Exp

$$f(x) = \begin{cases} x^{\frac{2}{3}}, & x \geq 0 \\ x^{\frac{1}{3}}, & x < 0 \end{cases}$$

0

① Is f cont. at $x=0$

$f(0) = \lim_{x\to 0} f(x)$

$0 = 0$

$f(0) = 0^{\frac{2}{3}} = \sqrt[3]{0^2} = 0$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \sqrt[3]{x^2} = \sqrt[3]{0^2} = 0$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \sqrt[3]{x} = \sqrt[3]{0} = 0$

Yes f is cont. at $x=0$

② Is f diff at $x=0$

$f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h}$

Check $f'_+(0) \stackrel{?}{=} f'_-(0)$

we will use the definition of derivative

$f(h) = \sqrt[3]{h^2}$

$$f'_+(0) = \lim_{h\to 0^+} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0^+} \frac{f(0+h)}{h} = \lim_{h\to 0^+} \frac{f(h)}{h}$$

0

$$= \lim \quad h^{\frac{2}{3}} \quad \lim \quad h^{\frac{2}{3}-1} = \lim \quad h^{-\frac{1}{3}}$$

$$= \lim_{h \to 0^+} \frac{h^{\frac{2}{3}}}{h^{1}} = \lim_{h \to 0^+} h^{3} = \lim_{h \to 0^+} h^{-3}$$

$$= \lim_{h \to 0^+} \frac{1}{\sqrt[3]{h}} = \frac{1}{\text{small} +} = +\infty$$

$$f'_-(0) = \lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^-} \frac{f(h) - 0}{h}$$

$\to 0$

-0.0001

0 h

$f(x) = \sqrt[3]{x}$

$$= \lim_{h \to 0^-} \frac{h^{\frac{1}{3}}}{h^{1}} = \lim_{h \to 0^-} h^{\frac{1}{3} - 1} = \lim_{h \to 0^-} h^{-\frac{2}{3}}$$

$$= \lim_{h \to 0^-} \frac{1}{\sqrt[3]{h^2}} = \frac{1}{\text{small} \#} = +\infty$$

not real #

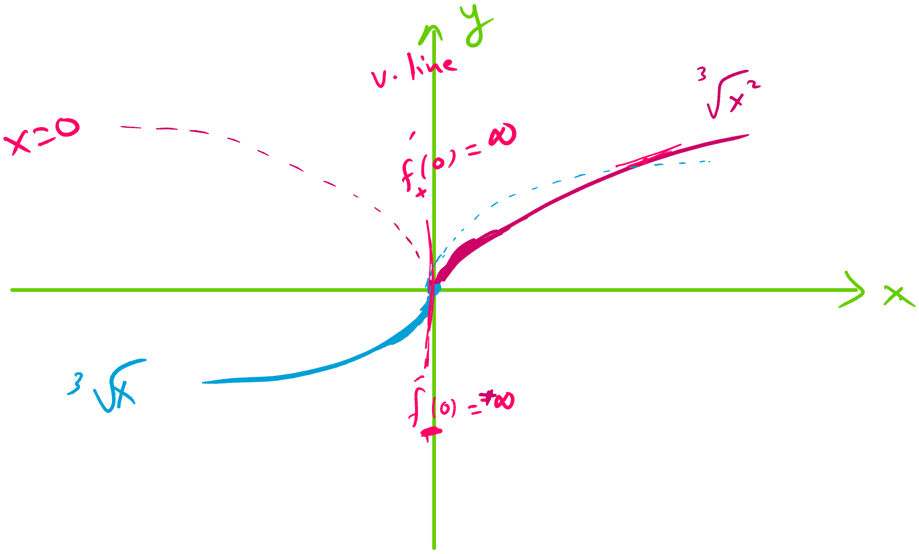
f is not diff at $x = 0$

$$f(x) = \begin{cases} \sqrt[3]{x^2} & \text{if } x \geq 0 \\ \sqrt[3]{} & \text{if } x < 0 \end{cases}$$

