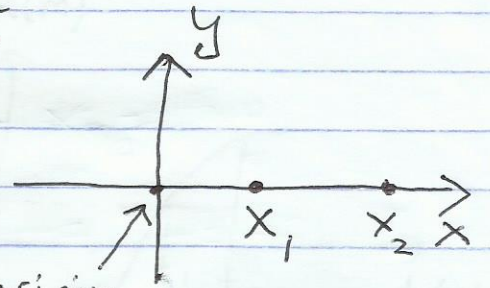
Chapter 2

Motion Along a Straight Line:

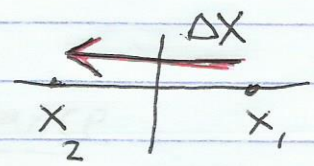
Position and Displacement

initial position $= x_1$
final position $= x_2$



Displacement = change in position origin
$\Delta x = x_2 - x_1$

example: suppose $x_1 = 5m$ at t_1
$x_2 = -5m$ at t_2

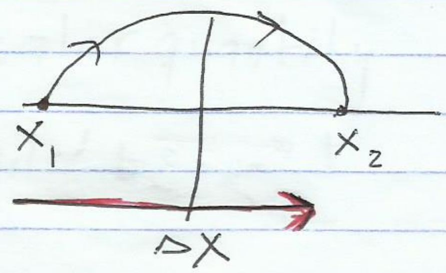


Displacement $= x_2 - x_1$

$\Delta x = -5 - 5 = -10m$

Displacement $\neq$ distance

example:- a particle moves
from $x_1 = -2m$ to
$x_2 = 2m$
along a semicicle (دائرة $\frac{1}{2}$)
find 1) The displacemt ?
2) The distance ?



1) Displacement = Change in Position
$\Delta x = x_2 - x_1$
$= 2 - {}^-2 = 4m$

distance $= 2\pi r/2 = \pi r$
$S = (3.14)(2) = 6.3\,m$

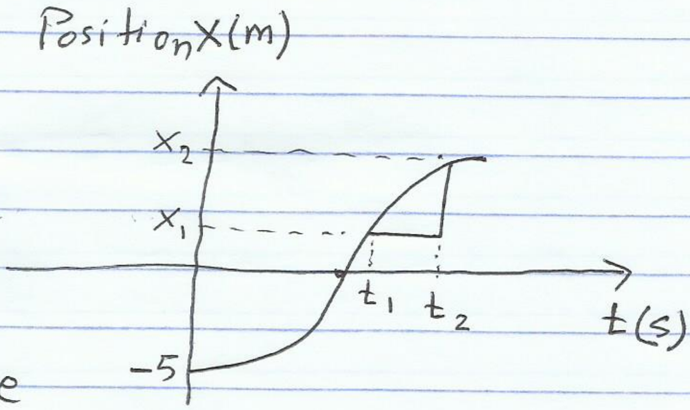
Solve checkpoint 1 Page 15

Average velocity and Average speed:-

To find Average velocity you have to know $x(t)$ graphically $\boxed{\text{or}}$ as equation

average velocity $= \frac{\Delta x}{\Delta t}$

$\vec{V}_{avg} = \frac{x_2 - x_1}{t_2 - t_1}$ from the graph



but the average speed can not be found from the graph, because

average speed (S_{avg}) $=$ $\frac{\text{tototal distance}}{\text{total time}}$

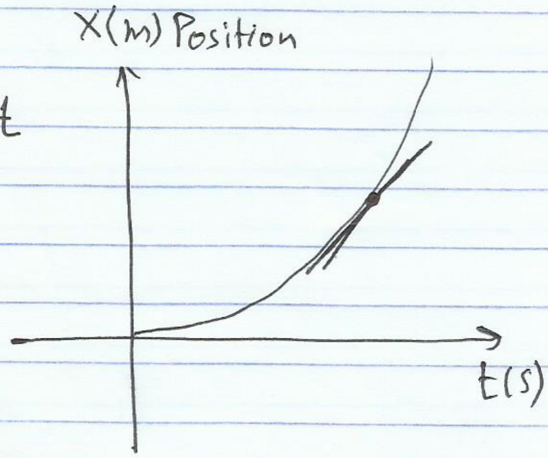
$S_{avg} = \frac{\text{total distance}}{\Delta \text{time}} = \frac{\text{total distance}}{t_2 - t_1}$

S_{avg} always positive scalar quantity

$\vec{V}_{avg}$ is vector quantity could be $+$ or $-$

Instantaneous velocity:

V_{inst} ① could be found from x Vs. t by plotting the tangent of the curve at a certain Point, then find the slop.



