9.1 Developing Null and Alternative Hypotheses

- Null Hypotheses : H_0 (Always contain equality)
- Alternative Hypotheses: A_a (what the test is attempting to establish) "Research Hypotheses"

- Three forms of hypotheses tests used to test the population parameters μ and p :

① $H_0 : \mu \geq \mu_0$
$H_a : \mu < \mu_0$
lower tail test

② $H_0 : \mu \leq \mu_0$
$H_a : \mu > \mu_0$
upper tail test

One-tailed tests

③ $H_0 : \mu = \mu_0$
$H_a : \mu \neq \mu_0$

Two-tailed test

- In the three hypotheses tests above;
  1. If Ho can be rejected, then the research hypotheses Ha is supported.
  2. If Ho can not be rejected, then there is no evidence that the research hypothesis Ha is supported. → In test ② this means accept Ho.

Example (Q2 page 336) The manager of an automobile dealership is considering a new plan designed to increase sales volume. Currently, the mean sales volume is 14 automobile per month.

(a) Develop the null and alternative hypothesis?

$H_0 : \mu \leq 14$
$H_a : \mu > 14$

(b) Comment on the conclusion when Ho cannot be rejected.

There is no evidence that the new plan increases sales

(c) Comment on the conclusion when Ho can be rejected.

The research hypotheses $\mu > 14$ is supported.

"The new plan increases the sales volume"

- Null hypotheses(H_0): is the Hypotheses assumed to be true in the hypotheses testing procedure.
- Alternative hypotheses (H_a): is the Hypotheses concluded to be true if Ho is rejected.

9.2 Type I and Type II Errors

	Population Condition: H_0 True	Population Condition: H_a True
Conclusion: Accept H_0	Correct Conclusion	Type II Error
Conclusion: Reject H_0	Type I Error	Correct Conclusion

- Type I error: The error of rejecting H_0 when H_0 is true
- Type II error: The error of accepting H_0 when H_0 is false.

Example (Q6 page 338) The claim: the orange juice contains an average of 1 gm of fat or less.

[a] Develop the null and alternative hypothesis? $H_0: \mu \le 1$, $H_a: \mu > 1$

[b] What is Type I error?

Claiming $\mu > 1$ when it is false..
" reject $(\mu \le 1)$ when it is true "

[c] What is Type II error?

Claiming $\mu \le 1$ when it is false.

* level of significance (alpha α): is the prob. of making a Type I error when H_0 is true as an equality.

$\Downarrow$

$H_0: \mu = 1$
$H_a: \mu \ne 1$ used in two tailed test.

* usually α is 5% or 1%.

* If the cost of making type I error is high, then small α are prefered.
* " " " " " " " " small, " large " " ".

* Applications of hypotheses testing only control for type I error are called significant tests.

* If the sample data are consistent with H_0, to conclude we use "do not reject" H_0.
$\Rightarrow$ This conclusion is prefered over "accept H_0" if we use type I error
$\Rightarrow$ The conclusion "accept H_0" is prefered over " do not reject" H_0, if we control by type II error.

9.3 Hypothesis Testing about the Population Mean (μ) when σ is known

Test statistics: A statistic whose value helps to determine whether H_0 should be rejected.

For example: the test statistic for hypothesis tests about population mean when σ is known is

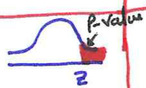
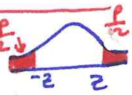
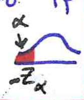
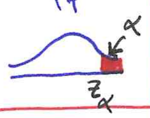
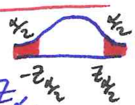
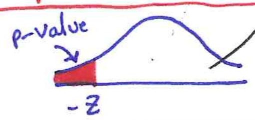
$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

There are two approaches in Hypothesis Testing:

1. p-value approach: uses the value of the test statistic z to compute p-value.

   p-value: is the prob. that provides a measure of the evidence against H_0 provided by the sample. "Smaller p-values indicate more evidence against H_0".

2. Critical value approach: uses a critical value (z_α or $z_{\alpha/2}$) to compare with the test statistic z in order to determine wheather H_0 should be rejected.

