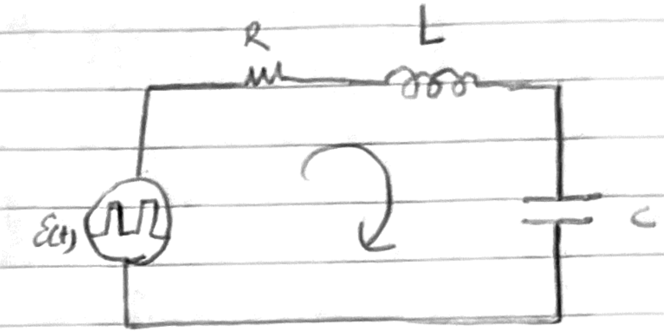
ساره يار

Experiment 7

Damped Oscillations

RLC circuit



$$\varepsilon = IR + L\frac{dI}{dt} + \frac{Q}{C}$$

$$\varepsilon = R\frac{dQ}{dt} + L\frac{d^2Q}{dt^2} + \frac{Q}{C}$$

The solution of this 2nd order linear differential equation is mathematically involved, therefore we solve it by this way (see appedix B)

$$Q(t) = A_1 e^{\lambda_+ t} + A_2 e^{\lambda_- t} \quad (1)$$

$$V(t) = A e^{\lambda_+ t} + B\, e^{\lambda_- t}$$

Where A_1 and A_2 are constants and

$$\lambda_+ = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \quad (2)$$

$$\lambda_- = \frac{-R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

سارم نجار

Case 1 critical damping

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$$

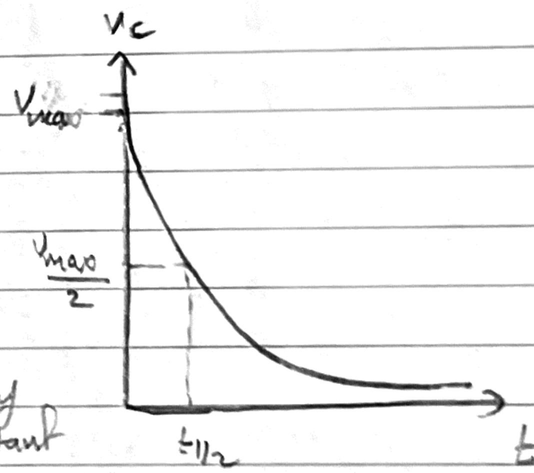
$$\Rightarrow \frac{R^2}{4L^2} = \frac{1}{LC} \Rightarrow \frac{R^2}{4L} = \frac{1}{C}$$

$$\Rightarrow R^2 = \frac{4L}{C} \Rightarrow \boxed{R_{critical} = 2\sqrt{\frac{L}{C}}}$$

in λ the term under the square root $\left(\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right)$

vanishes so

$$\lambda_+ = \lambda_- = -\frac{R}{2L}$$



* $t_{1/2} = \frac{2L}{R} \ln 2 = \frac{\ln 2}{-\lambda}$

الثابت

Let $\delta = \frac{R}{2L}$ $\boxed{\delta \text{ or } \lambda}$ decay constant

$$Q(t) = Q_0 e^{-\delta t_{1/2}}$$

$$\frac{Q_0}{2} = Q_0 e^{-\delta t_{1/2}}$$

$$\frac{1}{2} = e^{-\delta t_{1/2}}$$

$$\ln 1 - \ln 2 = -\delta t_{1/2}$$

$$-\ln 2 = -\delta t_{1/2}$$

$$\ln 2 = \delta t_{1/2}$$

$$t_{1/2} = \frac{\ln 2}{\delta} = \frac{2L \ln 2}{R}$$

③

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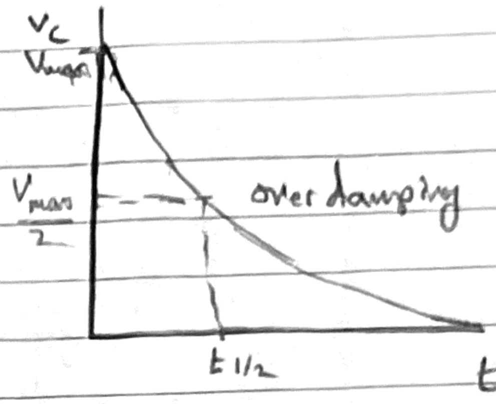
case (2) over damping

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$$



both terms in eq (1) decay exponentially with time and the voltage across the capacitor is said to be over damped

case (3) under damping

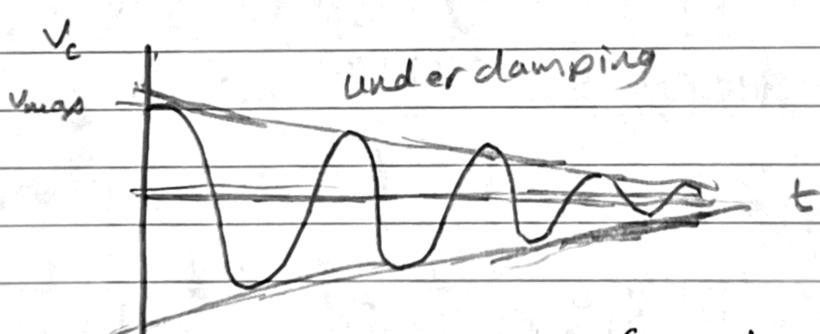
$$\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$$

the term under the squre root in eq (2) becomes negative

$$Q(t) = Q_0 e^{-\delta t} \cos(\omega' t + \phi)$$

$$\delta = \frac{R}{2L}$$

$$\omega = 2\pi f = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} \Rightarrow f = \frac{\omega}{2\pi}$$



$$\delta_{under} < \delta_{over} < \delta_{critical}$$

$$\lambda_{under} < \lambda_{over} < \lambda_{critical}$$