$$f(x) = \begin{cases} \ldots \\ \sqrt[3]{x} & \text{if } x<0 \end{cases}$$

f cont. at $x=0$

but f is not diff $x=0$



In general if f is diff at $x=a$

then f is cont. at $x=a$

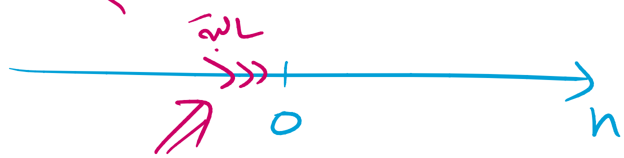
$$f(x) = \begin{cases} x^{\frac{2}{3}} & \text{if } x \geq 0 \\ x^{\frac{1}{3}} & \text{if } x < 0 \end{cases}$$

$$f'(x) = \begin{cases} \frac{2}{3} x^{-\frac{1}{3}} & \text{if } x > 0 \\ \frac{1}{3} x^{-\frac{2}{3}} & \text{if } x < 0 \end{cases}$$

$$f'_+(0) = \lim_{h \to 0^+} \frac{2}{3} \frac{1}{\sqrt[3]{0}} = \frac{1}{\text{small} +} = +\infty$$

$f'_+(0) = \lim_{h \to 0^+} \frac{1}{3} \cdot \frac{1}{\sqrt[3]{0}}$ small +

$$f'_-(0) = \lim_{h \to 0^-} \frac{1}{3} \cdot \frac{1}{\sqrt[3]{x^2}} = \frac{1}{\text{small}+} = +\infty$$



$$\frac{d}{dx}\left(a x^n\right) = a\, n\, x^{n-1}$$

Exp Find $y'(1)$ if $y(x) = (x^3 + 2x)^4$

$$y'(x) = 4(x^3 + 2x)^3(3x^2 + 2)$$

$$y'(1) = 4(1+2)^3(3+2)$$

$$= 4(3)^3(5)$$

$$= 4(27)(5)$$

$$= (20)(27)$$

$$= 540$$

Exp show that $\frac{d}{dx}(\sin x) = \cos x$

$f(x) = \sin x$

$\frac{d}{dx}(\sin x) = \lim \frac{f(x+h) - f(x)}{}$

$$\frac{d}{dx}(\sin x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$f(x) = \sin x$

$\sin(A+B) = \sin A \cos B + \sin B \cos A$

$$= \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cosh + \sinh \cos x - \sin x}{h}$$

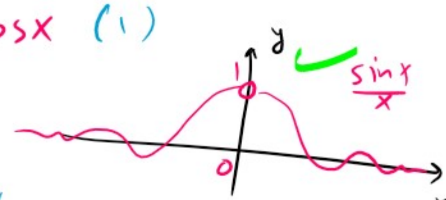
$$= \lim_{h\to 0} \frac{\sin x(\cosh - 1) + \sinh \cos x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x(\cosh - 1)}{h} + \lim_{h\to 0} \frac{\sinh \cos x}{h}$$

$$= \sin x \left(\lim_{h\to 0} \frac{\cosh - 1}{h} \right) + \cos x \left(\lim_{h\to 0} \frac{\sinh}{h} \right) \to 1$$

??

$$= \sin x \lim_{h\to 0} \frac{-2\sin^2 \frac{h}{2}}{h} + \cos x\,(1)$$



$$= -\sin x \lim_{h\to 0} \frac{\sin^2 \frac{h}{2}}{\frac{h}{2}} + \cos x$$

$\cos 2x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$= \cos^2 x - \sin^2 x$

$$= -\sin x \left(\lim_{h\to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}} \right) \sin\frac{h}{2} + \cos x$$