V_{inst} = the slope of the tangent

② $V_{inst} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$ from equation $x(t)$

Average Acceleration and instanteous Acceleration:

$$a_{avg} = \frac{\Delta V}{\Delta t}$$

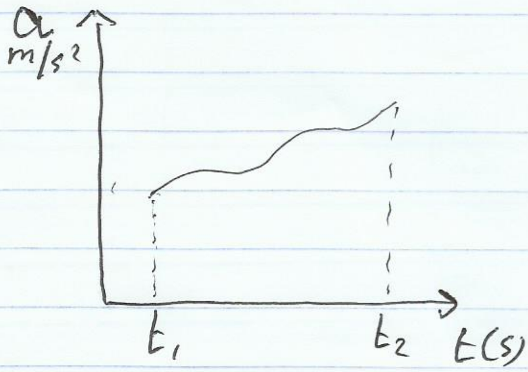
$$= \frac{V_2 - V_1}{\Delta t} \quad m/s/s = m/s^2$$

$$a_{inst} = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t}$$

$$a_{inst} = \frac{dv}{dt} \quad m/s^2$$

Finding v from a

$$a = \frac{dv}{dt} \Rightarrow dv = a\,dt$$



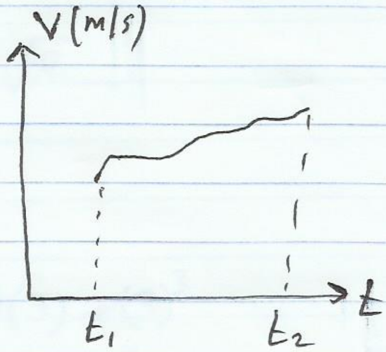
Integrate both sides

$$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt$$

$$v_2 - v_1 = \int_{t_1}^{t_2} a\,dt$$

= the area under the curve of a versus t from $t_1 \longrightarrow t_2$

Finding x from v :



$$v = \frac{dx}{dt} \Rightarrow dx = v\,dt$$

Integrate both sides

$$\int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v\,dt$$

$$x_2 - x_1 = \int_{t_1}^{t_2} v\,dt$$

= the area under the curve of v versus t between $t_1 \longrightarrow t_2$

Solve Sample Problem 2.01
Solve Sample Problem 2.02
Solve Sample Problem 2.06

Sample Problem 2.03:-

$x = 4 - 27t + t^3$, x in m ; t in s

a) Find $V(t)$ + $a(t)$?

