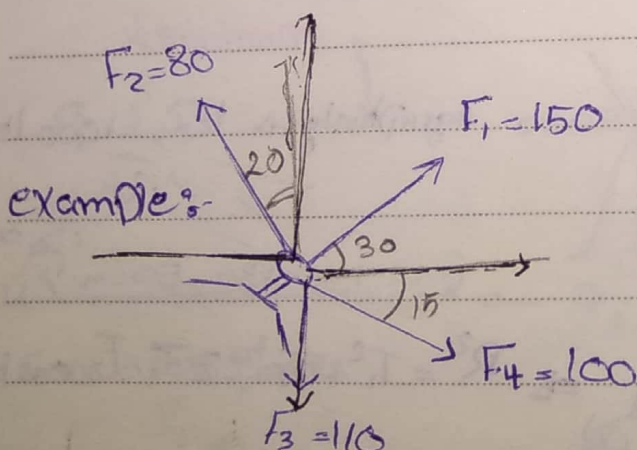
magnitude: $|F|$

$$* \vec{F} = F_x \vec{i} + F_y \vec{j}$$

$$(\sum F_x) \vec{i} + (\sum F_y) \vec{j}$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\theta = \tan^{-1} \frac{R_y}{R_x}$$



	F_x	R_x	F_y	R_y
1	$F_1 \cos 30 (N)$	$F_1 \sin 30 (N)$		
	129.9	75		
2	$-80 \sin 20$	$80 \cos 20$		
	-27.4	75.2		
3	0	-110		

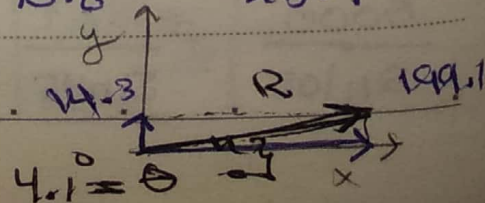
$$R = 199.1 \vec{i} + 14.3 \vec{j}$$

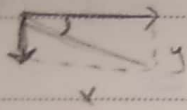
$$R = \sqrt{(199.1)^2 + (14.3)^2}$$

$$= 199.6 N$$

$$\theta = \tan^{-1} \frac{14.3}{199.1}$$

	$F \cos 15$	$F \sin 15$
4	96.6	-25.9





equilibrium of a particle

$$\sum F = 0$$

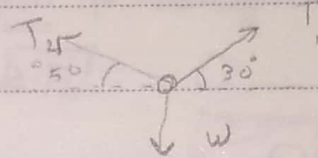
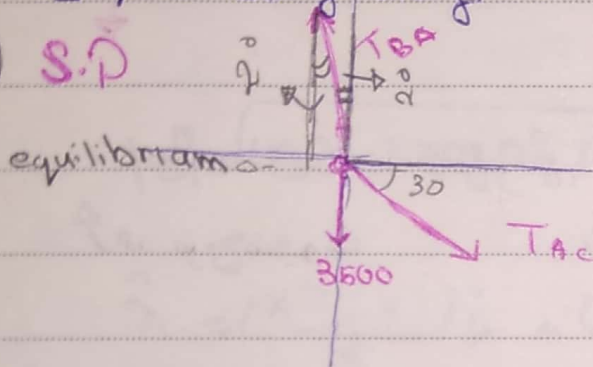
بإحداثيات
الزاوية

$$\sum F_x = 0$$

$$\sum F_y = 0$$

Free body diagram

S.P



$T_{AC}, T_{BC} ?$

$$\sum F_x = 0$$

Free

$$\sum F_y = 0$$

$$T_{AC} \sin 30 - T_{AB} \sin 2 = 0$$

$$T_{AC} \cos 30 = T_{AB} \cos 2$$

$$0.86 T_{AC} = 0.999 T_{AB}$$

$$T_{AB} = 0.86 T_{AC}$$

$$0 = T_{AB} \cos 2 - 3500 - T_{AC} \sin 30$$

$$0.86 T_{AC} \cos 2 - 3500 - T_{AC} \sin 30 = 0$$

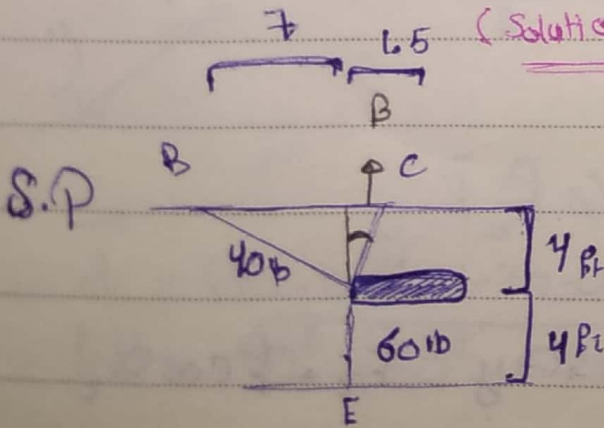
$$T_{AC} = 141.18 \text{ lb}$$

$$T_{AB} = 3570 \text{ lb}$$

3500 = T_{AB} \sin 2 + T_{AC} \sin 30 * الفرق بين الجيوب

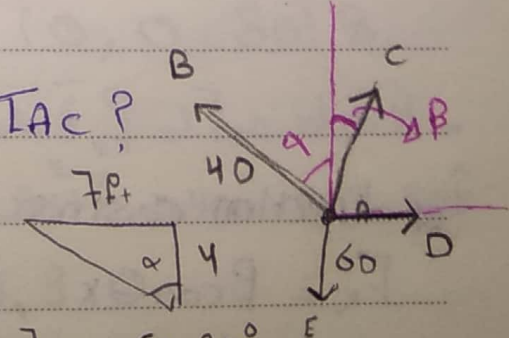
(solution check)

حل الجيوب

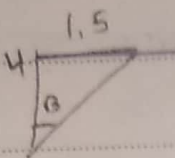


PP TAC ?

$$\alpha =$$



$$\tan^{-1} \frac{7}{60} = 6.52^\circ$$

$$B = 4 \quad 1.5 \quad \theta = \tan^{-1} \frac{1.5}{4} = 20.5$$


$$\sum F_x = 0$$

$$40 \sin 40.25 + T_{AC} \sin 20.55 + D = 0$$

$$D = 19.6 \text{ lb}$$

$$\sum F_y = 0$$

$$40 \cos 40.25 + T_{AC} \cos 20.55 - 60 = 0$$

$$T_{AC} = 43.8 \text{ lb}$$

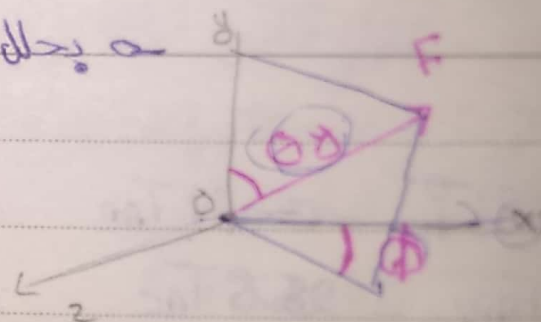
↑ توتر السلك

* Rectangular component of a force in space.

Plan XZ θ_y θ_x θ_z θ

$$F_y = F \cos \theta_y$$

$$F_{xz} = F \sin \theta_y$$



$$F_x = F_{xz} \cos \theta \quad , \quad F_z = F_{xz} \sin \theta$$

$$F \sin \theta_y$$

$$F = F_x \bar{i} + F_y \bar{j} + F_z \bar{k}$$

* direction cosines

$$F_x = F \cos \theta_x \quad , \quad F_y = F \cos \theta_y \quad , \quad F_z = F \cos \theta_z$$

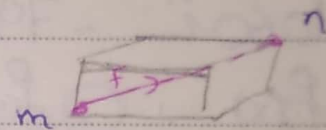
$$\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$$

$$= F \hat{n} \Rightarrow \hat{n} = \cos \theta_x \vec{i} + \cos \theta_y \vec{j} + \cos \theta_z \vec{k}$$

magnitud $\hat{n}$

$$\text{magnitud } \hat{n} = 1 \Rightarrow \cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$$

$$\vec{d} = dx \vec{i} + dy \vec{j} + dz \vec{k}$$



$$|\vec{d}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

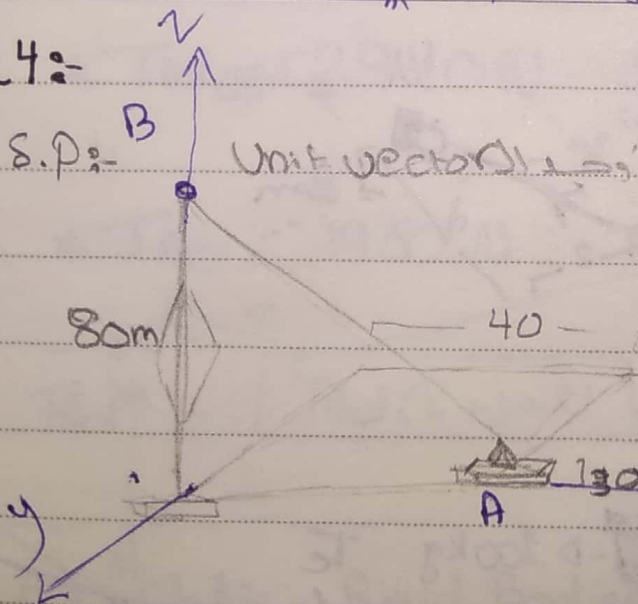
$$\hat{n} = \frac{(x_2 - x_1)}{d} \vec{i} + \frac{(y_2 - y_1)}{d} \vec{j} + \frac{(z_2 - z_1)}{d} \vec{k}$$

$$\cos \theta_x + \cos \theta_y + \cos \theta_z$$

L4:-

S.P:- B

Unit vector $\hat{n}$ is directing towards B.