	Lower Tail Test	Upper Tail Test	Two Tailed Test
Hypotheses	$H_0: \mu \geq \mu_0$ $H_a: \mu < \mu_0$	$H_0: \mu \leq \mu_0$ $H_a: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_a: \mu \neq \mu_0$
Test statistic	$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$	$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$	$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$
Rejection Rule using p-value approach	Reject H_0 if p-value $\leq \alpha$	Reject H_0 if p-value $\leq \alpha$ 	Reject H_0 if p-value $\leq \alpha$ 
Rejection Rule using Critical value approach	Reject H_0 if $z \leq -z_\alpha$ 	Reject H_0 if $z \geq z_\alpha$ 	Reject H_0 if $z \leq -z_{\alpha/2}$ or $z \geq z_{\alpha/2}$ 
			If the confidence interval $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ contains μ_0, do not reject H_0. Otherwise, reject H_0.

Example (Q9 page 350)

Consider the following hypothesis test:

$H_0: \mu \geq 20$

$H_a: \mu < 20$

lower Tail Test

A sample of 50 provided a sample mean of 19.4

The population standard deviation is 2

[a] Compute the value of the test statistic?

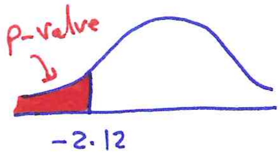
$n = 50,\ \sigma = 2$

$\bar{x} = 19.4,\ \mu_0 = 20$

$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{19.4 - 20}{\frac{2}{\sqrt{50}}} = \frac{-0.6}{0.283} = -2.12$$

[b] what is the p-value?

From the standard normal table, we have

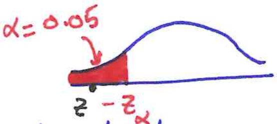


p-value = 0.0170

[c] Using $\alpha = 0.05$, what is your conclusion?

Reject H_0 since p-value $= 0.0170 \leq \alpha = 0.05$

[d] what is the rejection rule using the critical value? what is your conclusion?



Reject H_0 if $z \leq -z_\alpha = -1.645$

From the standard normal table $\Rightarrow -z_\alpha = -1.645$

since $-2.12 \leq -1.645$, we reject H_0

~~Example~~ (Q 10 page 351) Consider the following hypothesis test

$H_0: \mu \leq 25$

$H_a: \mu > 25$

Upper Tail Test

A sample of 40 provided a sample mean of 26.4.

The population standard deviation is 6.

[a] Compute the value of the test statistic.

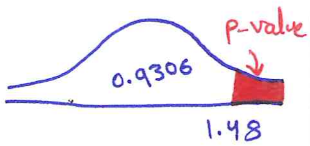
$n = 40,\ \sigma = 6$

$\bar{x} = 26.4,\ \mu_0 = 25$

$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{26.4 - 25}{\frac{6}{\sqrt{40}}} = \frac{1.4}{0.949} = 1.48$$

[b] what is the p-value?

From the standard normal table, we have



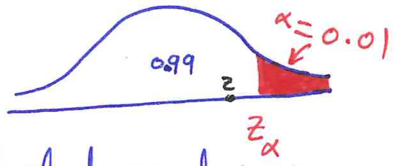
p-value $= 1 - 0.9306 = 0.0694$

[c] At $\alpha = 0.01$, what is your conclusion?

Do not reject H_0 since p-value $> \alpha$ i.e. $0.0694 > 0.01$

d) what is the rejection rule using the critical value? what is your conclusion?

Reject H_0 if $z \geq z_\alpha = 2.33$



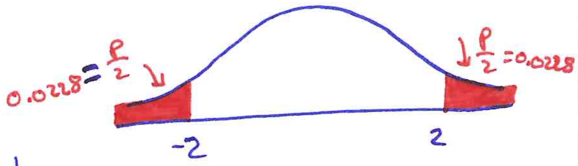
since $1.48 < 2.33$, do not reject H_0 | from the standard normal table, we have $z_\alpha = z_{0.01} = 2.33$

Example (Q11 page 351) Consider the following hypothesis test $H_0: \mu = 15$, $H_a: \mu \neq 15$ Two Tailed Test

A sample of 50 provided a sample mean of 14.15. The population standard deviation is 3.

a) Compute the value of the test statistic $n = 50$, $\sigma = 3$, $\bar{x} = 14.15$, $\mu_0 = 15$

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{14.15 - 15}{\frac{3}{\sqrt{50}}} = -2$$



b) Compute the p-value?