$V(t) = \frac{dx}{dt}$

$V(t) = -27 + 3t^2$

$a(t) = \frac{dV}{dt} = 6t$

b) When $V=0$? find t ? at $V=0$

$0 = -27 + 3t^2$

$3t^2 = 27$, $t^2 = 9$, $t = \sqrt{9}$

$t = \pm 3$ sec

$t = +3$ sec

Find a and x at $t = +3s$

$a = +18 m/s^2$, $x = 4 - 27(3) + (3)^3$

$= 4 - 81 + 27$

$= -50 m$

Note: at $t = 3s$ → $V_3 = 0$
→ $a_3 = +18 m/s^2$
→ $x_3 = -50 m$

At $t = 4s$

$x_4 = 4 - 27(4) + (4)^3$

$= -40 m$

$V_4 = -27 + 3(4)^2$

$= -27 + 48$

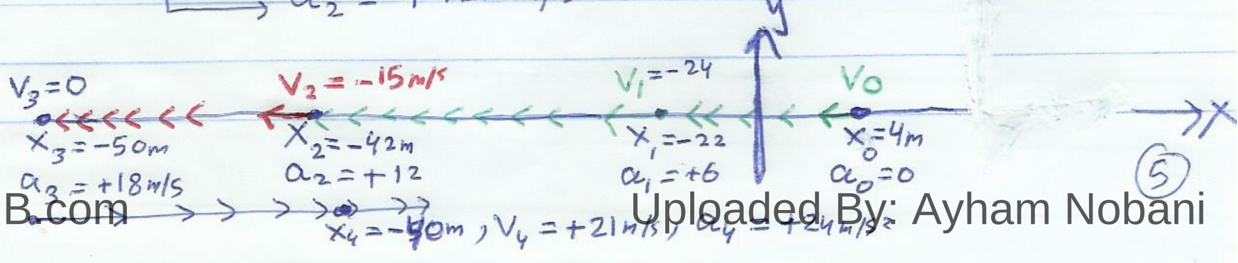
$= +21 m/s$

$a_4 = +24 m/s^2$

at $t = 0$ → $x_0 = 4 m$
→ $V_0 = -27 m/s$
→ $a_0 = 0$

at $t = 1s$ → $x_1 = 4 - 27 + 1 = -22 m$
→ $V_1 = -27 + 3 = -24 m/s$
→ $a_1 = +6 m/s$

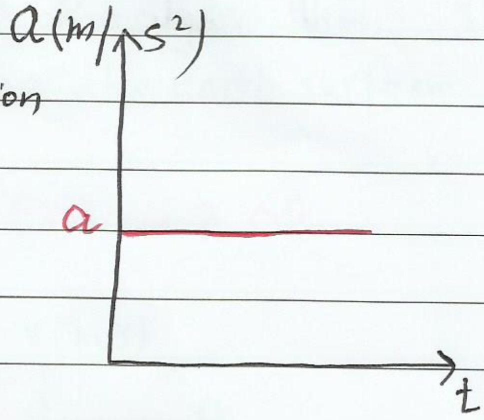
at $t = 2s$ → $x_2 = 4 - 27(2) + (2)^3 = -42 m$
→ $V_2 = -27 + 3(2)^2 = -15 m/s$
→ $a_2 = +12 m/s^2$



Constant Acceleration:-

A Particle moves from x_0, with V_0

$$a = \frac{dV}{dt}$$ at constant acceleration



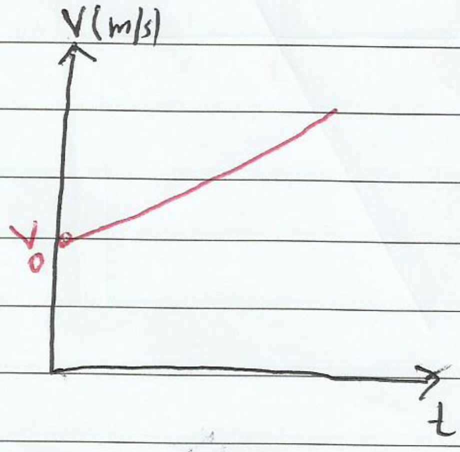
$$dV = a\,dt$$

$$\int_{V_0}^{V} dV = \int_0^t a\,dt = a\int_0^t dt$$

$$V - V_0 = at$$

$$V = V_0 + at \quad (1)$$ the velocity at time = t

find x at time = t



$$V = \frac{dx}{dt}$$

$$dx = V\,dt$$

$$\int_{x_0}^{x} dx = \int_0^t V\,dt$$

$$x - x_0 = \int_0^t (V_0 + at)\,dt = V_0 t + \frac{1}{2}at^2$$

$$x - x_0 = V_0 t + \frac{1}{2}at^2 \quad (2)$$

$$V_{avg} = \frac{V_0 + V}{2}$$

$$x - x_0 = V_{avg} \cdot t$$

$$x - x_0 = \left(\frac{V_0 + V}{2}\right)t \quad (3)$$, from (1) $t = \frac{V - V_0}{a}$

substitude (1) in (3) $x - x_0 = \left(\frac{V_0 + V}{2}\right)\left(\frac{V - V_0}{a}\right) = \frac{V^2 - V_0^2}{2a}$

$$V^2 = V_0^2 + 2a(x - x_0) \quad (4)$$

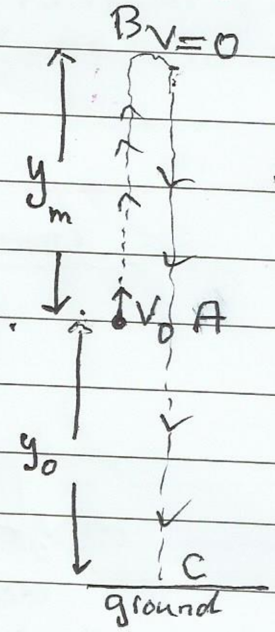
Free Fall: is the best example of motion object with Constant acceleration near the earth surface where $a_y = -9.8\ m/s^2$