لأرسم نقطه A و B

$$A(40, -30, 0)$$

$$B(0, 0, 80)$$

المتجه من A إلى B

$$\vec{d} = (40 - 0)\vec{i} + (-30 - 0)\vec{j} + (0 - 80)\vec{k} = 40\vec{i} - 30\vec{j} - 80\vec{k}$$

$$|\vec{d}| = \sqrt{40^2 + 30^2 + 80^2} = 94.3$$

$$\hat{\lambda} = \frac{40}{94.3} \hat{i} - 0.318 \hat{j} - 0.848 \hat{k}$$

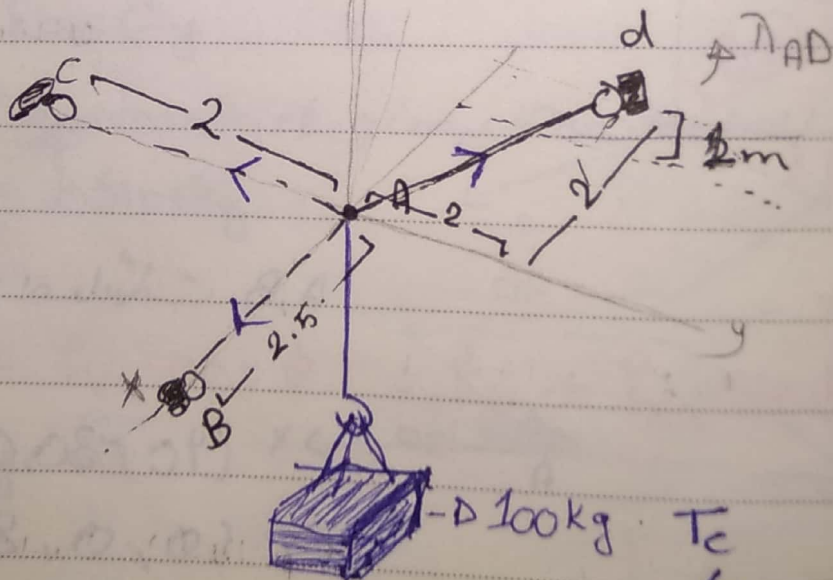
$$0.424$$

$$\begin{aligned} * T &= 2500 * \hat{\lambda} \rightarrow 2500 * (0.424 \hat{i} - 0.318 \hat{j} - 0.848 \hat{k}) \\ &\Rightarrow 1060 \hat{i} - 795 \hat{j} - 2120 \hat{k} \\ &\quad P_x \quad P_y \quad P_z \\ &\quad 1060 \text{ N} \quad -795 \text{ N} \quad -2120 \text{ N} \end{aligned}$$

$$\Rightarrow \cos^{-1} 0.424 = \theta_x, \quad \cos^{-1} 0.318 = \theta_y$$

$$\cos^{-1} 0.848 = \theta_z$$

S.P.:-

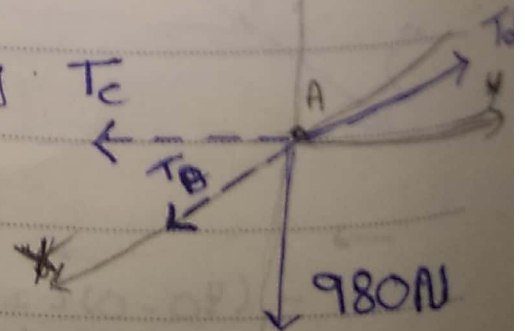


T AB, AC, AD P

$$* \vec{T}_{AD} = (-2\hat{i} + 2\hat{j} + 1\hat{k})$$

$$\hat{\lambda}_{AD} = \frac{-2}{3} \hat{i} + \frac{2}{3} \hat{j} + \frac{1}{3} \hat{k}$$

$$T_{AD} = -2 T_{AD} \left(\frac{-2}{3} \hat{i} + \frac{2}{3} \hat{j} + \frac{1}{3} \hat{k} \right)$$



$$\vec{T}_{AC} = -T_{AC} \hat{j}, \quad T_{AB} = T_{AB} \hat{i}, \quad \vec{W} = 980 \hat{k}$$

مركب القوى (T) في اتجاه.

$$\sum \epsilon P_x = 0, \quad \epsilon P_y = 0, \quad \epsilon P_z = 0$$

3

$$* -\frac{2}{3} T_{Ad} + T_{AB} = 0 \rightarrow \textcircled{1} \quad (x)$$

$$* \frac{2}{3} T_{Ad} - T_{AC} = 0 \rightarrow \textcircled{2} \quad (y)$$

$$* \frac{1}{3} T_{Ad} - 980 = 0 \rightarrow \textcircled{3}$$

2

$$* T_{Ad} = 2940 \text{ N}$$

عوض
الاصحوف
على

مبلغ العواء كماله
لوح طالب (R)

$$* T_{AC} = 1960 \text{ N}, \quad T_{AB} = 1960 \text{ N}$$

$$\vec{T}_{Ad} = [T_{Ad}(\hat{i} + \hat{j} + \hat{k})]$$

L5:- ch8:- Rigid Bodies "equivalent system of force"

* deformable body

* Rigid body

- Rotation moment
- Translation

حركة اذ دوران



NO.

DATE / /

SUN.

MON.

TUES.

WED.

THUR.

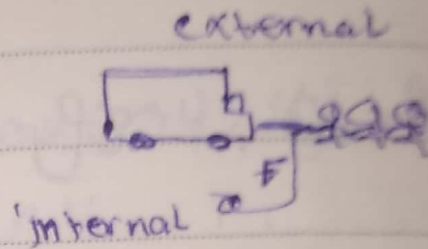
FRI.

SAT.

rigid body القوى على الجسم الصلب
moment. قوة واحدة و

External and Internal forces

section cut
القسم

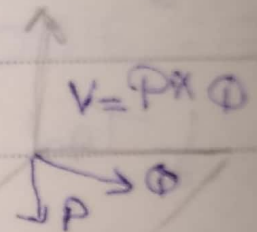


Line of action Force الحركة
نقطة التأثير

ويعتبر نقطة التأثير

منه دائما صحيح اذا لم يكن مع قوى داخلية

* Vector Product of two vectors.



مستوى دوران المستوى الى المستوى المتجهين

(P, Q) 3 magnitud , line of action

$|V| = P Q \sin \theta$, direction قاعدة اليد اليمنى

خارج المستوى

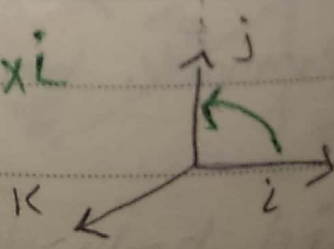
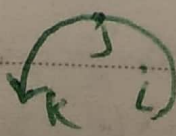
$\Phi \times P \neq (P \times \Phi) \rightarrow -(P \times \Phi)$ نفس المقياس بالاجد مختلف

$P(\Phi_1 + \Phi_2) = P \times \Phi_1 + P \times \Phi_2$

$(P \times \Phi) \times S \neq P \times (\Phi \times S)$

$$K = j \times i$$

Rectangular component



$$i \times i = 0$$

$$G \cdot P \times Q$$

$$\begin{matrix} & i & j & k \\ P & P_x & P_y & P_z \\ Q & Q_x & Q_y & Q_z \end{matrix} \Rightarrow \vec{V} = i(P_y Q_z - P_z Q_y) - j(P_x Q_z - P_z Q_x) + k(P_x Q_y - P_y Q_x)$$

٤.٣ Moment of a force about a point

لحظة القوة حول نقطة M_O هي القوة مضروبة في المسافة العمودية عن القوة إلى النقطة O .

$$\underline{M_O = \vec{r} \times \vec{F}}$$

$\rightarrow$ magnitude, direction $\uparrow$

$$r F \sin \theta$$

$$\rightarrow F d$$

المسافة العمودية بين القوة والنقطة O

القوة ونقطة التأثير.

$$\vec{r} \times (\vec{F}_1 + \vec{F}_2 + \dots) = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 + \dots$$

3D vectors

The moment of the group of forces about a point, equal the moment of the resultant about the point.

$$\begin{matrix} & i & j & k \\ \begin{vmatrix} x & y & z \\ P_x & P_y & P_z \end{vmatrix} \end{matrix}$$

3d.

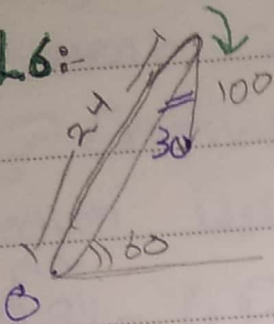
$$= (y F_z - z F_y) i - (x F_z - z F_x) j + (y F_x - x F_y) k$$

$$3d \rightarrow F(i, j, k), r(i, j, k)$$

$$M \rightarrow k$$

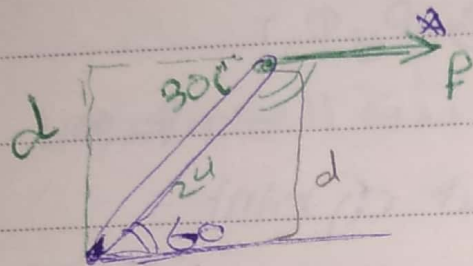
$$\begin{bmatrix} + & - \\ - & + \end{bmatrix}$$

L6:

 $d = 24$

$$M = r_o \times F = 24 \times 100 \times \sin 30 = 1200 \text{ lb in}$$

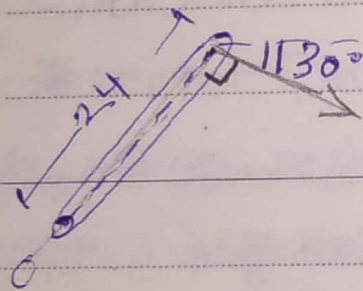
Moment of force



$$M = Fd \Rightarrow 1200 = P \times 24 \cos 30$$

$$P = 57.7 \text{ lb}$$

* Smallest force to produce same M

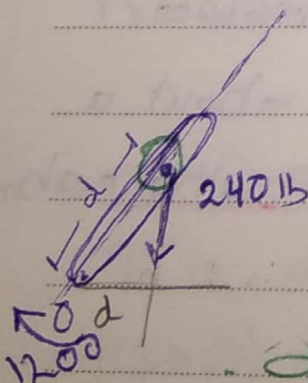


$$M = F \times d$$

direction $\theta = 30^\circ \perp OA$

$$1200 = F \times 24 \Rightarrow F = 50 \text{ lb}$$

* Location of vertical force 240 lb



$$M = F \times d \quad 1200 = 240 \times d$$

$$d = 5$$

distance $d = OB \cos 60$

$$OB = \frac{5}{\cos 60} = 10 \text{ in}$$

SUN.

MON.

TUES.

WED.

THUR.

FRI.

