8.1	21, 28	2, 6, 11, 16, 22, 25, ~~33, 39, 46, 50, 58~~

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

⑥ $\int_1^e \underline{x^3}\ \underline{\ln x}\ \underline{dx}$

$u = \ln x \qquad dv = x^3\,dx$

$du = \dfrac{dx}{x} \xrightarrow{\int} v = \dfrac{x^4}{4}$

$= \int_1^e u\,dv$

$= uv\Big|_1^e - \int_1^e \underline{\underline{v\,du}} \quad = (\ln x)\dfrac{x^4}{4}\Big|_1^e - \int_1^e \dfrac{x^{\cancel{4}\,3}}{4}\dfrac{dx}{\cancel{x}}$

$= \underset{1}{(\ln e)}\dfrac{e^4}{4} - \cancel{(\ln 1)}^{\,0}\dfrac{1}{4} - \dfrac{1}{4}\dfrac{x^4}{4}\Big|_1^e$

$= \dfrac{e^4}{4} - 0 - \dfrac{1}{16}\left[e^4 - 1\right]$

$= \dfrac{e^4}{4} - \dfrac{e^4}{16} + \dfrac{1}{16}$

⑯ $\int p^4 e^{-p}\,dp = \int x^4 e^{-x}\,dx$

$\frac{d}{dx}$		$\int$
x^4		e^{-x}
	+	
$4x^3$		$-e^{-x}$
	−	
$12x^2$		e^{-x}
	+	
$24x$		$-e^{-x}$

$= -x^4 e^{-x} - 4x^3 e^{-x} - 12x^2 e^{-x} - 24x e^{-x} - 24e^{-x} + C$

$^{-x}\,[\;^{4} \quad ^{3} \quad ^{2} \quad]$

$$= -e^{-x}\left[x^4 + 4x^3 + 12x^2 + 24x + 24\right] + C$$

$$\begin{tikzcd}
24x \arrow[dr, "-"] & -e^{-x} \\
24 \arrow[dr, "+"] & e^{-x} \\
\boxed{0} & -e^{-x}
\end{tikzcd}$$

㉒ $\int e^{-y}\cos y \, dy$

$$\begin{tikzcd}
\frac{d}{dx} & \int \\
e^{-y} \arrow[dr, "+"] & \cos y \\
-e^{-y} \arrow[dr, "-"] & \sin y \\
e^{-y} \arrow[r, "\text{stop}"] \arrow[uu, bend left] & -\cos y \arrow[uu, bend right]
\end{tikzcd}$$

$$\int e^{-y}\cos y \, dy = e^{-y}\sin y - e^{-y}\cos y - \int e^{-y}\cos y \, dy$$

$$2\int e^{-y}\cos y \, dy = e^{-y}(\sin y - \cos y)$$

$$\int e^{-y}\cos y \, dy = \frac{e^{-y}}{2}(\sin y - \cos y) + C$$

㉝ $\int x (\ln x)^2 \, dx$

$$u = \ln x \rightarrow e^u = e^{\ln x} \Rightarrow e^u = x$$

$$du = \frac{dx}{x}$$

$$x \, du = dx$$

$$e^u du = dx$$

$$\int e^u \, u^2 \, e^u \, du$$

$$\int u^2 e^{2u} \, du$$

$$\begin{tikzcd}
u^2 \arrow[dr, "+"] & e^{2u} \\
2u \arrow[dr, "-"] & \frac{1}{2}e^{2u} \\
2 \arrow[dr, "+"] & \frac{1}{4}e^{2u} \\
0 & \frac{1}{8}e^{2u}
\end{tikzcd}$$

$$= \frac{u^2}{2}e^{2u} - \frac{u}{2}e^{2u} + \frac{1}{4}e^{2u} + C$$

$$= e^{2u}\left[\frac{u^2}{2} - \frac{u}{2} + \frac{1}{4}\right] + C$$

$$= e^{2\ln x}\left[\frac{(\ln x)^2}{2} - \frac{\ln x}{2} + \frac{1}{4}\right] + C$$

$$\downarrow$$

$$e^{\ln x^2} \quad (\quad)^2 \quad \ln x \quad + C$$

$\boxed{e^{\ln x^2}} \rightarrow x^2\left(\frac{(\ln x)^2}{2} - \frac{\ln x}{2} + \frac{1}{4}\right) + C$

---

(46) $\int \sqrt{x}\, e^{\sqrt{x}}\, dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\, dx$

