تكاملات غير محددة

Exp $f(x) = 2x$. Find $F(x)$ such that $F'(x) = f(x)$

Anti-derivative $F_1(x) = x^2$ ✓ $\Rightarrow F_1'(x) = 2x = f(x)$

" " $F_2(x) = x^2 - 5$ ✓ $\Rightarrow F_2'(x) = 2x = f(x)$

" " $F_3(x) = x^2 + \pi$ ✓ $\Rightarrow F_3'(x) = 2x = f(x)$

$F(x) = x^2 + c \Rightarrow F'(x) = 2x$

The set of all anti-derivatives → Indefinite Integral

$\int \cancel{F'(x)} = \int f(x)$

$$\boxed{F(x) = \int f(x)dx + c}$$

Integration Rule

1) $\int \underbrace{K}_{f} dx = \underbrace{KX + c}_{F}$

$F' = f$ ✓

Exp $\int 5dx = 5x + c$

Exp $\int \sqrt{3}\, dx = \sqrt{3}\, x + c$

Exp $\int -\frac{3}{4}\,dx = -\frac{3}{4}x + c$

$\boxed{2}$ $\int x^{n}\,dx = \frac{x^{n+1}}{n+1} + c$, $n+1 \neq 0$ $\rightarrow$ $n \neq -1$

Exp $\int \underset{f}{\underline{x^{5}}}\,dx = \frac{x^{5+1}}{5+1} + c = \frac{x^{6}}{6} + c \rightarrow F$

$F' = f$

Exp $\int \sqrt[3]{x^{2}}\,dx = \int x^{\frac{2}{3}}\,dx = \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + c$

$\frac{2}{3} + 1 = \frac{2}{3} + \frac{3}{3} = \frac{5}{3}$

$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + c$

$= \frac{3}{5}\,x^{\frac{5}{3}} + c$

$= \frac{3}{5}\sqrt[3]{x^{5}} + c$

$\boxed{3}$ $\int \left(f(x) \pm g(x)\right) dx = \int f(x)\,dx \pm \int g(x)\,dx$

Exp $\int (x^{2} - 3)^{2}\,dx$

$(a+b)^{2} = a^{2} + 2ab + b^{2}$

$\int \left(x^{4} + 2(x^{2})(-3) + (-3)^{2} \right) dx$

$\int \left(x^{4} - 6x^{2} + 9 \right) dx$

$(x^{2}-3)^{2} = (x^{2}-3)(x^{2}-3)$

$= x^{4} - 3x^{2} - 3x^{2} + 9$

$= x^{4} - 6x^{2} + 9$

$$\int \left(x^4 - 6x^2 + 9\right) dx \qquad = x^4 - 6x^2 + 9$$

$$\frac{x^5}{5} - \cancel{6}^{\,2} \frac{x^3}{\cancel{3}} + 9x + C$$

$$\frac{x^5}{5} - 2x^3 + 9x + C$$

Exp
$$\int \frac{x+1}{\sqrt{x}}\,dx = \int \frac{x+1}{x^{\frac{1}{2}}}\,dx$$

$$x^1 \cdot x^{-\frac{1}{2}} = x^{1+-\frac{1}{2}}$$
$$= x^{1-\frac{1}{2}}$$
$$= x^{\frac{2}{2}-\frac{1}{2}}$$
$$= x^{\frac{1}{2}}$$

$$= \int \left(x^1+1\right) x^{-\frac{1}{2}}\,dx = \int \left(x^{1-\frac{1}{2}} + x\right) dx$$

$$= \int \left(x^{\frac{1}{2}} - x^{1}\right) dx$$

$$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{x^{1+1}}{1+1} + C$$

$$\frac{1}{2} + 1 = \frac{1}{2} + \frac{2}{2}$$
$$= \frac{3}{2}$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^2}{2} + C$$

$$\frac{2}{3}\sqrt{x^3} \qquad \frac{x^2}{2} + C$$

$$= \frac{2}{3}\sqrt[2]{x^3} - \frac{x^2}{2} + c$$

<u>Exp</u> Assume $\int g(x)\,dx = 4x^3 - 5x^2 + 7x + c$

Find $g(x)$

$$\left(\cancel{\int} g(x)\,dx\right)' = \left(4x^3 - 5x^2 + 7x + c\right)'$$

$$\boxed{g(x) = 12x^2 - 10x + 7}$$

Applications

$$\int R'(x) = \int \overline{MR} \Rightarrow \underset{\text{Total}}{R(x)} = \int \underset{\text{given}}{\overline{MR}}\,dx$$

$$\int C'(x) = \int \overline{MC} \Rightarrow C(x) = \int \overline{MC}\,dx$$

$$\int P'(x) = \int \overline{MP} \Rightarrow P(x) = \int \overline{MP}\,dx$$

<u>Exp</u> Given $\overline{MR} = 300 - 0.2x$, x quantity sold (# of)

1) Find Total Revenue

when 1000 units are being

① Find Total Reven...

② Find total revenue when 1000 units are being sold

① $R(x) = \int \overline{MR}\, dx = \int (300 - 0.2x^{1})\, dx$

$= 300x - 0.2\dfrac{x^2}{2} + C$

$$\boxed{R(x) = 300x - 0.1x^2 + C}$$

To find C we use $R(0) = 0$

$R(x) = px$

$R(0) = p(0) = 0$

$\boxed{R(0) = 0} \rightarrow$ دائماً

$R(0) = 300(0) - 0.1(0)^2 + C$

$0 = 0 - 0 + C$

$\boxed{0 = C}$

$\Downarrow$

$$\boxed{R(x) = 300x - 0.1x^2}$$

② $R(1000) = 300(1000) - 0.1(1000)^2$

$= 300{,}000 - 100{,}000$

$= \$\,200{,}000$

$$\int k \, dx = kx + c$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$