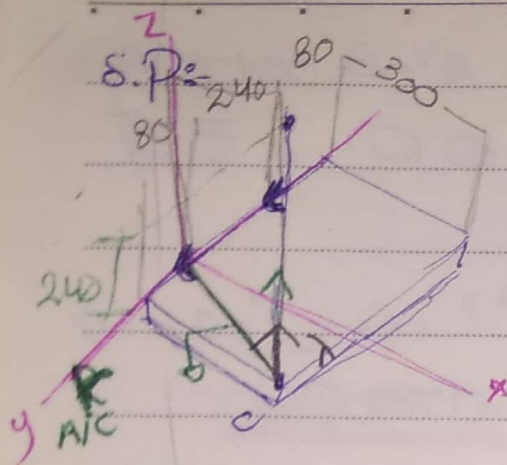
SAT.

NO.

DATE

/

/



$$M = \vec{r}_{AC} \times \vec{F}$$

$$A = (0, 0, 0), C = (300, 80, 0)$$

$$\vec{r}_{AC} = 300\hat{i} + 80\hat{j}$$

$$\vec{F}$$

المتجه القوة

$$\vec{F} = 200 \vec{\lambda} \rightarrow \vec{\lambda} = \frac{d_{CD}}{|d_{CD}|}$$

$$D = (0, -240, 240)$$

$$\vec{d}_{CD} = -300\hat{i} - 320\hat{j} + 240\hat{k}$$

$$|d_{CD}| = 500 \text{ mm}$$

$$\vec{\lambda} = -0.6\hat{i} - 0.64\hat{j} + 0.48\hat{k}$$

$$\vec{F} = \sqrt{3} 200 (\vec{\lambda}) = -120\hat{i} - 128\hat{j} + 96\hat{k}$$

$$\vec{M} = \vec{r}_{AC} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 300 & 80 & 0 \\ -120 & -128 & 96 \end{vmatrix} = \begin{matrix} \hat{i} (7680) \\ -\hat{j} (28800) \\ +\hat{k} (-28800) \end{matrix}$$

$$\vec{M} = 7680\hat{i} + 28800\hat{j} - 28800\hat{k}$$

$$|M| = 141447 \text{ Nmm}$$

$$\theta_x = \frac{\cos^{-1} 7680}{41447}$$

$$\theta_y = \frac{\cos^{-1} 22800}{41447}$$

$$\theta_z = \frac{-28880}{41447}$$

17:- Scalar product of two vectors

$$\vec{P} \cdot \vec{Q} = PQ \cos \theta$$

أستخدم غالباً "كيفية استخراج زاوية"

$$\vec{P} \cdot \vec{Q} = \vec{Q} \cdot \vec{P}, \quad P(\vec{Q}_1 + \vec{Q}_2) = \vec{Q}_1 P + \vec{Q}_2 P$$

$$(\vec{P} \cdot \vec{Q}) \cdot \vec{S} \rightarrow \text{كمية قياسية}$$

↳ Scalar quantity

$$\vec{i} \cdot \vec{i} = 1, \quad \vec{j} \cdot \vec{j} = 1, \quad \vec{k} \cdot \vec{k} = 1$$

$$0 = \vec{k} \cdot \vec{j} \quad \text{و} \quad \vec{i} \cdot \vec{j} = 0 \quad \text{لأنه}$$

$$\vec{P} \cdot \vec{Q} = P_x Q_x + P_y Q_y + P_z Q_z$$

$$\vec{P} \cdot \vec{P} = P^2 \rightarrow PP \cos 0 = P^2$$

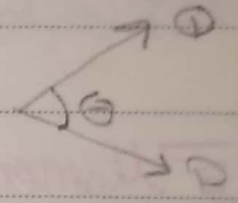
2 vector unknown.

$$\vec{P} \cdot \vec{Q} = PQ \cos \theta = P_x Q_x + P_y Q_y + P_z Q_z$$

باستخدام $(\vec{P} \cdot \vec{Q})$ بعد ايجاد $\vec{P}$ و $\vec{Q}$

components axis حول

3d



Mixed Triple Product of 3 vector.

الاجزاء الثلاثة حول axis واحد

$$\vec{S} \cdot (\vec{P} \times \vec{Q}) = \text{scalar result.}$$

$$\vec{S} \cdot (\vec{P} \times \vec{Q}) = \vec{P} \cdot (\vec{S} \times \vec{Q}) = \vec{Q} \cdot (\vec{P} \times \vec{S}) = (-1) \vec{Q} \cdot (\vec{S} \times \vec{P})$$

الاجزاء الثلاثة ال Mixed Triple

$$\begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

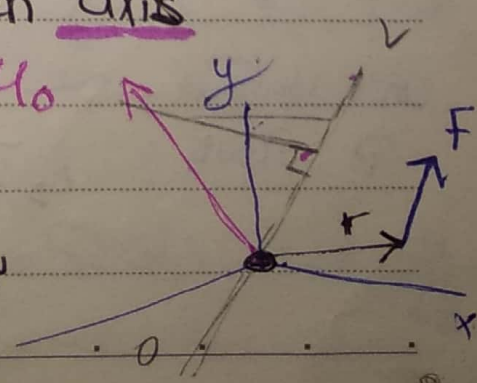
Moment of a force about a given axis

يوجد $\vec{M}_0$ حول نقطة O

عن $\vec{M}_0$ بوجهه حول

بعده النكز و بين لفتت البقطة O

رج يعطين بنفس ال $\vec{M}_0$ حول ال $axis$



$$1) * \mu_0 = \vec{r} \times \vec{r}$$

2.) * $M_{OL} = \vec{r}_O \times \vec{F} = \vec{r} (\vec{r} \times \vec{F})$

→ Moment of F about coordinate axes

$$U_x = y\sqrt{z} - 2\sqrt{y}$$

$$M_y = 2F_x - \lambda F_2$$

$$M_z = xF_y - yF_x$$

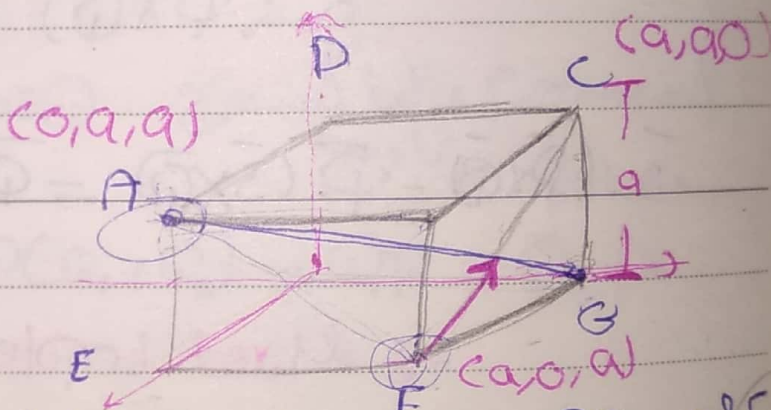
Example:-

a) about A (eu)

b) about AB

c) about AG

d) perpendicular distance between AG and FC



as $H_A = \bar{r}_{AF} \times \bar{p}$

* $\vec{P} = P \times \vec{\lambda}$ $\rightarrow \vec{\lambda} = \frac{Fc}{|Fc|} = \frac{0i + aj - ak}{\sqrt{a^2 + a^2}} = \frac{aj - ak}{\sqrt{2}a}$
unit vector $\vec{a}$

Unit Vector

پ یازجاہ

$$L \rightarrow \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}} - \frac{1}{\sqrt{2}} K$$

$$r_{af} = a_i - a_j - OK$$

$$\Rightarrow \vec{r} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & -a & 0 \\ 0 & \frac{p}{\sqrt{2}} & -\frac{p}{\sqrt{2}} \end{vmatrix} = \frac{ap}{\sqrt{2}} (\hat{i} + \hat{j} + \hat{k})$$

b) $\hat{i} \cdot \hat{u}_{AB} = \hat{i} \cdot \frac{ap}{\sqrt{2}} (\hat{i} + \hat{j} + \hat{k}) = \frac{ap}{\sqrt{2}}$
 X axis.

$$\hat{i} \cdot \hat{u}_A = \hat{u}_{AB} = \hat{i} \cdot \left(\frac{ap}{\sqrt{2}} (\hat{i} + \hat{j} + \hat{k}) \right) = \frac{ap}{\sqrt{2}}$$

reference point B → $\hat{u}_{AB}$ axis Point

c) $\hat{u}_{AG}$ axis Unit vector

$$\hat{B} \cdot \hat{u}_A = \hat{u}_{AG}$$

$$\hat{B} \cdot \frac{a}{|GA|} = \frac{1}{\sqrt{3}} (\hat{i} - \hat{j} - \hat{k})$$

$$\frac{1}{\sqrt{3}} (\hat{i} - \hat{j} - \hat{k}) \cdot \left(\frac{ap}{\sqrt{2}} (\hat{i} + \hat{j} + \hat{k}) \right) = \frac{-ap}{\sqrt{6}}$$

$$d) \vec{F} \times \vec{d} = \tau = \frac{-ap}{\sqrt{6}} = \tau d \Rightarrow d = \frac{a}{\sqrt{6}}$$

NO.

DATE

SUN.

☐

MON.

☐

TUES.

☐

WED.

☐

THUR.

☐

FRI.

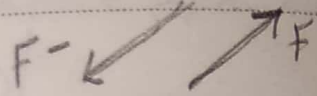
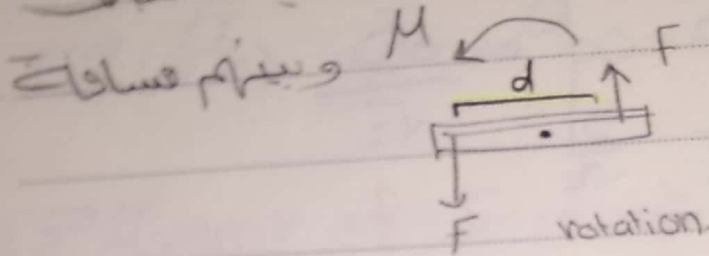
☐

SAT.