$$= \cos^2 x - \sin^2 x$$

$\cos 2x = 1 - 2\sin^2 x$

$\cos 2h = 1 - 2\sin^2 h$

$\cosh = 1 - 2\sin^2 \frac{h}{2}$

$\cosh - 1 = -2\sin^2\frac{h}{2}$

$$= -\sin x \quad (1) \boxed{\lim_{h\to 0} \sin\frac{h}{2}} +$$

$\sin 0 = 0$

$$= \cancel{-\sin(x)\,(1)}\,(0) + \cos x$$

$$= \cos x$$

$$\frac{d}{dx}(\sin x) = \cos x$$

Exp Given the curve $xy + 2x - y = 0$

① write this curve in the form $y = f(x)$

$y(x-1) + 2x = 0$

$y(x-1) = -2x \implies y = \frac{-2x}{x-1} = \frac{2x}{1-x}$

$$\boxed{f(x) = \frac{2x}{1-x}}$$

② Find $D(f)$

$D(f) = \mathbb{R} \setminus \{1\} = (-\infty, 1) \cup (1, \infty)$

$x \in$

$1 - x \neq 0$

$x \neq 1$

③ Find Asy.

(3) Find H Asy.

$f(x) = \dfrac{2x}{1-x}$ no O. Asy. $\Rightarrow \exists$ H. Asy.

H. Asy $\Rightarrow \lim\limits_{x \to \infty} f(x) = \lim\limits_{x \to \infty} \dfrac{2x}{1-x} = \lim\limits_{x \to \infty} \dfrac{2}{\frac{1}{x} - 1} = \dfrac{2}{0-1}$

$= -2$

✓ $\boxed{y = -2}$ is H. Asy.

$\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} \dfrac{2x}{1-x} = \lim\limits_{x \to -\infty} \dfrac{2}{\frac{1}{x} - 1} = \dfrac{2}{0-1} = -2$