$= \frac{1}{2u}\, dx$

$2u\,du = dx$

$\int u\, e^{u}\, (2u\,du)$

$= \int 2u^2 e^u\, du$

$= 2u^2 e^u - 4u e^u + 4e^u + C$

$= 2x\, e^{\sqrt{x}} - 4\sqrt{x}\, e^{\sqrt{x}} + 4e^{\sqrt{x}} + C$

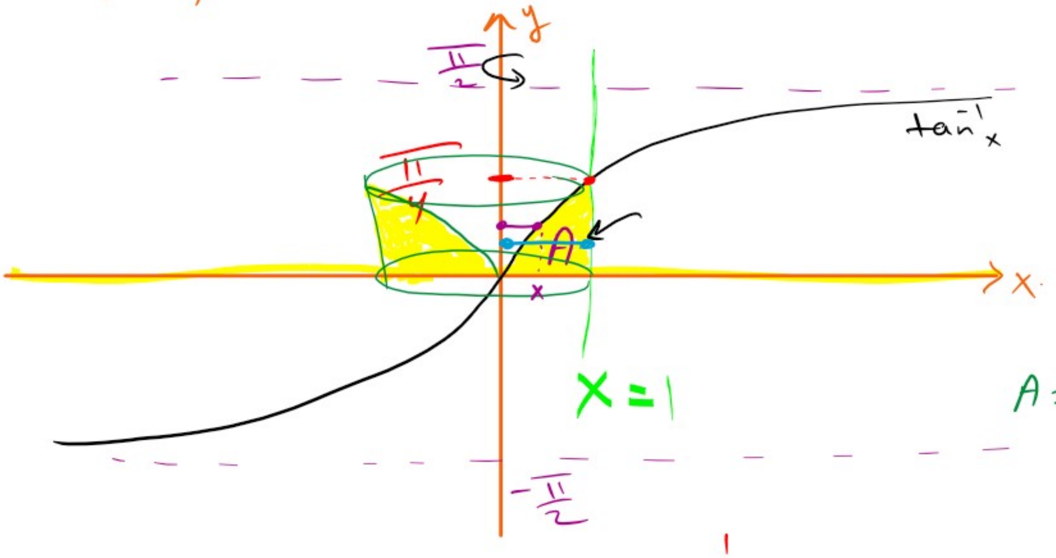
$2u^2$	e^u
$4u$	e^u
4	e^u
0	e^u

$+$, $-$, $+$

---

58 Region bounded $y=0$, $x=1$, $y=\tan^{-1}x$ ✓

(A) Find the area of this region



$A = \int_0^1 \tan^{-1}x\, dx$

$u = \tan^{-1}x \qquad dv = dx$

$du = \frac{dx}{1+x^2} \rightarrow v = x$

$A = x\tan^{-1}x \Big|_0^1 - \int_0^1 \frac{x}{1+x^2}\, dx$

1

$$\Big| -\frac{\pi}{2}$$

$$A = 1\tan^{-1}1 - \cancel{0\tan^{-1}0} \xrightarrow{} 0 - \frac{1}{2}\int_0^1 \frac{2x}{\boxed{1+x^2}}\,dx$$

$$= \frac{\pi}{4} - \frac{1}{2}\ln(\underbrace{1+x^2}_{t})\Big|_0^1$$

$$= \frac{\pi}{4} - \frac{1}{2}(\ln 2 - \underbrace{\ln 1}_{0}) \quad = \underline{\underline{\frac{\pi}{4} - \frac{1}{2}\ln 2}}$$

(B) Find $V_{\downarrow}$ of solid generated if A is rotated about y-axis

$$V = \pi\int_c^d \left(R^2(y) - r^2(y)\right)dy = \pi\int_0^{\frac{\pi}{4}} [1^2 - \underset{\tan y}{\underset{\downarrow}{x^2}}]\,dy$$

$y = \tan^{-1}x$

$\underline{\underline{\tan y}} = \cancel{\tan}(\cancel{\tan^{-1}}x) = x$

$1 + \tan^2 y = \sec^2 y$

$\tan^2 y = \sec^2 y - 1$

$$= \pi\int_0^{\frac{\pi}{4}} (1 - \tan^2 y)\,dy$$

$$= \pi\int_0^{\frac{\pi}{4}} \left(1 - (\sec^2 y - 1)\right)dy$$

$$= \pi\int_0^{\frac{\pi}{4}} (2 - \sec^2 y)\,dy$$

$$= \pi(2y - \tan y)\Big|_0^{\frac{\pi}{4}}$$

$$= \pi\left(\frac{\pi}{2} - 1\right) - 0$$

$$= \pi \left(\frac{\pi}{2} - 1 \right)$$