☐

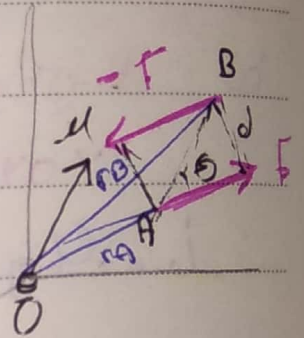
13 Moment of a couple

قوة متساوية في المقدار ومتعاكسات في الاتجاه



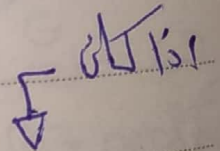
$$\begin{aligned} M &= \vec{r}_A \times \vec{F} + \vec{r}_B \times (-\vec{F}) \\ &= (\vec{r}_A - \vec{r}_B) \times \vec{F} \\ &= r F \sin \theta = [d] \end{aligned}$$

الدورانية



Ps: Moment vector $\vec{M}$ originen $\vec{r}$ و $\vec{F}$ $\times$ $\vec{F}$

couple forces (M و $\vec{r}$)



in the same plane $F_1 d_1 = F_2 d_2$
rotation. $\vec{r}$ و $\vec{F}$ $\times$ $\vec{F}$

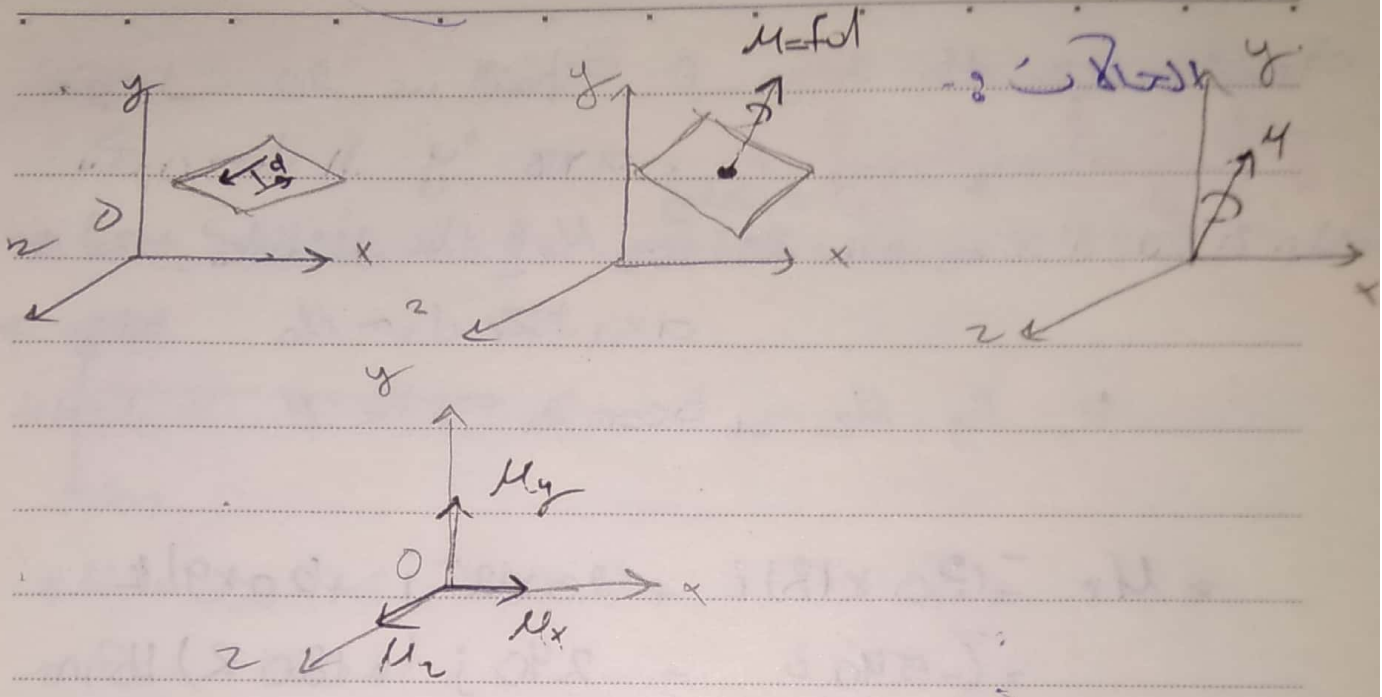
إذا كانا في نفس المستوى couple forces $\vec{M}$ و $\vec{r}$ $\times$ $\vec{F}$

$$M_1 = \vec{r} \times \vec{F}_1 \text{ in plane } P_1$$

$$M_2 = \vec{r} \times \vec{F}_2 \text{ in plane } P_2$$

نفس اتجاه الدوران وواحد $\vec{r}$ و $\vec{F}$ $\times$ $\vec{F}$

$$M = \vec{r} \times \vec{R} = \vec{r} \cdot (\vec{F}_1 + \vec{F}_2)$$



L8:-

The Moment is the effect of a force to rotate a body about a point or axis.

يتمثل M_o بنفس اتجاه $\vec{F}$ أو عكس اتجاه $\vec{F}$.

Point of interest. ينقل القوة على نقطة الاهتمام. ينقل كل القوة على الجسم لنقله.

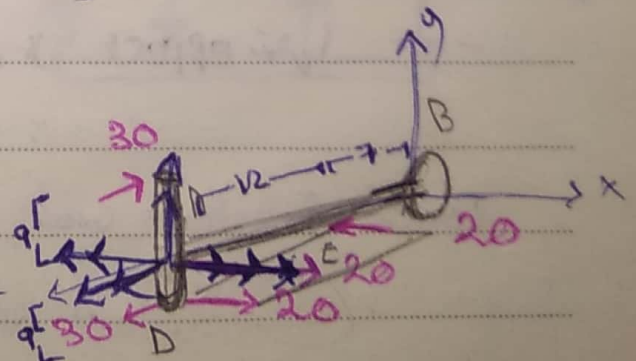
$$\vec{M}_o = \vec{r} \times \vec{F} \rightarrow \text{free vector}$$

نقطة الاهتمام هي نقطة على الجسم.

SP:-

أولاً: عننا قوتين بار 20 و 30 فال M_o

علين حول ال x -axis.



القوتين 20 و 30 متجهتوانيت بنفس المستوى.

فال M_o رج يكون عمودي على ال xy Plan.

* محور 20 عن الـ A مع الـ M_0 تبعاً عن الـ $axis$ y
 * محور 30 في M_0 مع الـ $axis$ z
 * محور الـ M مع الـ 3 comp M_x, M_y, M_z

$$M_x = (30 \times 18)i + (20 \times 12)j + (20 \times 9)k$$

$$= (-540i + 240j + 180k) \text{ lb in}$$

System of forces to a force and couple.

Resultant force $\rightarrow$ M_0 $\rightarrow$ Resultant Moment

" M_0 Free vector " $\rightarrow$ M_0 $\rightarrow$ F vector

(R) (R)

في حالة الـ F والـ $Moment$ عوياً على يكون مقدار الـ $effect$

إذا كانت الـ $Forces$ P لها على نفس الـ $Line of action$
 أو نفس الـ $Plane$ أو الـ $Plane$ موازياً

$$M = r \times F \rightarrow r = \frac{M}{F}$$

SUN.

☐

MON.

☐

TUES.

☐

WED.

☐

THUR.

☐

FRI.

☐

SAT.

☐

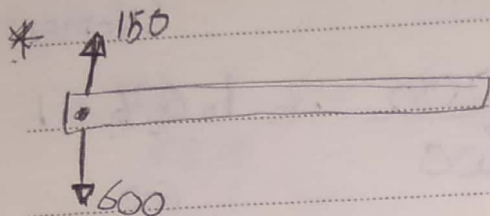
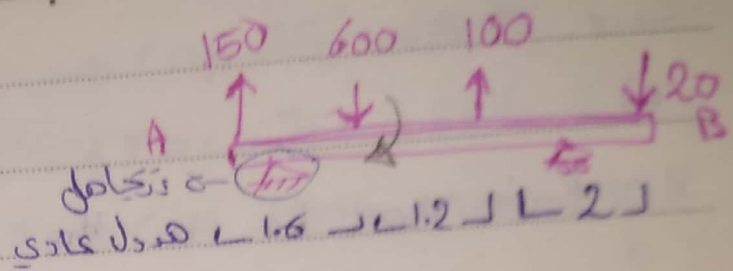
NO.

DATE

/

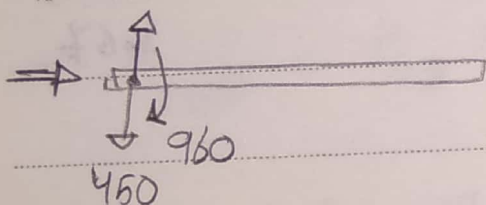
/

S.P:-

a) force and M_0 at A

$$* M_1 = 600 \times 1.6 = 960 \text{ Nm}$$

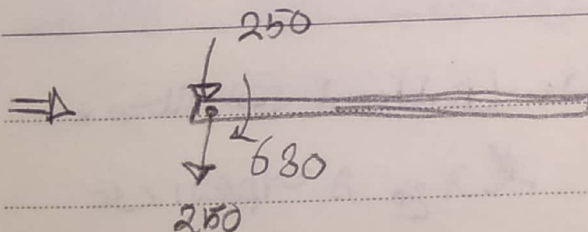
$$* R_1 = -450 \text{ N}$$



$$* M_2 = 100 \times 2.8 = 280 \text{ Nm}$$

$$M_R = -680 \text{ Nm}$$

$$F_R = -380 \text{ N}$$



$$M_3 = 250 \times 4.8 = 1200$$

$$M_R = -1880 \text{ Nm}$$

$$F_R = -600 \text{ N}$$

b) F and M at B

$$* R = 600$$

$$M_1 = -200 \text{ Nm}$$

$$M_3 = 720$$

$$M_2 = 1920$$

$$\rightarrow R = 600 \text{ N}, M = 1000 \text{ ccw}$$

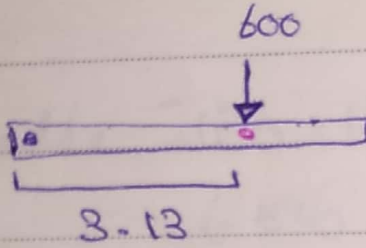


c)