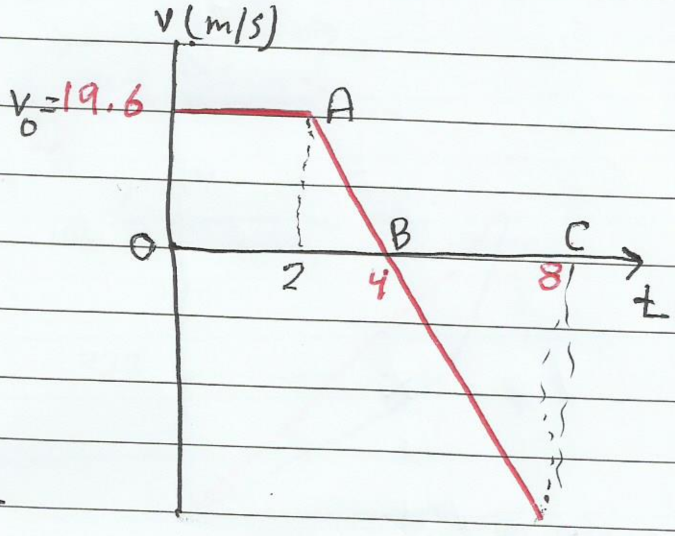
Solve sample Problem 2.05

(2-51)

a) $V_0 = 19.6\ m/s$ upward

at, $t = 4s$, $V = 0$





from A ⟶ B

$V_y^2 = V_{oy}^2 + 2a_y \Delta y$

$0 = (19.6)^2 + 2(-9.8) y_m$

$y_m = \frac{(19.6)^2}{2(9.8)} = 19.6\ m$

$y_m = 19.6\ m$ above point A (the begining of free fall)

b) find y_0?

from A ⟶ C (ground) ⟶ $t = 8 - 2 = 6s$

⟶ $V_0 = +19.6\ m/s$

⟶ $\Delta y = -y_0$

⟶ $a_y = -9.8\ m/s^2$

$\Delta y = V_{oy} t + \frac{1}{2} a_y t^2$

$-y_0 = (+19.6)(6) + \frac{1}{2}(-9.8)(6)^2$

$-y_0 = +117.6 - 176.4$

$-y_0 = -58.8\ m$

the heigt ~~of~~ of starting free fall $= 58.8\ m$

Note: maximum height from the ground $= 58.8 + 19.6$
$= 78.4\ m$

* find the velocity of the object when reached the ground

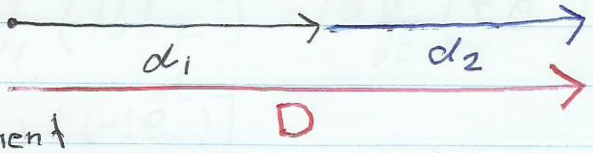
A ⟶ C $\quad V_y = V_{oy} + a_y t \Rightarrow V_y = +19.6 + (-9.8)(6)$
$= -39.2\ m/s$

Solving Lecture Problems (chapter 2)

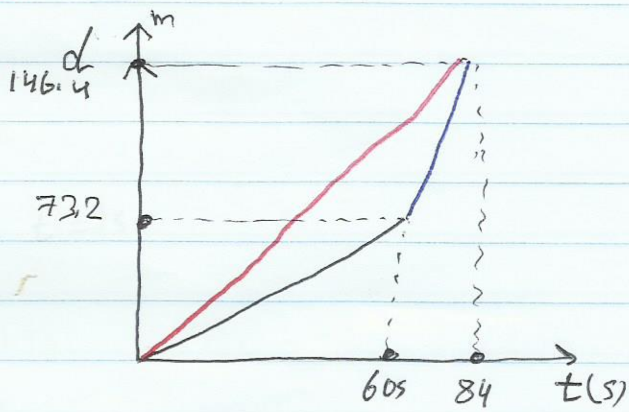
(2-2) you walk $d_1 = 73.2m$ at $V_1 = 1.22 m/s$

