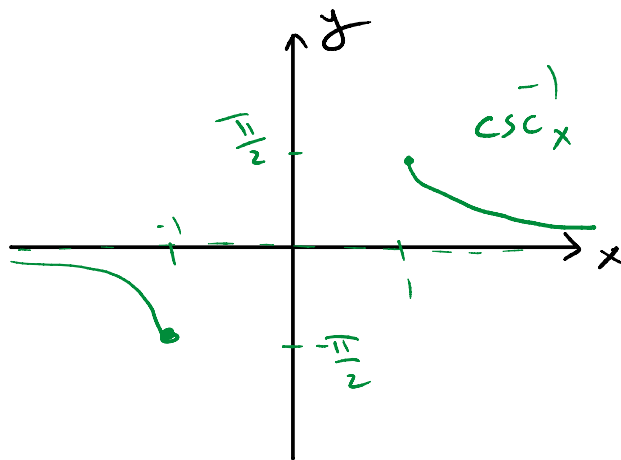
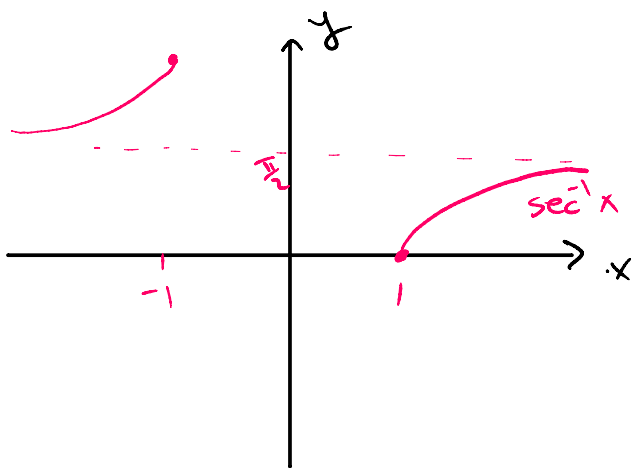
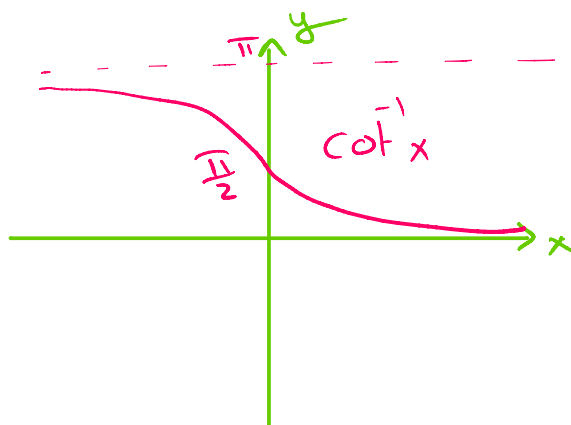
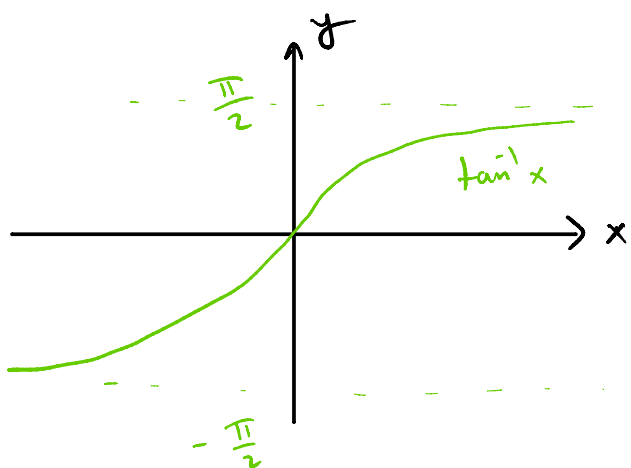
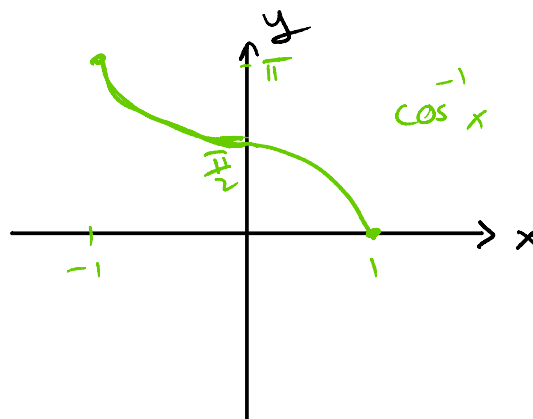
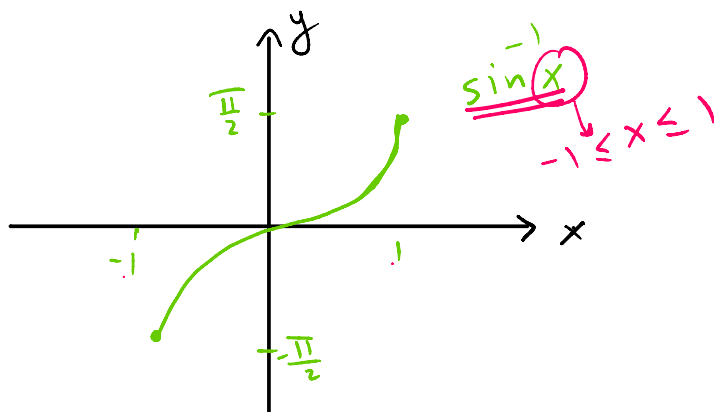