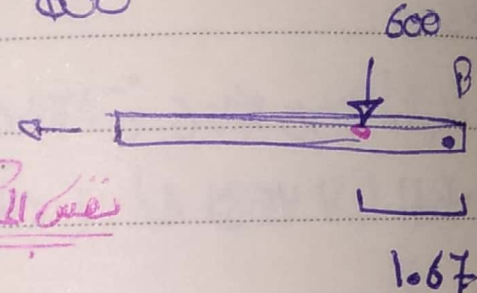
1.67

force must be to the right to produce effect (Moment)

$$M = P \times d \rightarrow d = \frac{1000}{600} = 1.67$$



نفس المبدأ



L9:-

SiP → الرتبة من الساعات 2d

* نحتاج القوى لنجد على الـ vector form

على الـ A مع الـ

$$P_B = 700(\bar{\lambda})$$

$$\bar{\lambda} = \frac{\vec{r}_{B/E}}{|\vec{r}_{B/E}|}$$

$$\frac{75i - 150j}{167.7}$$

$$B = (75, 100, 100)$$

$$E = (150, -50, 100)$$

$$C = (75, 100, 0)$$

$$= 700(0.44i - 0.89j) \cdot D(100, 0, 0)$$

$$= (308i - 623j) \text{ N} \quad A(0, 100, 150)$$

$$P_x = 1000 \cos 45 = 707.1 \text{ N}$$

$$P_z = 1000 \cos 45 = 707.1 \text{ N}$$

$$707.1i + 707.1j$$

$\cdot F_{Dx} = 600 \rightarrow r_{AB} = 75j + 50k \text{ mm}$

$\cdot F_{Dy} = 1039.2 \rightarrow r_{AC} = 75j - 50k \text{ mm}$

$600i + 1039.2j + r_{AD} = 100j - 100i - 50k$

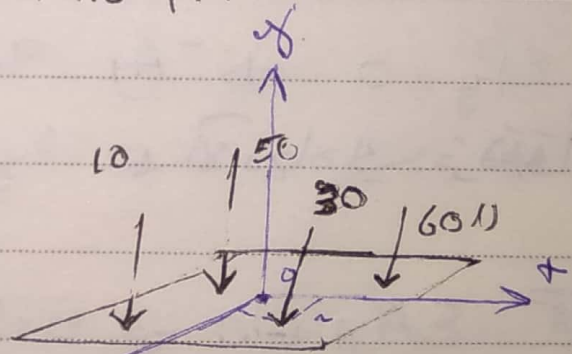
$\cdot M_{F_{DA}} = r_{AB} \times F_B = \begin{vmatrix} i & j & k \\ 75 & 50 & 0 \\ 300 & -600 & 0 \end{vmatrix} = 30i - 45k \text{ N}$

$\cdot M_{F_{CA}} = r_{AC} \times F_C = 17.68j \text{ Nm}$

$\cdot M_{F_{DA}} = r_{AD} \times F_D = 118.9k$

$\rightarrow M_A = 30i + 17.68j + 118.9k \text{ Nmm}$

$\rightarrow R_A = 1815.1 \text{ l}$



القوة 300 تنقلها 2 م واحصول

z، وواحصول الـ y

بحسب الـ M فالتحريك المسافة لـ z تحركها

ch4 :- equilibrium of rigid body.

له قانونان، الأول

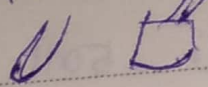
$M_x = 0, M_y = 0, M_z = 0$

$\Rightarrow \sum F = 0$

$\sum M = 0$

$F_x = 0, F_y = 0, F_z = 0$

Supp Dots:



$\vec{r}, \vec{F} \leftarrow \text{reaction}$

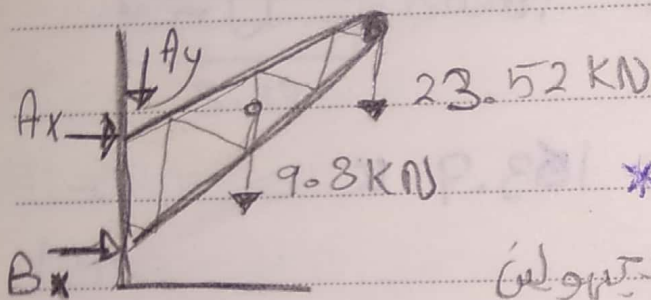
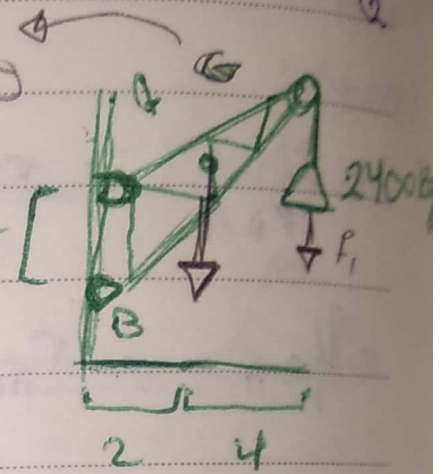


L10:

$$9.8 \times 1000$$

$$* P = 2400 \times 9.8$$

$$23520 \text{ N} \rightarrow 23.5 \text{ KN}$$



$$* \sum F_x = 0, \sum F_y = 0$$

الموازاة

الموازية

$$\sum M = 0$$

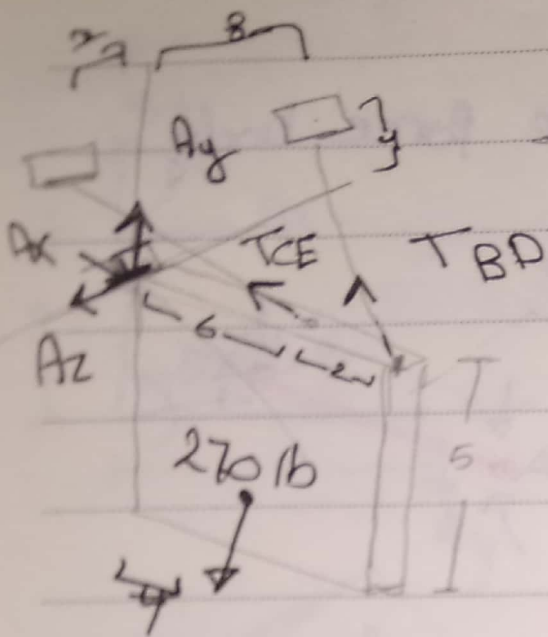
$$\sum F_y = 0 \Rightarrow -A_y - 9.8 - 23.52 = 0$$

