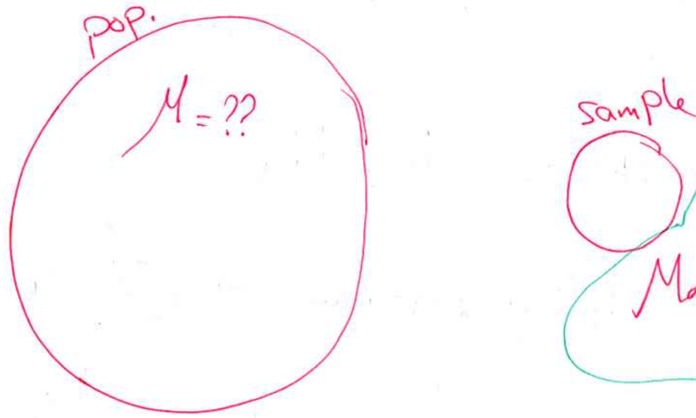
Summary:

Chapter 9: Hypothesis testing

9.1
* We will use sample to test hypothesis about μ.



التلخيص :-

Use sample statistics to make conclusion (prediction) about population

In any hypothesis testing, there are two hypotheses:

1. The Null hypothesis: H_0: is the researcher claim
2. The Alternative hypothesis :- H_a :- is the opposite of H_0

9.2 Type I and Type II

Researcher \ Conclusion	H_0 True	H_0 False
Reject H_0	Type I Error	Correct conclusion
Accept H_0	Correct conclusion	Type II error

Def * Def:- Type I error:- Rejecting H_0 when H_0 is True

significance level α = the prob. of making type I error

Type II error: Accepting H_0 when H_0 is False

9.3 مطلوب σ Known

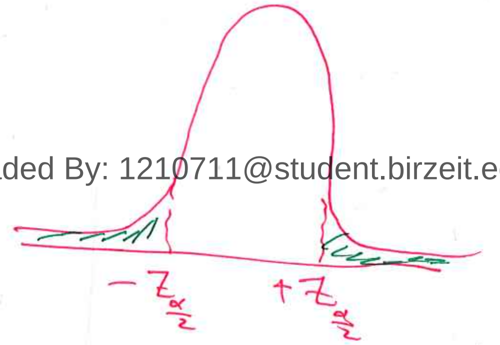
9.4 غير مطلوب σ Un Know

In general, the steps for testing are:-

1. Write the hypotheses H_0, H_a.
2. Find the test Statistics: a value of Z (مطلوب) or t (غير مطلوب)
3. Test using Critical value or P-value Approach
4. Conclusion

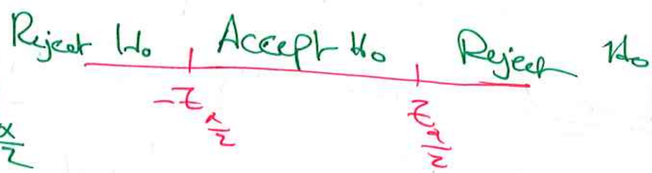
σ: known
test statistics is $Z = \dfrac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$

*** Hypothesis:- $H_0: \mu = \mu_0$
$H_a: \mu \neq \mu_0$ (two tailed test)



Test Statistics :- $Z = \dfrac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$

Critical value : $\pm Z_{\frac{\alpha}{2}}$

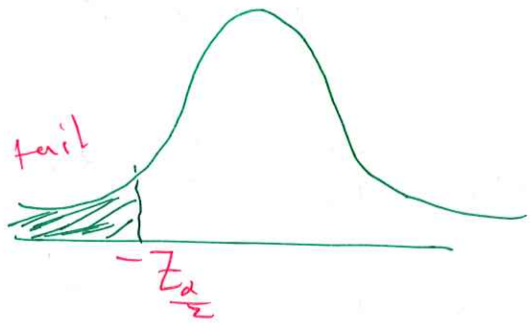


Conclusion :- Reject $H_0 \rightarrow |Z| \geq Z_{\frac{\alpha}{2}}$

Accept $H_0 \rightarrow |Z| < Z_{\frac{\alpha}{2}}$

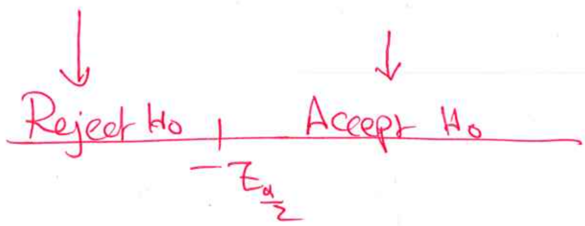
Hypothesis:- $H_0 : \mu \geq \mu_0$ Lower tailed test