ⓐ then you run $d_2 = 73.2m$ at $V_2 = 3.05 m/s$

Fing Vavg?



$$V_{avg} = \frac{\text{displacement}}{\text{time}}$$



$$t_1 = \frac{d_1}{V_1} = 60s$$

$$t_2 = \frac{d_2}{V_2} = 24s$$

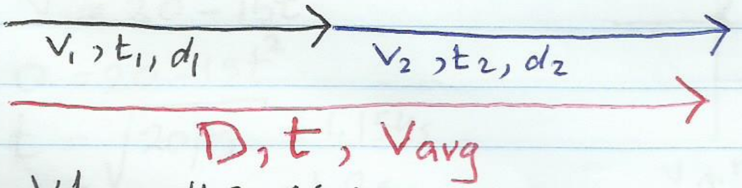
$$D = d_1 + d_2 = 146.4m$$

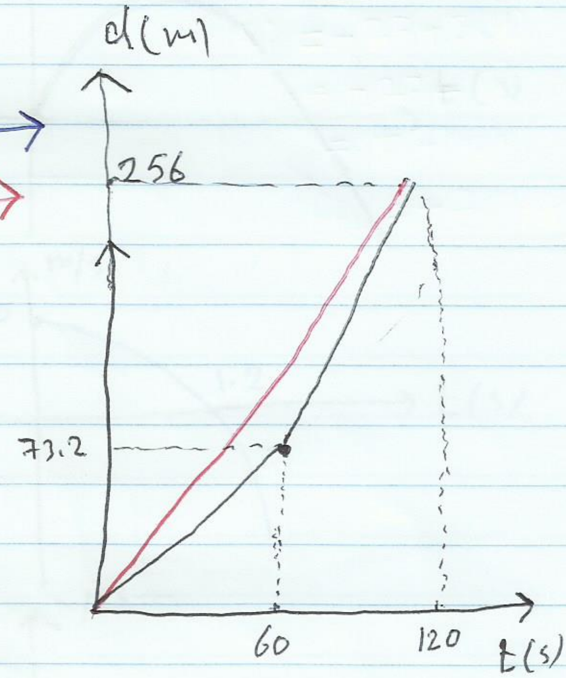
$$t_{tot} = t_1 + t_2 = 84 s$$

$$V_{avg} = \frac{D}{t_{tot}} = \frac{146.4}{84} = \underline{1.74 m/s}$$

b) You walk for $t_1 = 1min = 60s$ at $V_1 = 1.22 m/s$, then you run for $t_2 = 1min = 60s$ at $V_2 = 3.05 m/s$

Find V_{avg}?





$$d_1 = V_1 t_1 = (1.22)(60) = 73.2m$$

$$d_2 = V_2 t_2 = (3.05)(60) = 183 m$$

$$t = t_1 + t_2 = 120s$$

$$D = d_1 + d_2 = 73.2 + 183$$

$$D = 256.2m$$

$$V_{avg} = \frac{D}{t} = \frac{256.2}{120} = 2.14 m/s$$

(2-14) $X = 16t\,e^{-t}$

$x_0 = 0$

a) How far is the electron from the origin when it momentarily stop?