V. Asy $\Rightarrow$ check zeros of denominator.

$\Rightarrow$ check $x = 1$

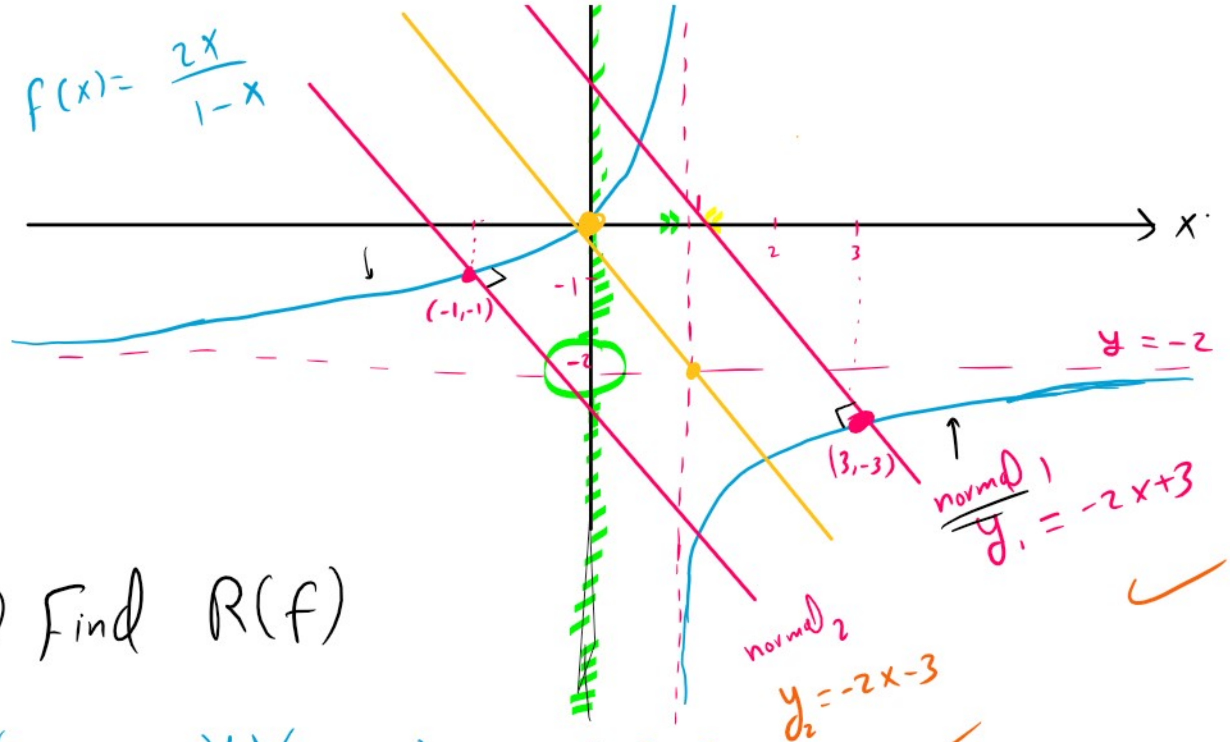
✓

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} \dfrac{2x}{1-x} = \dfrac{2}{\text{small} -} = -\infty$

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} \dfrac{2x}{1-x} = \dfrac{2}{\text{small} +} = +\infty$

$\boxed{x = 1}$ is V. Asy.

(4) sketch $f(x) = \dfrac{2x}{1-x}$ keypoint $(0,0)$



④ Find $R(f)$

$(-\infty, -2) \cup (-2, \infty) = \mathbb{R} \setminus \{-2\}$

y غ

⑤ Draw the line $2x + y = 0$ on the graph

$\boxed{y = -2x}$

(0, 0)

(1, -2)

⑥ Find normal lines to the curve $f(x)$ that are parallel to the line $y = -2x$

- $y = -2x$ has slope $m_1 = -2$
- Normal line has slope $m_1 = -2$

$\underline{\underline{\text{Normal line}}}$

L

$L \perp f(x) = \frac{2x}{1-x}$

ميل

m_2

(ميل المماس)(ميل الـ

m_2

$-1 =$ (ميل العمودي)(ميل المماس)

$-1 = (-2)\ (m_2)$

$m_2 = \frac{1}{2}$

ميل المماس

Tangent

$$y' = f'(x) = \frac{(1-x)(2) - (2x)(-1)}{(1-x)^2}$$

$$\frac{1}{2} = \frac{2 \cancel{-2x} \cancel{+2x}}{(1-x)^2}$$

$$\frac{1}{2} = \frac{2}{(1-x)^2}$$

$$(1-x)^2 = 4$$

$$1 - 2x + x^2 = 4$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3 \quad , \quad x = -1$$

$$f(3) = \frac{2(3)}{1-(3)} = \frac{6}{-2} = -3 \qquad f(-1) = \frac{2(-1)}{1-(-1)} = \frac{-2}{2} = -1$$

$(3,-3)$ $\qquad$ $(-1,-1)$

Normal 1 $\quad \overset{x_0}{3}, \overset{y_0}{-3}$: $(3,-3)$, $m_1 = -2$

$$y - y_0 = m_1 (x - x_0)$$

$$y - -3 = -2(x - 3)$$

$$y + 3 = -2x + 6$$

$$\boxed{y_1 = -2x + 3}$$

Normal 2 $\quad (x_0, y_0) = (-1,-1)$, $m_1 = -2$

$$y - y_0 = m_1 (x - x_0)$$

$$y - -1 = -2(x - -1)$$

$$y + 1 = -2x - 2$$

$$\boxed{y_2 = -2x - 3}$$

8) Find tangent at $x = -1$ $\quad ((-1,-1))$

Tangent $\quad y - y_0 = m_2 (x - x_0)$

$$y - -1 = \frac{1}{2}(x - -1)$$

$y - -1 = \frac{1}{2}(x - -1)$

$y + 1 = \frac{1}{2}x + \frac{1}{2}$

$\boxed{y = \frac{1}{2}x - \frac{1}{2}}$ $\Rightarrow$ slope $\frac{1}{2}$ $\times$ $= -1$

Tangent

$\Rightarrow$ slope -2

Tangent $\perp$ $y = -2x$

But $y = -2x$ not normal on the curve $f(x)$