$H_a : \mu < \mu_0$



Test Statistics $Z = \dfrac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$

Critical value: $-Z_{\frac{\alpha}{2}}$

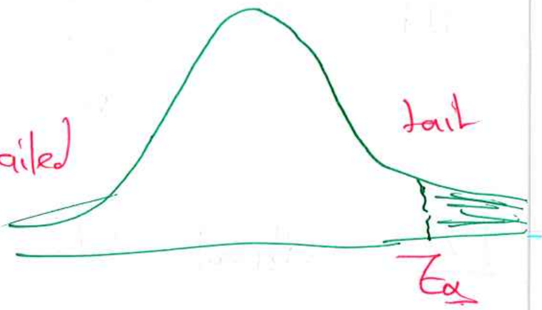


Conclusion $Z \leq -Z_{\frac{\alpha}{2}}$ Reject Ho

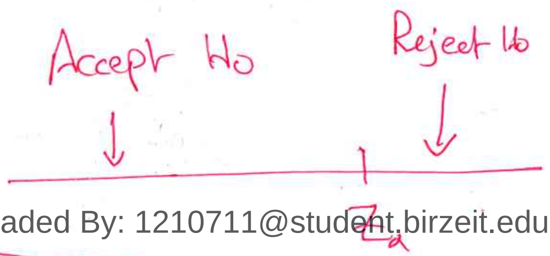
$Z > Z_{\frac{\alpha}{2}}$ Accept Ho

Hypothesis : $H_0 : \mu \leq \mu_0$ upper tailed test

$H_a : \mu > \mu_0$



Test Statistics :- $Z = \dfrac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$



Critical value

$C.V. = \boxed{Z_\alpha}$

Conclusion

$Z \geq Z_\alpha$ Reject Ho

$Z < Z_\alpha$ Accept Ho

Ex:- $H_0 = \mu \leq 20$ given that $n = 35$

$H_a: \mu > 20$ $\bar{x} = 23$

$\sigma = 4.5$ (known)

$\alpha = 10\%$

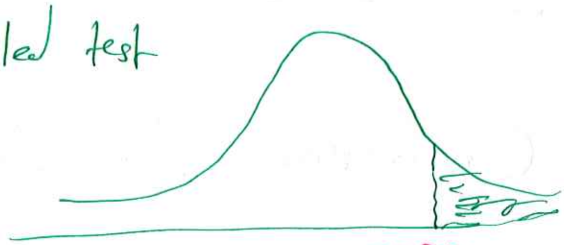
[1] Find the test statistics.

[2] Find the critical value

[3] what is your conclusion

Hypothesis $H_0 = \mu \leq 20$ upper tailed test

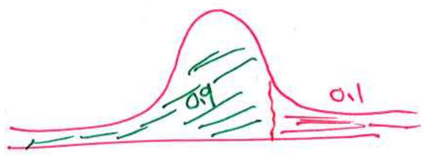
$H_1 = \mu > 20$



[1] $Z = \dfrac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \dfrac{23 - 20}{\frac{4.5}{\sqrt{35}}} = 3.94$ $\mu_0 = 20$