$$v = \frac{dx}{dt} = \frac{d}{dt}(16t\,e^{-t}) = 16\frac{d}{dt}(t\,e^{-t})$$

$$= 16\left[(1)e^{-t} + t(-1e^{-t})\right]$$

$$v = 16e^{-t}(1-t)$$

$$v = 0 = 16e^{-t}(1-t)$$

$v = 0 \Rightarrow t = 1s$,

$X = 16t\,e^{-t}$, find x? at $t = 1s$

$$= 16(1)e^{-1}$$

$$= 16(0.37)$$

$$= 5.89m$$

[2-20] $X = 20t - 5t^3$

a) find t? at $v = 0$

$$v = \frac{dx}{dt}$$

$$v = 20 - 15t^2$$

$$0 = 20 - 15t^2$$

$$t = \sqrt{20/15} = 1.154s$$

$$= 1.2s$$

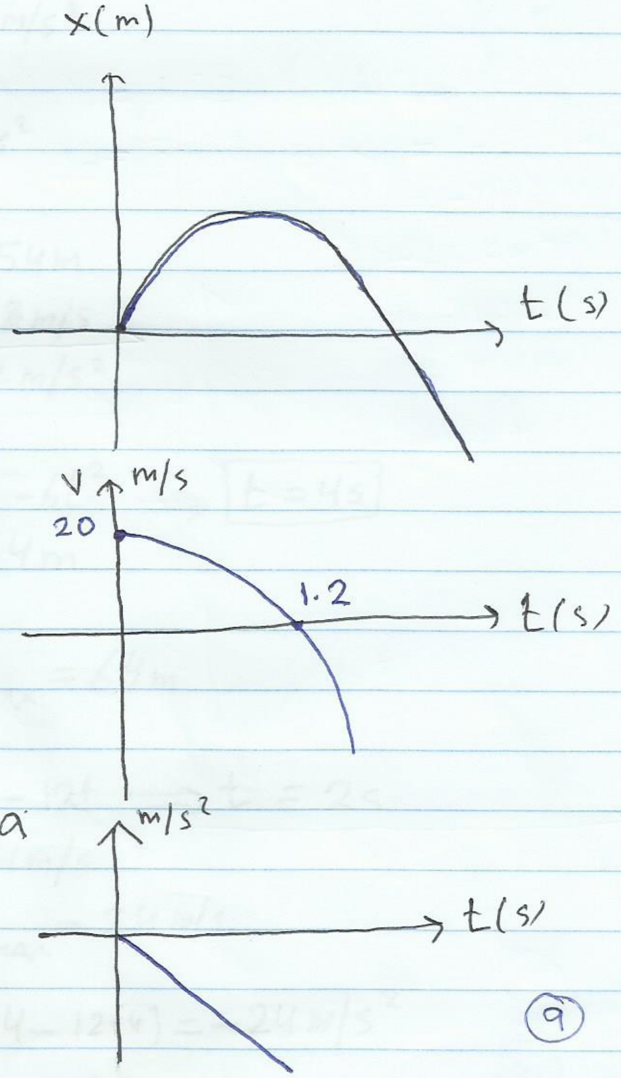
b) find t? at $a = 0$

$$a = \frac{dv}{dt} = -30t$$

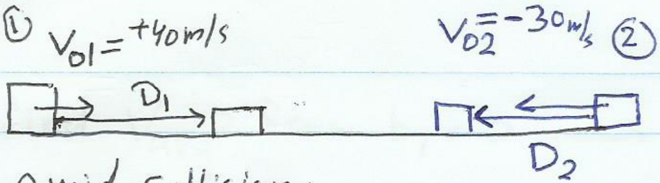
$a = 0$, at $= 0$

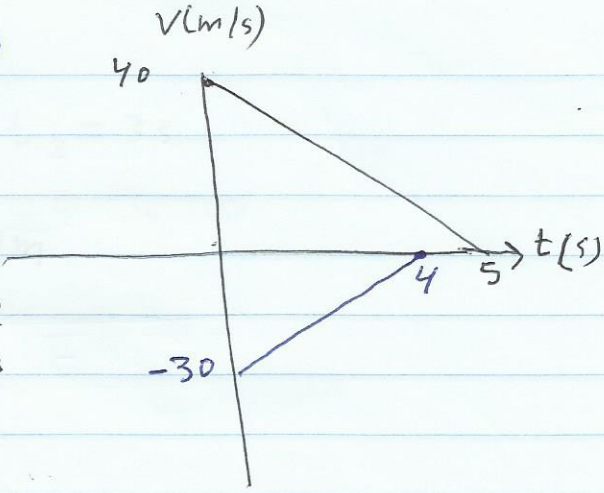
c) a is negative for $t > 0$

d) a is positive for $t < 0$



(2-41)





to Avoid collision:

train ① stopped after 5s

Displacement = Area und (V-t) curve from $t_1 = 0 \longrightarrow t_2 = 5s$

$D_1 = \frac{1}{2}(5)(40) = +100m$

train ② stopped after 4s

D_2 = Area under (V-t) curve from $t_1 = 0 \longrightarrow t_2 = 4s$

$D_2 = \frac{1}{2}(4)(-30) = -60m$ (-) for direction

Distance between stopping train = 200 - 160 = 40m

(2-18) $X = 12t^2 - 2t^3$ m

$V = \frac{dx}{dt} = 24t - 6t^2$ m/s

$a = \frac{dv}{dt} = 24 - 12t$ m/s^2

at t = 3s

a) $X(3) = 12(3)^2 - 2(3)^3 = 54m$

b) $V(3) = 24(3) - 6(3)^2 = 18 m/s$

c) $a(3) = 24 - 12(3) = -12 m/s^2$

d) find X_{max}?