From the standard normal table, we have

p-value $= 0.0228 + 0.0228 = 0.0456$

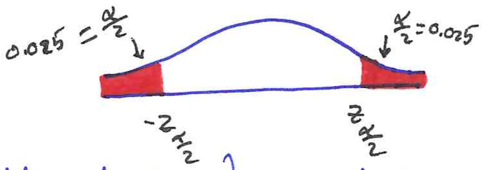
c) At $\alpha = 0.05$, what is your conclusion?

Reject H_0 since p-value $= 0.0456 \leq \alpha = 0.05$.

d) what is the rejection rule using the critical value? what is your conclusion.

Reject H_0 if $z \leq -z_{\frac{\alpha}{2}} = -1.96$

or if $z \geq z_{\frac{\alpha}{2}} = 1.96$



since $-2 \leq -1.96$, we reject H_0 | From the standard normal table, we have $-z_{\frac{\alpha}{2}} = -1.96$

Notes:

- If the sample size $n \geq 30$, then we can use hypothesis tests above
- If the sample size $n < 30$ and the population is normally distribution, then we can use the hypothesis tests above. (exact)
- If the sample size $n < 30$ and " " not " " but is symmetric, then sample size as small as 15 is good to be able to provid acceptable results.

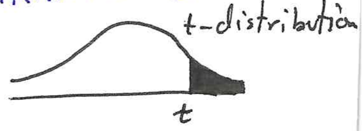
9.4 Hypothesis Testing about the population Mean (μ) when σ is Unknown.

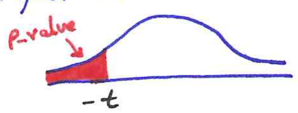
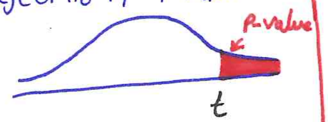
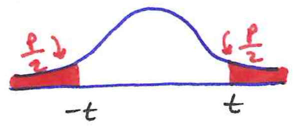
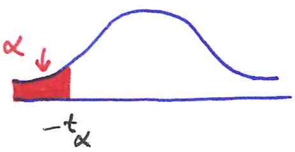
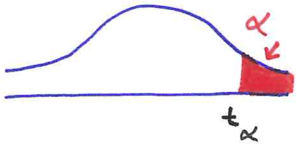
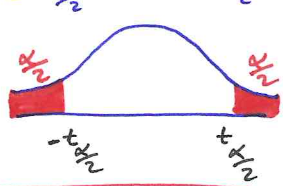
* When σ is Known, the sampling distribution of the test statistic z has a standard normal distribution. see pages 581-582

* when σ is unknown, the sampling distribution of the test statistic t has a t distribution see pages 583-585

⇒ The test statistic for hypothesis tests about the population mean μ when σ is unknown is

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$



	lower Tail Test	Upper Tail Test	Two Tailed Test
Hypothesis	$H_0: \mu \geq \mu_0$ $H_a: \mu < \mu_0$	$H_0: \mu \leq \mu_0$ $H_a: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_a: \mu \neq \mu_0$
Test statistic	$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$	$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$	$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$
Rejection Rule using p-value approach	Reject H_0 if P-value $\leq \alpha$ 	Reject H_0 if P-value $\leq \alpha$ 	Reject H_0 if P-value $\leq \alpha$ 
Rejection Rule using Critical value approach	Reject H_0 if $t \leq -t_\alpha$ 	Reject H_0 if $t \geq t_\alpha$ 	Reject H_0 if $t \leq -t_{\alpha/2}$ or $t \geq t_{\alpha/2}$ 

* when the population is normally distributed, the hypothesis tests provide exact results

* " " " " not " " " , " " " " approximation.