$$* \text{الموازاة في الاتجاه } \rightarrow A_y = 33.32 \text{ KN}$$

$$\sum M_A = 0 \Rightarrow B_x \times 1.5 - 9.8 \times 2 - 23.52 \times 6$$

$$B_x = 107.1 \text{ KN}$$

$$\sum F_x = 0 \Rightarrow A_x + B_x = 0 \rightarrow A_x = -107.1$$



$$\sum F_x = 0 \quad \sum F_y = 0$$

$$\sum F_z = 0$$

$$\sum M_x = 0 \quad \sum M_y = 0$$

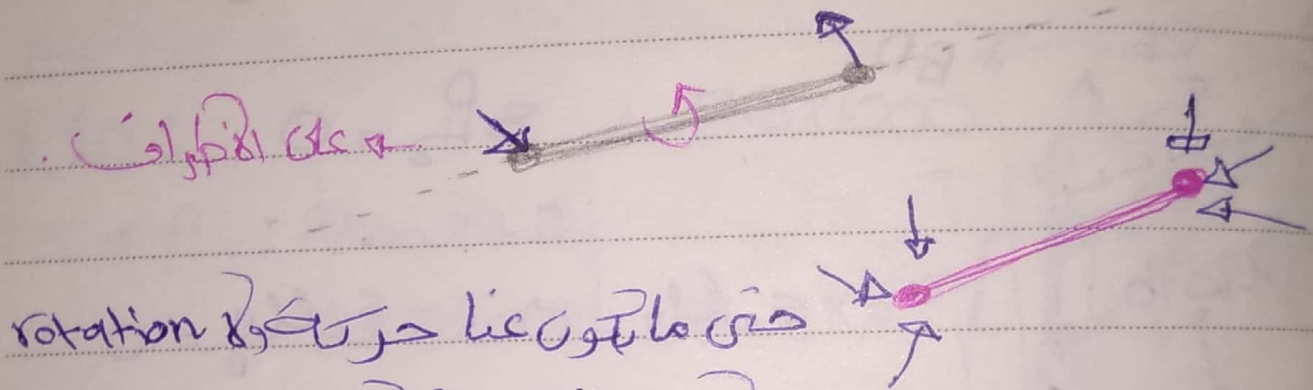
$$\sum M_z = 0$$

*reaction $\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$

$$*\vec{\omega} = -270 \vec{j}$$

$$\rightarrow T_{CE} \lambda_{CE} = \frac{d_{CE}}{|CE|} =$$

L11 :- Equilibrium of two force body



من مبادئ حركية
لا زيم ال R يتوزع
member و L of

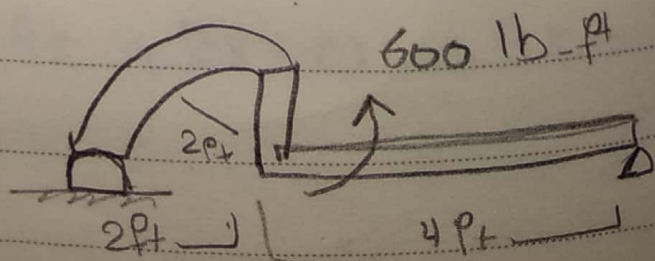
1. R و F متساوية
بالا حارة

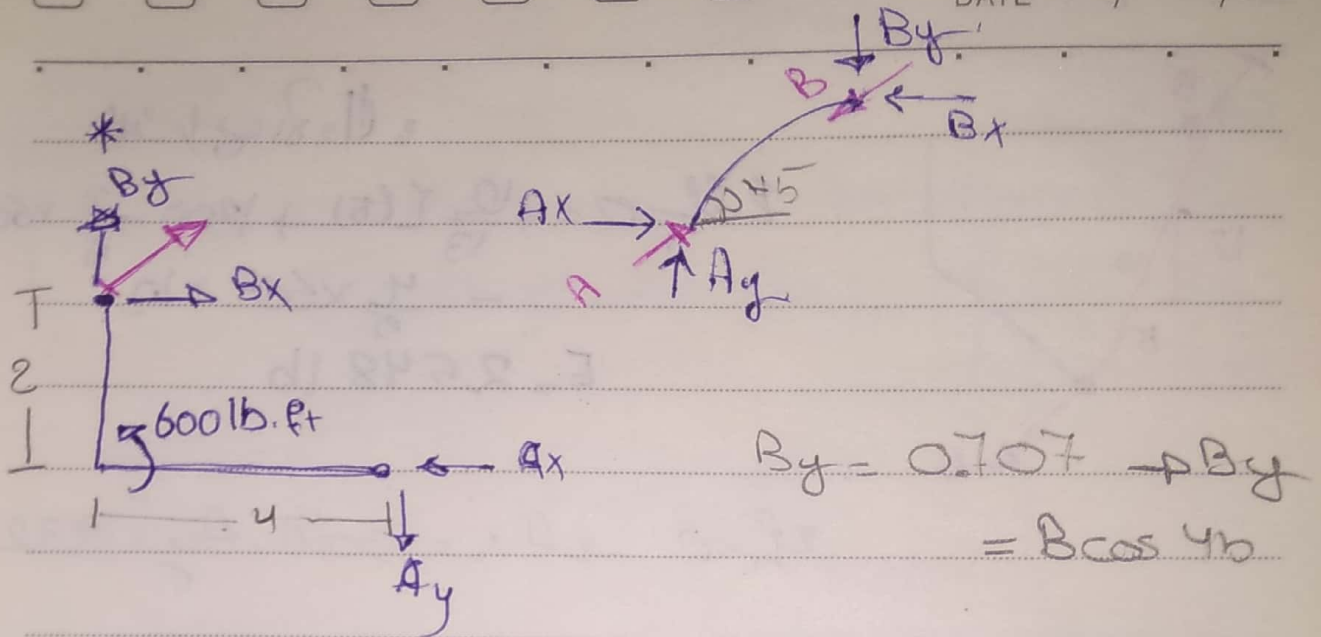
2. member و L مع
axis

Equilibrium of a 3 force body :-

- 1) concurrent forces (متساوية)
- 2) Parallel forces

* Example :-





$$\sum M_c = 0 \rightarrow 0.707B(4) - 0.707B(2) + 600$$

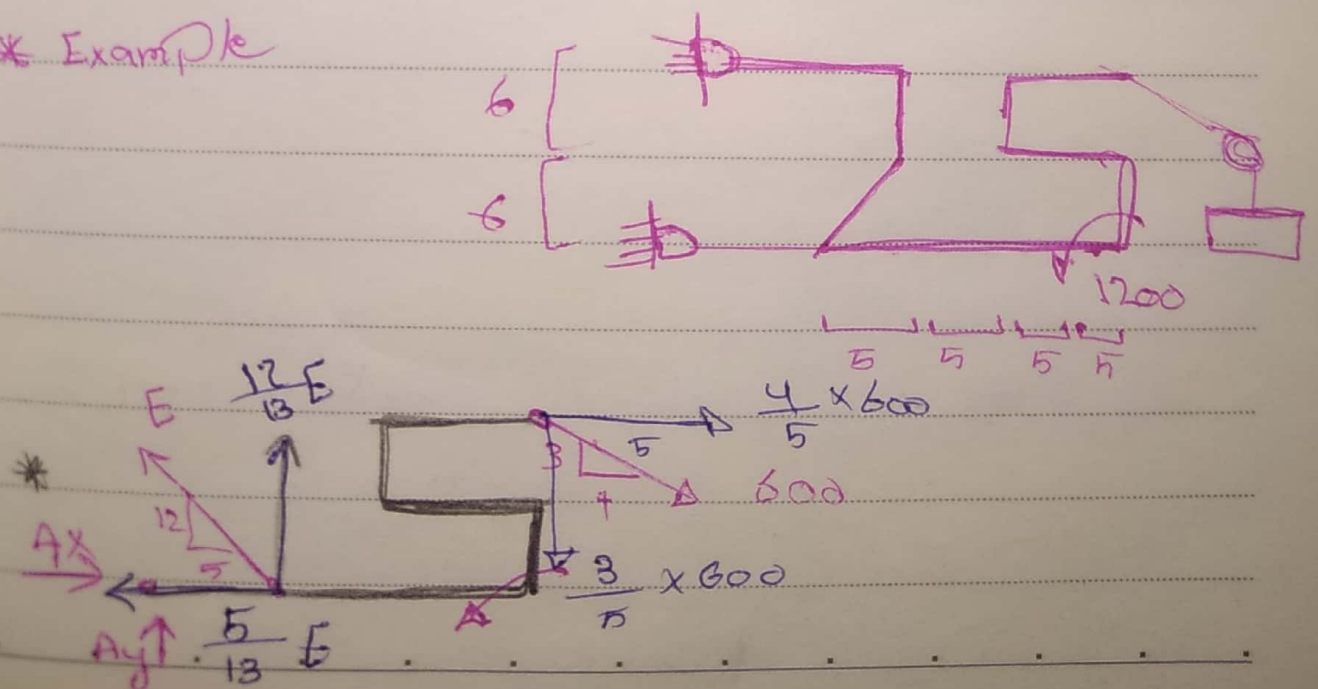
$$B = 141.4 \text{ lb}$$

$$\rightarrow B_x = 99.97 \text{ lb}, \quad B_y = 99.97$$

$$\sum F_x = 99.97 - C_x = 0 \rightarrow C_x = 99.97 \text{ lb}$$

$$\sum F_y = 99.97 - C_y = 0 \rightarrow C_y = 99.97 \text{ lb}$$

* Example



NO.

DATE

/

/

SUN.

☐

MON.

☐

TUES.

☐

WED.

☐

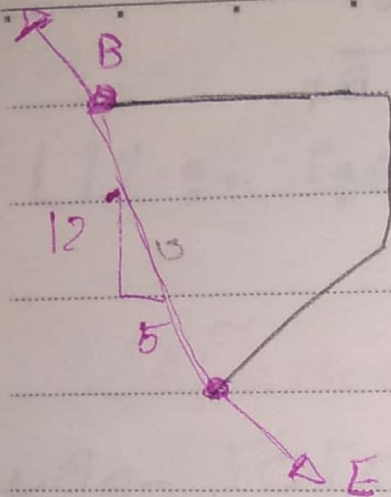
THUR.

☐

FRI.

☐

SAT.

☐

المساحة = 1200

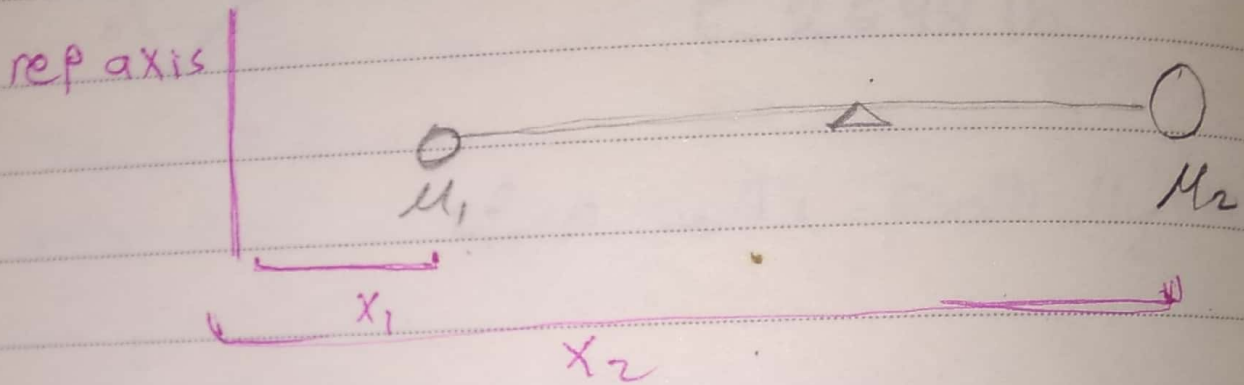
$$\sum M_A = 0 = +\frac{12}{13} E(5) + 1200 - \frac{3}{5} \times 600 - \frac{4}{5} \times 600 \times 12$$

$$E = 2548 \text{ lb}$$

$$\sum F_x = 0 \rightarrow A_x = 560 \text{ lb}, A_y = 1992 \text{ lb}$$

Ch 5:-

Distributed Forces centroid and center of gravity.



$$x_1 m_1 + x_2 m_2 = (m_1 + m_2) \bar{x}$$



$$\bar{x}_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

* center of gravity

$$\bar{x} = \frac{\int x dm}{\int dm}$$

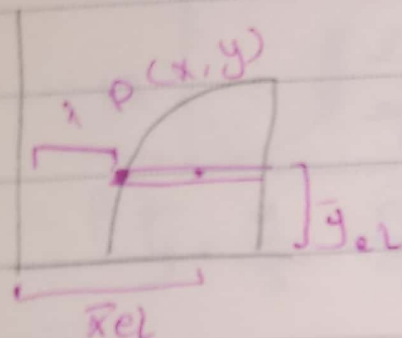
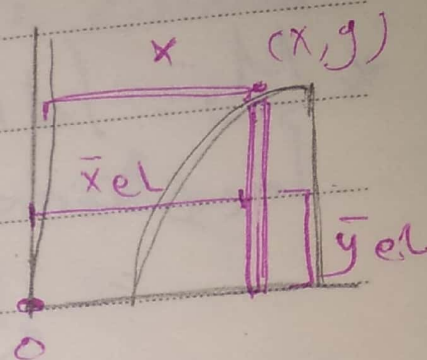
* center of gravity of a 2D Body

$$\sum m_y \bar{x} w = \sum x \Delta w \rightarrow \int x dw$$

$$\sum m_x \bar{y} w = \sum y \Delta w \rightarrow \int y dw$$

$$\bar{x}A = \int_{el} \bar{x} dA = \int x(y dx)$$

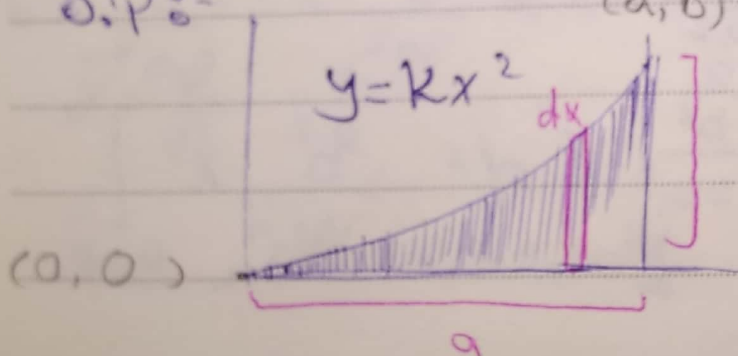
$$\bar{y}A = \int_{el} \bar{y} dA = \int \frac{y}{2} (y dx)$$



$$\bar{x}A = \int_{el} \bar{x}_{el} dA = \int \frac{a+x}{2}$$

S.P:-

(a, b) determine the centroid



b a, b a, b k

$$x = a, y = b$$

$$y = \frac{b}{a^2} x^2$$

$$b = k a^2 \rightarrow k = \frac{b}{a^2}$$

vertical strip

$$dA = \left(\frac{b}{a^2} x^2 \right) (dx)$$

Find M of A about y = A * x

$$A = \int_0^a \frac{b}{a^2} x^2 dx = \frac{b}{a^2} \left[\frac{x^3}{3} \right]_0^a = \frac{ba}{3}$$

$$* M_y = \int_0^a dM_y = \int_0^a \left(\frac{b}{a^2} x^2 \right) dx \cdot x$$

$$= \int_0^a \frac{b}{a^2} x^3 dx = \frac{b}{a^2} \left[\frac{x^4}{4} \right]_0^a$$

$$= \frac{b a^2}{4}$$

$$* \text{Location } \bar{x} = \frac{\frac{b a^2}{4}}{\frac{b a}{3}} = \frac{\epsilon L}{\epsilon A}$$

$$= \frac{3a}{4} \rightarrow \bar{x}$$

$\bar{y}$:

$$dM_x = dA \cdot x \cdot \frac{y}{2}$$

$$\frac{b}{a^2} x^2 dx \cdot \frac{b}{2a^2} x^2 = \frac{b^2}{2a^4} x^4 dx$$

$$M_x = \int_0^a \frac{b^2}{2a^4} x^4 dx = \frac{b^2}{10a^4} x^5 \Big|_0^a$$

$$= \frac{b^2 a}{10}$$

$$\bar{y} = \frac{M_x}{A} = \frac{\frac{b^2 a}{10}}{\frac{ba}{3}} = \frac{3b}{10}$$

SUN.