X_{max} is at $V = 0 = 24t - 6t^2 \Longrightarrow \boxed{t = 4s}$

$X_{max} = 12(4)^2 - 2(4)^3 = 64m$

e) At $t = 4s \Longrightarrow V = 0$, $X_{max} = 64m$

f) find V_{max}?

V_{max} is at $a = 0 = 24 - 12t \Longrightarrow t = 2s$

$V_{max} = 24(2) - 6(2)^2 = 24 m/s$

g) At $t = 2s \Longrightarrow a = 0$, $V_{max} = 24 m/s$

h) find a? at V = 0

$V = 0$ at $t = 4s \Longrightarrow a = 24 - 12(4) = -24 m/s^2$

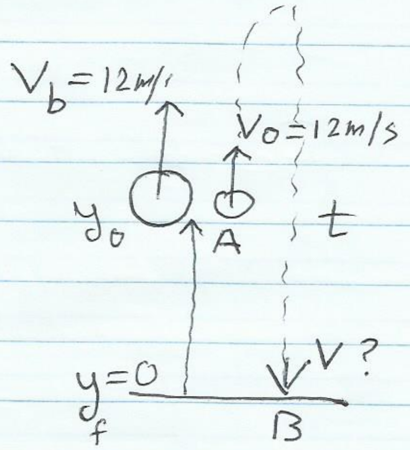
i) find V_{avg}? from $t_1 = 0 \longrightarrow t_2 = 3s$

$$X(0) = 0 \quad , \quad X(3) = 54m$$

$$V_{avg} = \frac{\Delta X}{\Delta t} = \frac{X(3) - X(0)}{\Delta t}$$

$$= \frac{54 - 0}{3}$$

$$= 18 m/s$$



(2-49) $y_0 = 80m$

$V_0 = +12 m/s$

a) Find the time for the package to reach ground?

from $A \longrightarrow B$

$$\Delta y = V_{oy} t + \frac{1}{2} a_y t^2$$

$$y_f - y_0 = V_{oy} t + \frac{1}{2} a_y t^2$$

$$0 - 80 = +12t + \frac{1}{2}(-10)t^2$$

$$-80 = +12t - 5t^2$$

$$5t^2 - 12t - 80 = 0$$

$$t = \frac{-{-12} \mp \sqrt{(12)^2 - 4(5)(-80)}}{2(5)}$$

$$t = 5.4s$$

b) find the velocity when the package hit the ground?

$$V_y = V_{oy} + a_y t$$

$$= +12 + -10(5.4)$$

$$= -42 m/s$$ downward

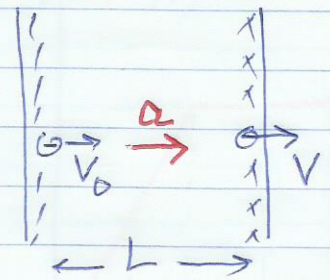
Problem 2.03 (Constant Acceleration)

$V_0 = 1.50\times10^5$ m/s

$L = 1.00$ cm

$V = 5.70\times10^6$ m/s

$a = ?$



$V^2 = V_0^2 + 2a\Delta X$

$(5.7\times10^6)^2 = (1.5\times10^5)^2 + 2a(1\times10^{-2})$

$$\frac{(5.7\times10^6)^2 - (1.5\times10^5)^2}{2(1\times10^{-2})} = a_x$$

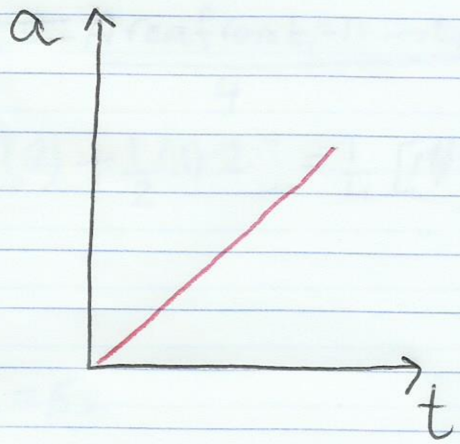
$a_x = 1.51\times10^{15}$ m/s^2

Problem (2.81) Variable Acceleration)