[2] Critical value $Z_\alpha = ??$

$\alpha = 0.10 \longrightarrow Z_{0.1}$



بالطريقه الاولى باستخدام Z table

لا احتاج لـ قيمه n في جدول Z

	0.08	0.09
1.2	0.8997	0.9015

$\boxed{Z_\alpha = Z_{0.1} = 1.28}$

الطريقه الثانيه . باستخدام t table

$\left[t_{\alpha \, Df=\infty} = Z_\alpha \right]$ يجوز اخاد قيمه

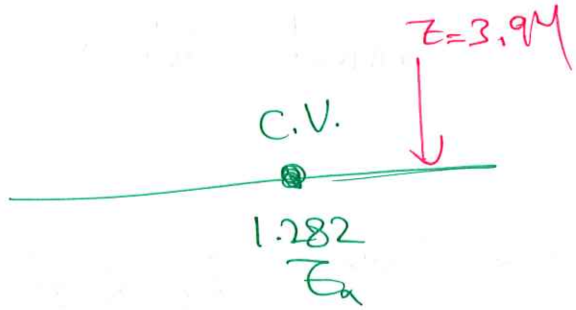
مهما كانت قيمه n

	0.1
∞	1.282

3 **Conclusion:** $Z \geq Z_\alpha$

$3.94 \geq 1.282$

Reject H_0



Exp: $H_0: \mu = 80.42$

$H_a: \mu \neq 80.42$

given than sample size = 100

$\bar{X} = 81$

$\sigma = 15.2$

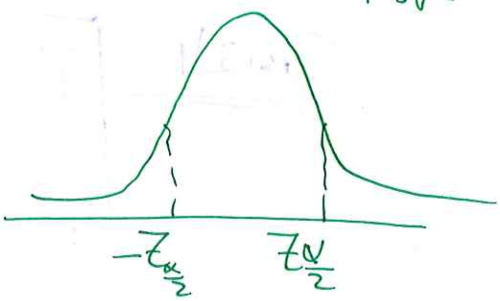
$\alpha = 1\%$

1 Find the test statistics

2 A 1% significance level, what is your conclusion By using critical value Approach

Solution • $Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{81 - 80.42}{15.2/\sqrt{100}} = 0.38$

two tailed test



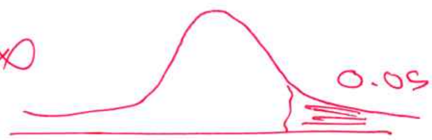
• Critical value $\pm Z_{\frac{\alpha}{2}}$

$\alpha = 0.01 \longrightarrow \frac{\alpha}{2} = 0.005$

$\boxed{Z_{0.005}}$ للكشف

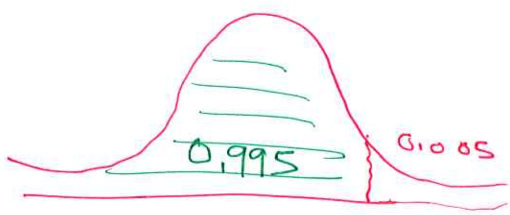
t table باستخدام

$D_f = \infty$



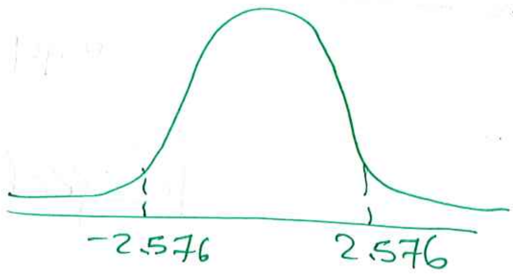
	0.005
∞	2.576

Z table باستخدام



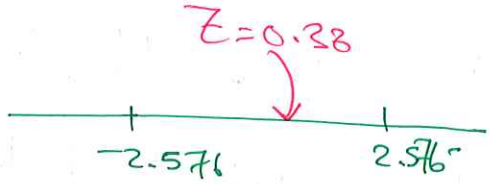
	0.07	0.08
2.5	0.9949	0.9951

Critical Value = ± 2.576



• conclusion: $-Z_{\frac{\alpha}{2}} < Z < Z_{\frac{\alpha}{2}}$

Accept Ho (Don't Reject Ho)



P-value Approach

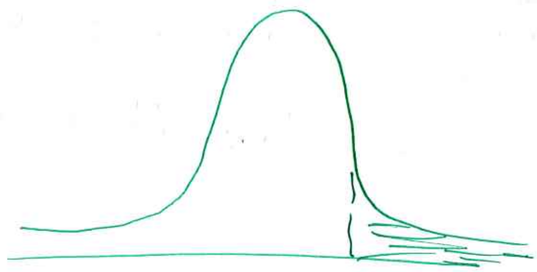
P-value = probability (area) value found using Z-table based on the test statistics and type of test.

ملاحظة

If P-value $\leq \alpha$ Reject
If P-value $> \alpha$ Accept

How to Find P-value?? Use Z-table

* Upper tailed Test



$\boxed{1}$ Hypothesis: $H_0 : \mu \leq \mu_0$

H_a $\mu > \mu_0$

$\boxed{2}$ Test statistics $Z = \dfrac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$