* If $n \geq 30$, then the hypothesis tests provid a good results.

* If the population is approximately normal, then small sample sizes ($n < 15$) will provid acceptable results.

* If the population is highly skewed or contains outliers, then $n \geq 50$ is recommended.

Example (Q 23 page 357) Consider the following hypothesis $H_0: \mu \leq 12$
$H_a: \mu > 12$

A sample of 25 provided a sample mean 14 and a sample standard deviation $s = 4.32$.

Upper Tail Test

a Compute the value of test statistics.

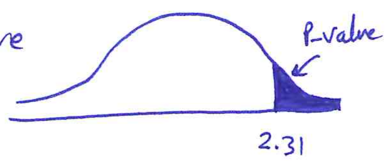
$\bar{x} = 14$, $\mu_0 = 12$
$s = 4.32$, $n = 25$, $d.f = 24$

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{14 - 12}{\frac{4.32}{\sqrt{25}}} = \frac{2}{0.864} = 2.31$$

b Compute the range for the p-value (use table of t-distribution)

$d.f = 24 \Rightarrow$ from the t table, we have

p is between 0.01 and 0.025

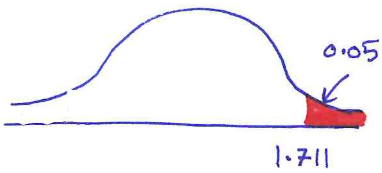


c At $\alpha = 0.05$, what is your conclusion?

Reject H_0 since p-value $\leq \alpha = 0.05$

d what is the rejection rule using the critical value?
what is your conclusion.

Reject H_0 if $t \geq t_\alpha = t_{0.05} = 1.711$



From the t table, we have $t_{0.05} = 1.711$

since $2.31 \geq 1.711$, so we reject H_0.

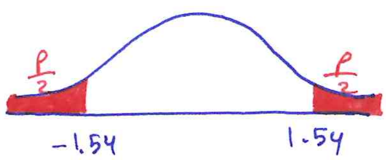
Example (Q24 page 357) Consider the following hypothesis $H_0: \mu = 18$
$H_a: \mu \neq 18$

A sample of 48 provided a sample mean $\bar{x} = 17$ and a sample standard deviation $s = 4.5$

Tow Tailed Test

a Compute the value of the test statistic. $n = 48$, $\bar{x} = 17$, $\mu_0 = 18$, $s = 4.5$

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{17 - 18}{\frac{4.5}{\sqrt{48}}} = \frac{-1}{0.65} = -1.54$$



b Use the t distribution table to compute a range for p-value?

From the t-table, we have $\frac{p}{2}$ is between 0.05 and 0.10

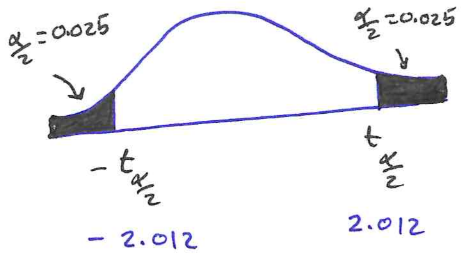
$\Rightarrow p$ is between 0.10 and 0.20

c At $\alpha = 0.05$, what is your conclusion?

Do not reject H_0 since p-value $> \alpha = 0.05$

d what is the rejection rule using the critical value? what is your conclusion?

$\alpha = 0.05$, $d.f = 47$



From the table, we have $t_{\frac{\alpha}{2}} = t_{0.025} = 2.012$

- Reject Ho if $t \le -t_{\frac{\alpha}{2}} = -2.012$ or

  $t \ge t_{\frac{\alpha}{2}} = 2.012$

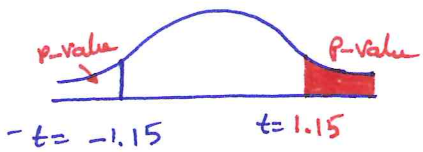
- since $t = -1.54 > -2.012$, we do not reject Ho.