Inverse of Trigonometric functions



$$f(x) = \sin^{-1} x \quad \Rightarrow \quad f^{-1}(x) ??$$

$\cos^{-1} x$

$$f(x) = \dots \Rightarrow f'(x) = \dots$$

$\cos^{-1} x$
 $\tan^{-1} x$
 $\sec^{-1} x$
 $\csc^{-1} x$
 $\cot^{-1} x$

If $u(x)$ is diff and

$$(1) f(x) = \sin^{-1} u \Rightarrow f'(x) = \frac{1}{\sqrt{1-u^2}} \cdot u'$$

$1-u^2 > 0 \Rightarrow 1 > u^2$
 $|u| < 1$

$$(2) f(x) = \cos^{-1} u \Rightarrow f'(x) = \frac{-1}{\sqrt{1-u^2}} \cdot u'$$

$$(3) f(x) = \tan^{-1} u \Rightarrow f'(x) = \frac{1}{1+u^2} \cdot u'$$

$$(4) f(x) = \cot^{-1} u \Rightarrow f'(x) = \frac{-1}{1+u^2} \cdot u'$$

$$(5) f(x) = \sec^{-1} u \Rightarrow f'(x) = \frac{1}{|u| \sqrt{u^2-1}} \cdot u'$$

$u^2-1 > 0 \Rightarrow u^2 > 1$
 $\Rightarrow |u| > 1$

$$(6) f(x) = \csc^{-1} u \Rightarrow f'(x) = \frac{-1}{|u| \sqrt{u^2-1}} \cdot u'$$

$$16) f(x) = \csc u \Rightarrow f'(x) = \frac{-1}{|u| \sqrt{u^2 - 1}} \cdot u$$

Exp $y = \sin^{-1} \sqrt{5}x$ Find $y'(0)$

$$y' = \frac{1}{\sqrt{1-u^2}} \cdot u' = \frac{1}{\sqrt{1-5x^2}} \cdot \sqrt{5}$$

$$\begin{aligned} u &= \sqrt{5}x \\ u^2 &= 5x^2 \\ \hline u' &= \sqrt{5} \end{aligned}$$

$$= \frac{\sqrt{5}}{\sqrt{1-5x^2}}$$

$$y'(0) = \frac{\sqrt{5}}{\sqrt{1-5(0)^2}} = \frac{\sqrt{5}}{\sqrt{1}} = \sqrt{5}$$

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2 $y = \ln(\tan^{-1}x)$ Find $y'(1)$

$$y' = \frac{\frac{1}{1+u^2} \cdot u'}{\tan^{-1}x}$$

$$\begin{aligned} u &= x \\ u' &= 1 \end{aligned}$$

$$= \frac{\frac{1}{1+x^2}}{\tan^{-1}x}$$

$$y'(1) = \frac{\frac{1}{1+1^2}}{\frac{\pi}{4}} = \frac{\frac{1}{2}}{\frac{\pi}{4}} = \frac{1}{2} \cdot \frac{4}{\pi} = \frac{2}{\pi}$$

$$y'(1) = \frac{\frac{1}{1+1^2}}{\tan^{-1} 1} = \frac{\frac{2}{2}}{\frac{\pi}{4}} = \frac{1}{2} \cdot \frac{4}{\pi} = \frac{2}{\pi}$$

(3) $y = \sec^{-1}(2x+1)$ Find $y'(1)$

$$u = 2x+1$$

$$u' = 2$$

$$y' = \frac{1}{|u| \sqrt{u^2 - 1}} \cdot u'$$

$$= \frac{1}{|2x+1| \sqrt{(2x+1)^2 - 1}} \cdot 2$$

$$y'(1) = \frac{2}{|2(1)+1| \sqrt{(2(1)+1)^2 - 1}} = \frac{2}{3 \sqrt{9-1}} = \frac{2}{3\sqrt{8}}$$