MON.

TUES.

WED.

THUR.

FRI.

SAT.

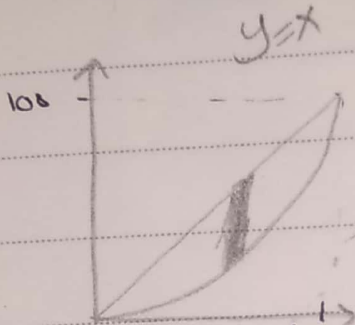
DATE

محل الدرس

Ch5 - center of mass

* center of area

لحساب النقطه في بياض عليها الجسم



$$A = \int dA$$

$$\bar{x} = \int \tilde{x} dA$$

$$\bar{y} = \int \tilde{y} dA$$

$$\tilde{y} = \frac{y_1}{2} + \frac{y_2}{2}$$

شريحة طولية

"بضوء كل y تقسمها بال x"

$$\tilde{x} = x, \quad dA = (y_1 - y_2) dx$$

شريحة بالعرض

$$\tilde{x} = \frac{x_1}{2} + \frac{x_2}{2}$$

$$\bar{y} = y, \quad dA = (x_1 - x_2) dy$$

$$* A = \int dA, \quad \bar{x} = \frac{\int \tilde{x} dA}{A}, \quad \bar{y} = \frac{\int \tilde{y} dA}{A}$$

$$* dA = y dx, \quad dA = x dy$$

بالعرض

بالطول

$$\tilde{x} = x, \quad \tilde{y} = \frac{y}{2}$$

$$\tilde{x} = \frac{x}{2}, \quad \tilde{y} = y$$

NO.

DATE

SUN.

☐

MON.

☐

TUES.

☐

WED.

☐

THUR.

☐

FRI.

☐

SAT.

☐

Wire :- $\bar{x} = \frac{\int x \, dL}{L}$, $\bar{y} = \frac{\int y \, dL}{L}$

$I_x = E \bar{x} A$, $I_y = E \bar{y} A$

⊙ Q_x , Q_y

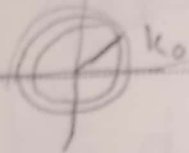
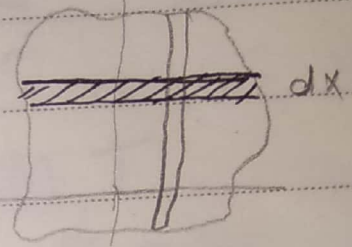
Ch6:-

Radius of gyration of an area :-

$$I_x = A(k_x)^2$$

↳ radius of gyration

$$I_y = K_y^2 A \rightarrow K_y = \sqrt{\frac{I_y}{A}}$$



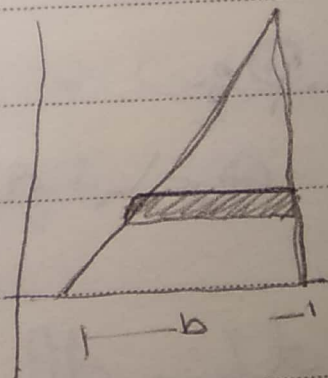
$$I_y = K_y^2 A \rightarrow K_y = \sqrt{\frac{I_y}{A}}$$

$$I_o = K_o^2 A \rightarrow K_o = \sqrt{\frac{I_o}{A}}$$

$$* K_o^2 = K_x^2 + K_y^2$$

S.P:-

determine the moment about the base.



بأخذ شريحة عرضية dx على مسافة y من القاعدة

$$dI_x = dA \times y^2 \rightarrow \int$$

المساحة المربعة + المساحة المربعة

NO.

DATE / /

SUN.



MON.



TUES.



WED.



THUR.



FRI.



SAT.



$$dI_x = dy \times \underbrace{L}_{\downarrow}$$

$$\frac{b}{h} = \frac{L}{h-y}$$

$$\downarrow L = b - \frac{b}{h}y$$

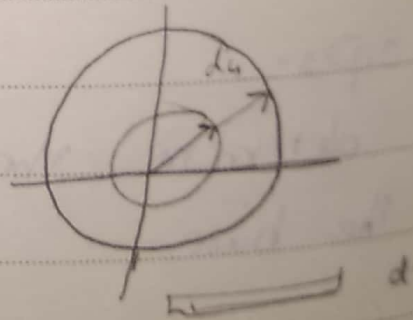
$$dI_x = y^2 dy \left(b - \frac{b}{h}y \right) \Rightarrow by^2 - \frac{b}{h}y^3 dy$$

$$I_x = \int_0^h \left(by^2 - \frac{b}{h}y^3 \right) dy = \left[\frac{b}{3}y^3 - \frac{b}{4h}y^4 \right]_0^h$$

$$= \frac{bh^3}{3} - \frac{b}{4}h^3 = \boxed{\frac{bh^3}{12}}$$

8.P3-

الحل: مساحة دائرة نصف قطرها r

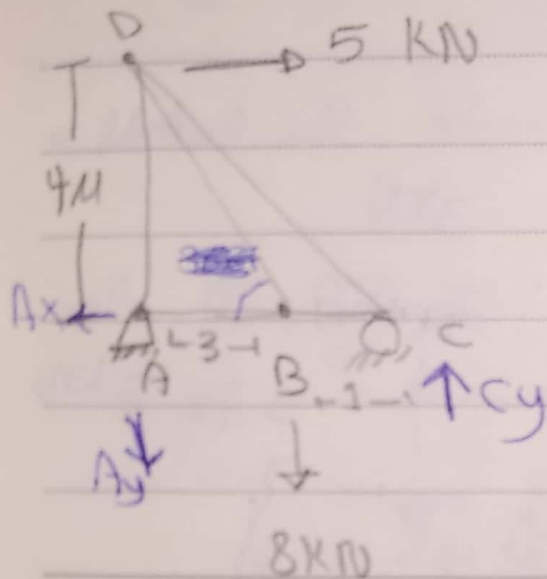


$$dI_o = \underbrace{dA}_{\downarrow} \times y^2$$

$$2\pi r$$

Analysis of Structure

Trusses "Method of joints"



$$AB = 5 \text{ kN T}$$

$$BC = 11 \text{ kN T}$$

$$AD = 3 \text{ kN T}$$

$$CD = 15.58 \text{ kN C}$$

$$BD = 10 \text{ kN T}$$

- 1) Find global equilibrium -
- 2) Find the reaction of the supports.

$$\sum M_A = 0 = -5(4) - 8(3) + C_y(4)$$

$$20 + 24 = 4C_y$$

$$C_y = 11$$

$$\sum F_y = 0 = 11 - 8 - A_y$$

$$A_y = 3 \text{ kN}$$

$$A_x = 5 \text{ kN}$$

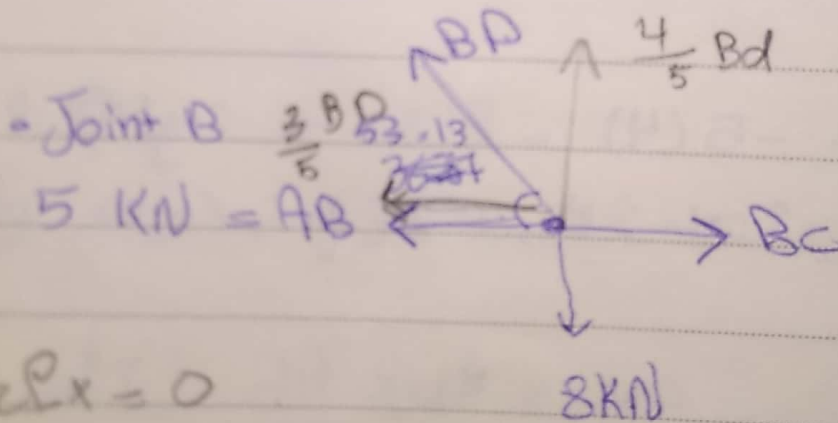
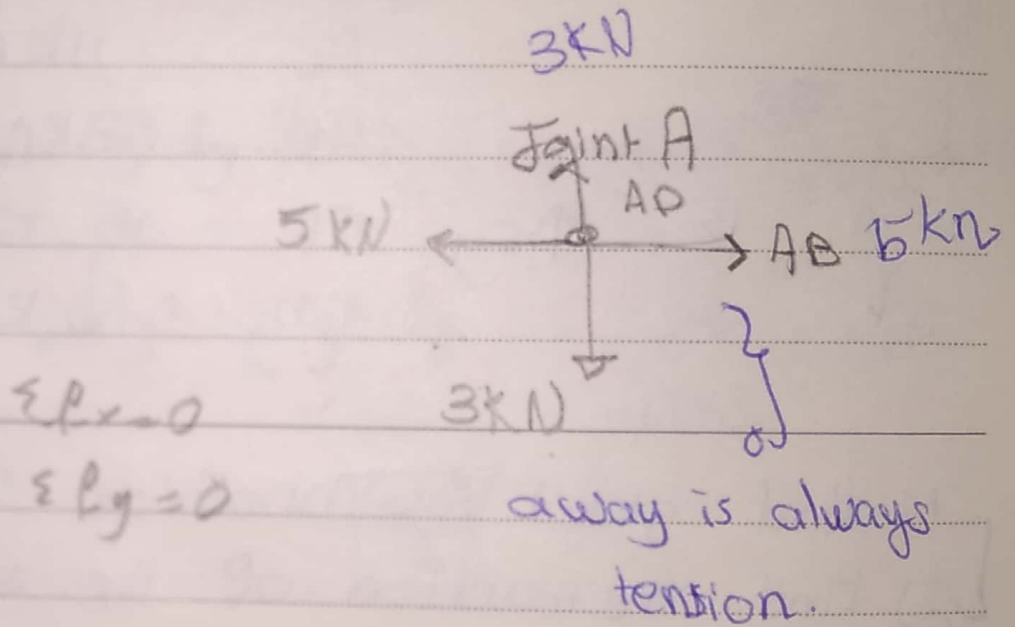
* select a joint