Example (Q 25 page 357) Consider the following hypothesis test $H_0: \mu \ge 45$
$H_a: \mu < 45$

A sample of 36 is used. Identify the p-value and state your conclusion for the following sample results: (Use $\alpha = 0.01$) lower Tail Test

[a] $\bar{x} = 44$ and $s = 5.2$ $n = 36$, $d.f = 35$, $\mu_0 = 45$

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{44 - 45}{\frac{5.2}{\sqrt{36}}} = \frac{-1}{0.87} = -1.15$$



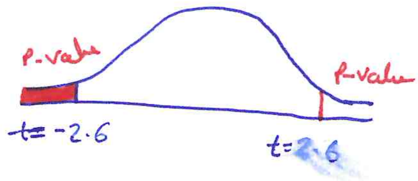
From the t-table we have p is between 0.10 and 0.20

$p \approx \frac{0.1 + 0.2}{2} = 0.15$

Don't reject Ho since p-value $= 0.15 > 0.01$

[b] $\bar{x} = 43$ and $s = 4.6$

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{43 - 45}{\frac{4.6}{\sqrt{36}}} = \frac{-2}{0.77} = -2.6$$



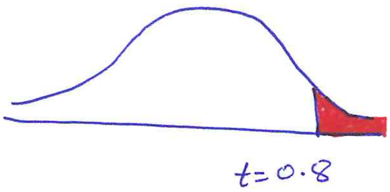
From the t-table, we have p is between 0.005 and 0.01

$p \approx 0.0075$

reject Ho since p-value $= 0.0075 < 0.01$

[c] $\bar{x} = 46$ and $s = 5$

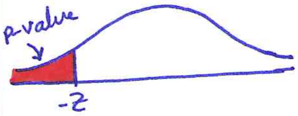
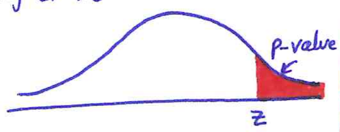
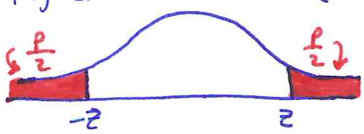
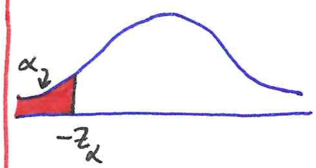
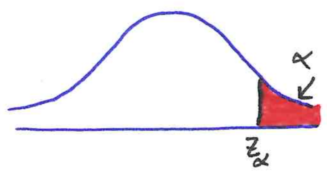
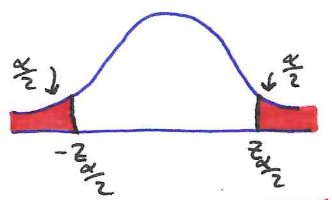
$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{46 - 45}{\frac{5}{\sqrt{36}}} = \frac{1}{1.25} = 0.8$$



From the t-table, we have p is between 0.20 and more

Don't reject Ho since p-value > 0.01

9.5 Hypothesis Testing about Proportion (p)

	Lower Tail Test	Upper Tail Test	Two Tailed Test
Hypothesis	$H_0: p \geq p_0$ $H_a: p < p_0$	$H_0: p \leq p_0$ $H_a: p > p_0$	$H_0: p = p_0$ $H_a: p \neq p_0$
Test statistic	$z = \dfrac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	$z = \dfrac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	$z = \dfrac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$
• Rejection Rule using p-value approach	Reject Ho if p-value $\leq \alpha$ 	Reject Ho if p-value $\leq \alpha$ 	Reject Ho if p-value $\leq \alpha$ 
• Rejection Rule using critical value approach	Reject Ho if $z \leq -z_\alpha$ 	Reject Ho if $z \geq z_\alpha$ 	Reject Ho if $z \leq -z_{\alpha/2}$ or $z \geq z_{\alpha/2}$ 

* The procedure used to construct hypothesis test about population proportion p is similar to the procedure used to construct hypothesis test about the population mean

* We assume $np \geq 5$ and $n(1-p) \geq 5$ so that the normal prob. dist. can be used to approximate the sampling distribution of $\bar{p}$ "which is a discreat binomial dist."