$$= \frac{2}{3 \cdot 2\sqrt{2}}$$

$$\sqrt{8} = \sqrt{2 \cdot 4} = \sqrt{2} \sqrt{4} = 2\sqrt{2}$$

$$= \frac{1}{3\sqrt{2}}$$

Exp Show that $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$ ✓

$$f(x) = \sin x$$

$$f'(x) = \cos x$$

$$\sin^{-1}$$

$$\Rightarrow \sin^{-1} x$$

$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ \cos^2 x &= 1 - \sin^2 x \\ \cos x &= \sqrt{1 - \sin^2 x} \end{aligned}$$

$$\frac{df^{-1}}{dx} = \frac{1}{f'(f^{-1}(x))} = \frac{1}{\cos(\sin^{-1}x)} \quad \cos x = \sqrt{1 - \sin^2 x}$$

$$= \frac{1}{\sqrt{1 - \sin^2(\sin^{-1}x)}} = \frac{1}{\sqrt{1 - x^2}}$$

$$\sin^2(\sin^{-1}x) = \left[\sin(\sin^{-1}x) \right]^2 = x^2$$

cos x similar

Exp show that $\frac{d}{dx}(\sec^{-1}u) = \frac{1}{|u|\sqrt{u^2-1}} \cdot u$

$$y = \sec^{-1}x$$

$$\sec y = \sec(\sec^{-1}x) = x$$

find y

$$\sec y = x$$

$$\sec y \tan y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\sec y \tan y}$$

$$1 + \tan^2 y = \sec^2 y$$

$$\tan y = \pm \sqrt{\sec^2 y - 1}$$

$$\frac{xy}{dx} = \frac{1}{\text{sec } y \tan y}$$

$\tan y = \pm \sqrt{\sec^2 y - 1}$

$$= \pm \frac{1}{\cancel{x} \sqrt{\sec^2 y - 1}}$$

$$= \pm \frac{1}{x \sqrt{x^2 - 1}}$$

$$= \frac{1}{|x| \sqrt{x^2 - 1}} \quad \checkmark$$

for any $a \neq 0 \Rightarrow$

$$\textcircled{1} \int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \left(\frac{u}{a} \right) + c \quad , \quad a^2 > u^2$$

$$\textcircled{2} \int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \left(\frac{u}{a} \right) + c$$

$$\downarrow \textcircled{3} \int \frac{du}{u \sqrt{u^2 - a^2}} = \left(\frac{1}{a} \right) \sec^{-1} \left| \frac{u}{a} \right| + c, \quad \underline{u^2 > a^2}$$

$$\textcircled{63} \int_0^{\ln \sqrt{3}} \frac{e^x}{1 + e^{2x}} dx$$

$\frac{du}{1 + u^2}$

$$u = e^x$$

$$du = e^x dx$$

$$u^2 = (e^x)^2 = e^{2x}$$

$$x=0 \Rightarrow u = e^0 = 1$$

$$x = \ln \sqrt{3} \Rightarrow u = e^{\ln \sqrt{3}} = \sqrt{3}$$

$$\int_1^{\sqrt{3}} \frac{du}{1^2 + u^2} = \frac{1}{a} \tan^{-1} \left(\frac{u}{a} \right) = \frac{1}{1} \tan^{-1} \left(\frac{u}{1} \right) \Big|_1^{\sqrt{3}}$$

$a=1$

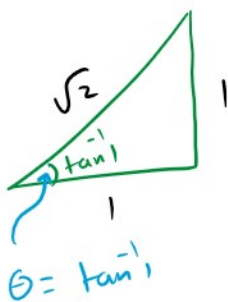
$$= \tan^{-1} u \Big|_1^{\sqrt{3}}$$

$$= \tan^{-1} \sqrt{3} - \tan^{-1} 1$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

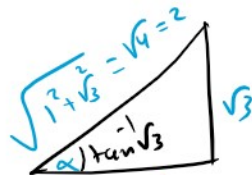
$$= \frac{4\pi - 3\pi}{12}$$

$$= \frac{\pi}{12}$$



$$\cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$$

$$\sin \theta = \frac{1}{\sqrt{2}}$$



$$\cos \alpha = \frac{1}{2}$$

$$\alpha = \frac{\pi}{3}$$

$$\sin \theta = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{12}$$

(75) $\int \frac{x+4}{x^2+4} dx$

$$\frac{A+B}{C} = \frac{A}{C} + \frac{B}{C}$$

$$\int \left(\frac{x}{x^2+4} + \frac{4}{x^2+4} \right) dx$$

$$\frac{1}{2} \int \frac{2x}{x^2+4} dx + \int \frac{4}{x^2+4} dx$$

$$\frac{1}{2} \ln |x^2+4| + 4 \int \frac{dx}{x^2+2^2}$$

$u=x \quad a=2$

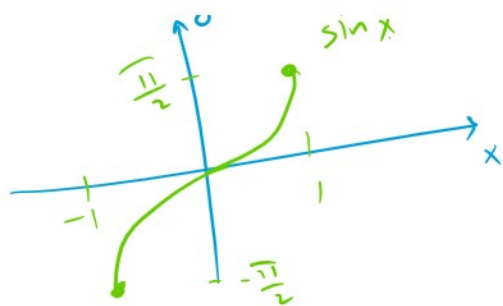
$$= \frac{1}{2} \ln(x^2+4) + 4 \left(\frac{1}{2} \right) + \tan^{-1} \left(\frac{x}{2} \right) + C$$