2 unknown as $\sum F_x = 0$ and $\sum F_y = 0$

* Draw FBD of joint

* solve

* Repeat



$$BC = 5 - \frac{3}{5} BD = 0$$

$$BC = 11 \text{ kN}$$

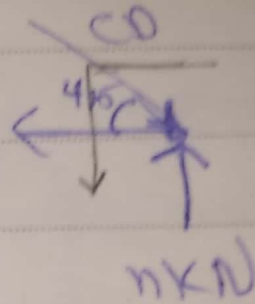
$$\frac{4}{5} BD - 8 = 0 \rightarrow 2) BD = 10 \text{ kN}$$

Joint C :-

$$11 \text{ kN} = BC$$

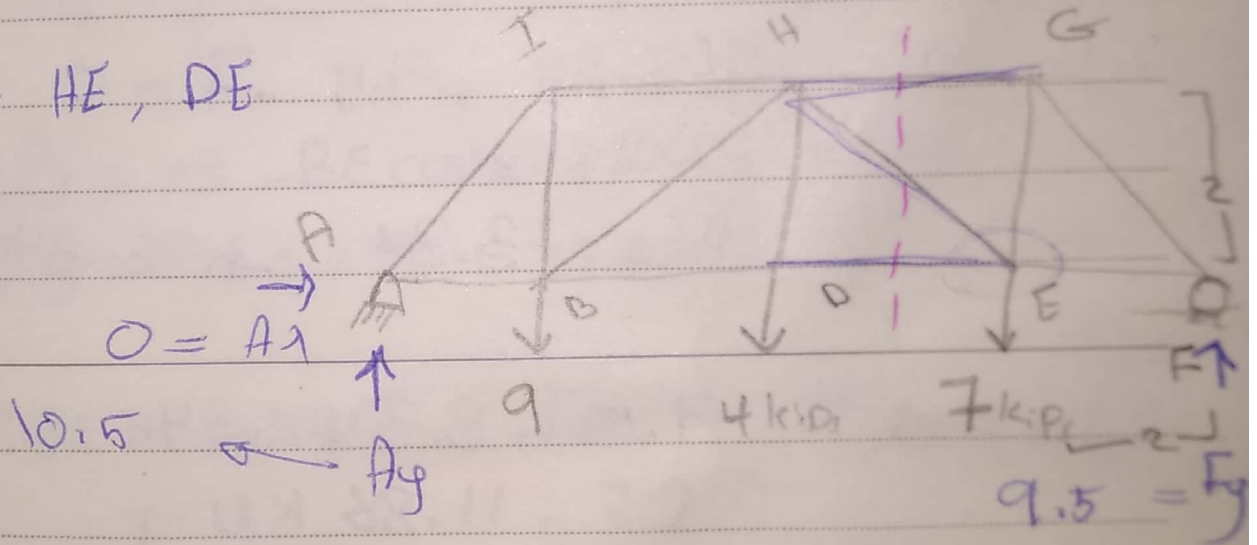
$$\sum F_y = 11 - CD \sin 45 = 0$$

$$CD = 15.56 \text{ kN}$$



* Method of sections

GH, HE, DE



$$\sum M_A = 0 = 8F_y - 7(6) - 4(4) - 9(2)$$

$$F_y = 9.5 \text{ kips.}$$

$$\sum F_y = 0, A_y = 10.5$$

Members 3 قطع الجزء المطلوب
3 قطع لا تتر من 3

$$\sum F_x = 0, \sum F_y = 0, \sum M = 0$$

ما رسم الجانب الآخر

NO.

DATE / /

$$9.5 = HG$$

SUM

MON.

TUES.

WED.

THUR.

FRI.

SAT.

$$3.34$$

HE

DE

45

7

9.5

$$\sum M_E = 0 = -HG(2) + 9.5(2) = 0$$

$$HG = -9.5 \Rightarrow 9.5 \text{ C}$$

$$\sum F_y = 0 = HE \sin 45 + 9.5 - 7 = 0$$

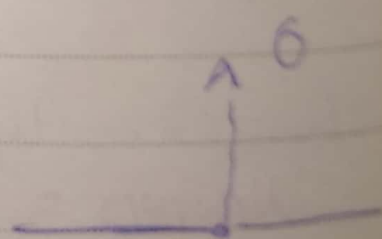
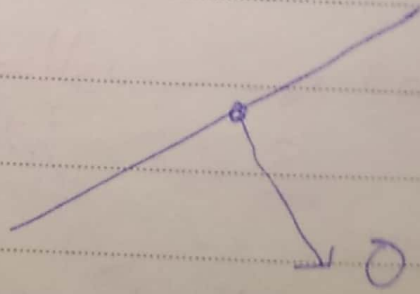
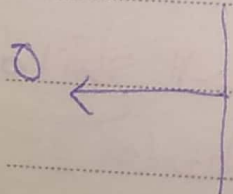
$$HE =$$

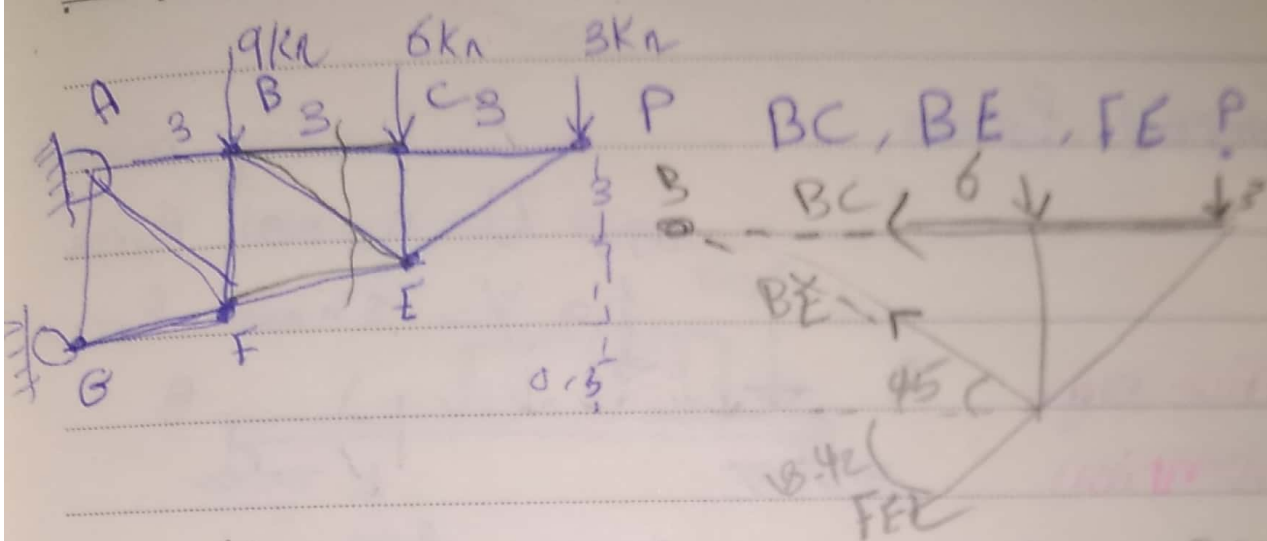
$$HE = -3.34 \Rightarrow 3.34 \text{ C}$$

$$\sum F_x = 0 = 9.5 + 3.34 \cos 45 - DE = 0$$

$$DE = 11.88 \text{ KN T}$$

Zero Force members





$$\sum M_E = 0 = BC(3) - 3(3) \rightarrow BC = 3 \text{ kN T}$$

$$\sum F_x = -3 - BE \cos 45 - FE \cos 18.42 = 0$$

$$\sum F_y = BE \sin 45 - FE \sin 18.42 - 6 - 3 = 0$$

$$BE = 8.49 \text{ T} \quad FE = -9.49 = 9.49 \text{ C}$$

NO.

DATE

SUN.

MON.

TUES.

WED.

THUR.

FRI.

SAT.

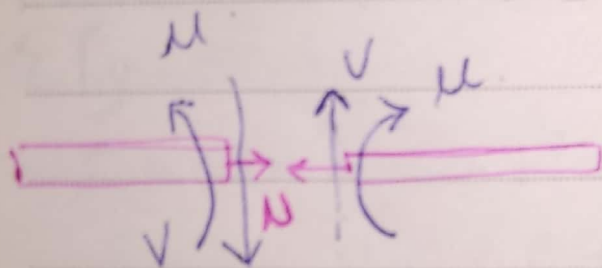
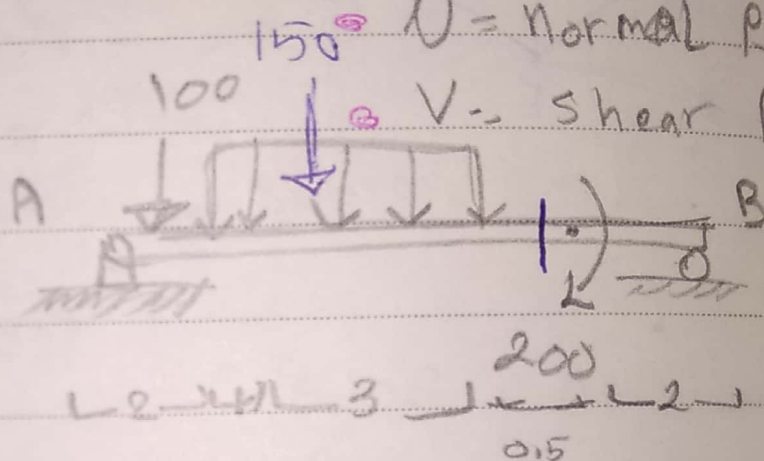
Internal Forces

μ = bending moment

U = Normal Force

V = Shear Force

Positive sign convention



$$\sum M_A = 0 \rightarrow B_y =$$