* The standard error of $\bar{p}$ is $\sigma_{\bar{p}} = \sqrt{\dfrac{p_0(1-p_0)}{n}}$

Example (Q 35 page 362) Consider the hypothesis test $H_0: p = 0.20$, $H_a: p \neq 0.20$

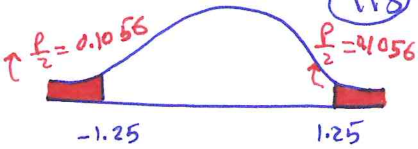
A sample of 400 provided a sample proportion $\bar{p} = 0.175$ Two Tailed Test

[a] Compute the value of the test statistic ? $p_0 = 0.2$, $\bar{p} = 0.175$, $n = 400$

$$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.175 - 0.20}{\sqrt{\frac{0.2(0.8)}{400}}} = \frac{-0.025}{0.02} = -1.25$$

[b] What is the p-value?

From the standard normal table, we have



p-value $= 0.1056 + 0.1056 = 0.2112$

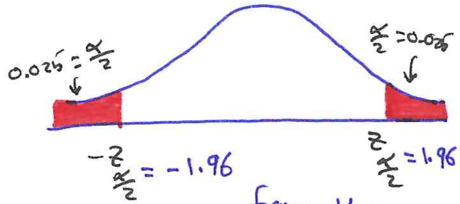
c) At $\alpha = 0.05$, what is your conclusion?

Do not reject Ho since p-value $= 0.2112 > 0.05 = \alpha$

d) what is the rejection rule using the critical value? what is your conclusion?

Reject Ho if $z \le -z_{\frac{\alpha}{2}} = -z_{0.025} = -1.96$ or

if $z \ge z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$



from the standard normal table.

Since $z = -1.25 > -1.96$, we do not reject Ho.

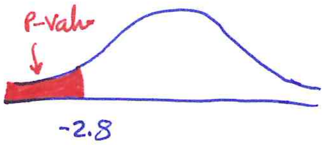
~~Example~~

Q 36 page 362 Consider the hypothesis test

Ho : $p \ge 0.75$
Ha : $p < 0.75$

A sample of 300 items was selected. Compute p-value and state your conclusion for each of the following results (use $\alpha = 0.05$). lower tail test

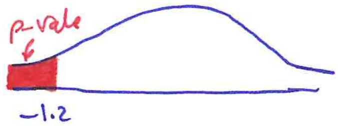
a) $\bar{p} = 0.68$ $z = \dfrac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \dfrac{0.68 - 0.75}{\sqrt{\frac{0.75(0.25)}{300}}} = -2.80$



From the standard normal table, we have p-value $= 0.0026$

Reject Ho since p-value $= 0.0026 \le \alpha = 0.05$.

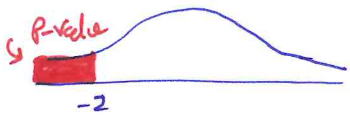
b) $\bar{p} = 0.72$ $z = \dfrac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \dfrac{0.72 - 0.75}{\sqrt{\frac{0.75(0.25)}{300}}} = -1.2$



From the standard normal table, we have p-value $= 0.1151$

Do not reject Ho since p-value $= 0.1151 > 0.05$

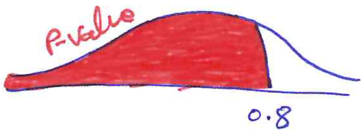
c) $\bar{p} = 0.70$ $z = \dfrac{0.70 - 0.75}{\sqrt{\frac{0.75(0.25)}{300}}} = -2$



From the standard normal table, we have p-value $= 0.0228$

Reject Ho since p-value ≤ 0.05

d) $\bar{p} = 0.77$ $z = \dfrac{0.77 - 0.75}{\sqrt{\frac{0.75(0.25)}{300}}} = 0.8$



From the standard normal table, we have p-value $= 0.7881$

Do not reject Ho since p-value ≥ 0.05.