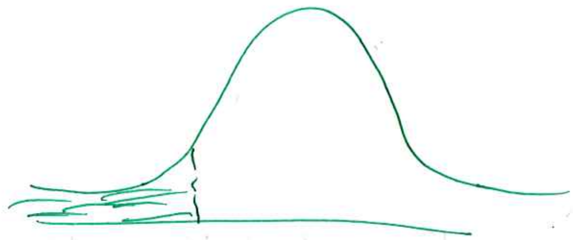
$\boxed{3}$ P_value = Area above Z

$\boxed{4}$ Conclusion

* lower tailed Test

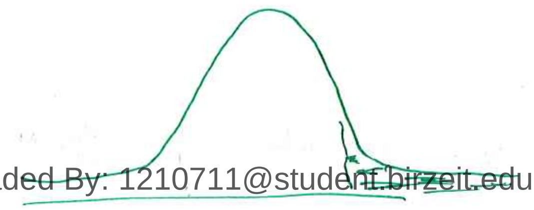
H_0 : $\mu \geq \mu_0$

H_a = $\mu < \mu_0$



* P_value : Area below Z

* Two tailed Test

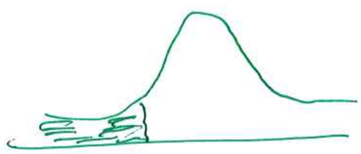


H_0 : $\mu = \mu_0$

H_a : $\mu \neq \mu_0$

* P_value : $2\left(\text{Area above } Z\right)$ if $Z > 0$

$2\left(\text{Area below } Z\right)$ if $Z < 0$



Exp:

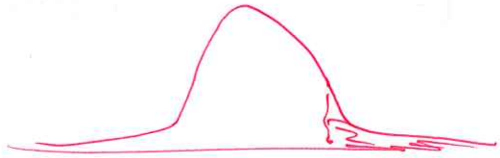
$H_o: \mu \leq 30.5$ $\quad n = 70$

$H_a: \mu > 30.5$ $\quad \bar{X} = 31.5$

$\sigma = 6.02.$

* Hypothesis : $H_o: \mu \leq 30.5$
  $H_a \quad \mu > 30.5$

* Upper tailed



* Test Statistics $Z = \dfrac{31.5 - 30.5}{6.02/\sqrt{70}} = 1.39$

* P-value $P(Z \geq 1.39) = 1 - P(Z \leq 1.39)$
  $1 - 0.9177 = \boxed{0.0823}$

* If $\alpha = 10\%$, what is your conclusion

Since P-value $\leq \alpha$ Reject H_o

* what is your conclusion if $\alpha = 5\%$

Since P-value $\geq \alpha \longrightarrow$ Don't Reject H_o.

Ex: The Mean weight of high school student is 70 Kg. A sample of 50 student is taken, This sample produced a mean of 73, Assume the pop. standard deviation is $\boxed{20}$

$\boxed{1}$ Use significance level of 1% to test the claim that the mean weight is greater than 70 (Critical Value)

$\boxed{2}$ Use significance level of 10% to test the claim that the Mean weight is different from 70 (P_Value)

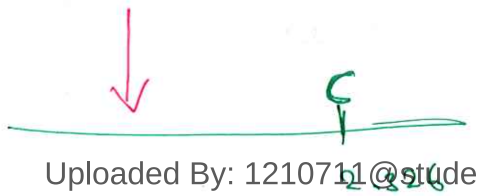
Short Answers

$\boxed{1}$ * Hypothesis $H_0: \mu \leq 70$

$H_a: \mu > 70$

* $Z = 1.06$

* Critical Value $Z_\alpha = 2.326$



* Accept H_0

$\boxed{2}$ * Hypothesis $H_0: \mu = 70$

$\mu \neq 70$

* $Z = 1.06$

* P_value $= 2\,(P \geq 1.06)$

$= 0.2892$

$\boxed{P > \alpha}$

Dont Reject

Exp:

$H_0 : \mu \geq 1220$ $\quad n = 95$ $\quad \bar{X} = 1210$

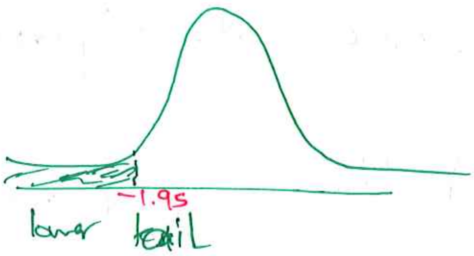
* $\quad \alpha = 1\%$ $\quad \sigma = 50$

$H_a \quad \mu < 1220$

[1] Find test Statestics.

[2] Find P-value

[3] What is your conclusion



$\boxed{*}$ $Z = \dfrac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \dfrac{1210 - 1220}{50 / \sqrt{95}} = -1.95$

$\boxed{**}$ P-value : Area below (-1.95)

$$P(Z \leq -1.95) = P(Z \geq 1.95)$$

$$= 1 - P(Z \leq 1.95)$$

$$= 1 - 0.9744$$

P-value $= 0.0256$

* Conclusion

P-value $> \alpha$ Don't Reject H_0

$0.0256 \geq \alpha$ Accept H_0