$a = 5t$

At $t = 2$s, $V_1 = +17$ m/s

At $t = 4$s, find V_2?



$a = \frac{dv}{dt}$

$dv = a\,dt$

$\int dv = \int(5t\,dt)$

$V = \frac{5t^2}{2} + C$

$+17 = \frac{5}{2}(2)^2 + C \Rightarrow C = 17 - 10 = 7$ m/s

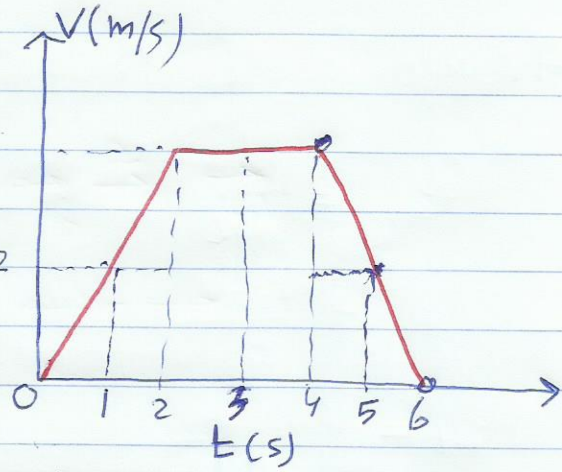
$\boxed{V = \frac{5}{2}t^2 + 7}$

At $t = 4$s, $V = \frac{5}{2}(4)^2 + 7 = 47$ m/s

Problem (2.90)

At $t=0$, $V_0=0$

$x_0 = 0$



a) find x? at $t=5s$

$\Delta x = \int v\,dt$

$=$ Area under the curve

V versus t, from $t_1 \rightarrow t_2$

$x_5 - x_0 =$ Area from $t_1=0 \rightarrow t_2=5s$

$= \frac{1}{2}(2)(4) + (4-2)4 + \frac{1}{2}(1)(2) + 1(2) = 4+8+1+2 = 15$

$= 15\,m$, $x_5 = 15m$

$x_5 = 15m$

b) At $t=5s \rightarrow V_5 = 2\,m/s$

c) $\longrightarrow a_5 =$ slope $= \frac{\Delta V}{\Delta t} = \frac{0-4}{6-4} = \frac{-4}{2} = -2\,m/s^2$

$\rightarrow x_5 = 15\,m$

d) find V_{avg} between $t_1 = 1s \longrightarrow t_2 = 5s$?

$V_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_5 - x_1}{5-1} = \frac{\text{Area from } t_1=1s \rightarrow t_2=5s}{4}$

$V_{avg} = \frac{1}{4}\left[1(2) + \frac{1}{2}(1)(2) + 2(4) + 1(2) + \frac{1}{2}(1)2\right] = \frac{1}{4}[14]$

$V_{avg} = \frac{14}{4} = 3.5\,m/s$

e) find a_{avg}? from $t_1 = 1s \longrightarrow t_2 = 5s$

$a_{avg} = \frac{\Delta V}{\Delta t} = \frac{V_5 - V_1}{5-1} = \frac{2-2}{4} =$ zero

$a_{avg} = 0\,m/s^2$

(extra) find the avarage velocity of the total motion from $t_1 = 0 \longrightarrow t_2 = 6s$?

$V_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_6 - x_0}{6-0} = \frac{\text{the area from } t_1=0 \rightarrow t_2=6}{6}$

$V_{avg} = \frac{1}{6}\left[\frac{1}{2}(2)(4) + 2(4) + \frac{1}{2}(2)(4)\right] = \frac{1}{6}[4+8+4] = \frac{1}{6}[16]\,m/s$

$\frac{8}{3} = 2.67\,m/s$