$$= \frac{1}{2} \ln(x^2+4) + 2 \tan^{-1} \frac{x}{2} + C$$

Exp Find the domain of $f(x) = \sin^{-1}(\ln x)$

$\frac{\pi}{2}$ y $\sin^{-1} x$

$$-1 \leq \ln x \leq 1$$



$$-1 \leq \ln x \leq 1$$

$$\frac{1}{e} \leq \frac{\ln x}{e} \leq e$$

$$\frac{1}{e} \leq x \leq e$$

$$D = \left[\frac{1}{e}, e \right]$$

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$$\int \frac{dx}{(2x-1) \sqrt{(2x-1)^2 - 4}}$$

sec

$$\begin{aligned} u &= 2x-1 \\ du &= 2 dx \\ \frac{du}{2} &= dx \end{aligned}$$

$$\int \frac{\frac{du}{2}}{u \sqrt{u^2 - 4}} = \frac{1}{2} \int \frac{du}{u \sqrt{u^2 - 2^2}}$$

a=2

$$= \frac{1}{2} \cdot \frac{1}{a} \sec^{-1} \left| \frac{u}{a} \right| + C$$

$$= \frac{1}{2} \left(\frac{1}{2} \right) \sec^{-1} \left| \frac{2x-1}{2} \right| + C$$

$$= \frac{1}{4} \sec^{-1} \left| \frac{2x-1}{2} \right| + C$$

(70) $\int \frac{6 dt}{\sqrt{3+4t-4t^2}}$

$$1t^2 - 6t + 3 = (t - 3)^2 + \boxed{-6}$$

$\frac{-6}{2} = -3$

$$1t^2 + 4t - 3 = (t + 2)^2 + \boxed{-7}$$

$\frac{4}{2} = 2$

$4 - 7 = -3$

$$3 + 4t - 4t^2$$

$$= - \left[\frac{4t^2}{2} - \frac{4t}{2} - 3 \right]$$

$$= - \left[(2t - 1)^2 - 4 \right]$$

$(2t)^2 = 4t^2$

$$2(2t)(-1) = 2(2t)(-1) = -4t$$

$$(-1)^2 = 1$$

$$= 4 - (2t - 1)^2$$

$$\int \frac{6 dt}{\sqrt{3+4t-4t^2}} = \int \frac{6 dt}{\sqrt{4 - (2t-1)^2}}$$

$a=2$

$$u = 2t - 1$$

$$du = 2 dt$$

$$\frac{du}{2} = dt$$

$$\int_0^1 \frac{6^3 \frac{du}{2}}{\sqrt{2^2 - u^2}}$$

$$a=2$$

$$u=2t-1$$

$$t = \frac{1}{2} \Rightarrow u = 2\left(\frac{1}{2}\right) - 1 = 1 - 1 = 0$$

$$t = 1 \Rightarrow u = 2(1) - 1 = 2 - 1 = 1$$

$$3 \int_0^1 \frac{du}{\sqrt{2^2 - u^2}} = 3 \sin^{-1} \frac{u}{a} \Big|_0^1 = 3 \sin^{-1} \frac{u}{2} \Big|_0^1$$

$$= 3 \left[\sin^{-1} \frac{1}{2} - \sin^{-1} \frac{0}{2} \right]$$

$$= 3 \left[\frac{\pi}{6} - 0 \right]$$

$$= 3 \frac{\pi}{6}$$

$$= \frac{\pi}{2}$$

Exp

$$\int_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} \frac{dx}{\sqrt{1 - x^2}} = \sin^{-1} \frac{u}{a} \Big|_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}} = \sin^{-1} x \Big|_{\frac{1}{\sqrt{2}}}^{\frac{\sqrt{3}}{2}}$$

$a=1$ $u=x$ \sin^{-1}

$$= \sin^{-1} \frac{\sqrt{3}}{2} - \sin^{-1} \frac{1}{\sqrt{2}}$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{\pi}{12}$$

~~Ex 8~~

$$\int \frac{dx}{\sqrt{4x - x^2}}$$

\swarrow
 $\sin^{-1} x$

$$\int \frac{dx}{\sqrt{4 - (x-2)^2}}$$

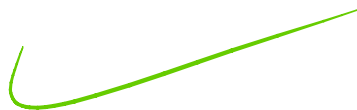
\uparrow \downarrow
 2 u
 \searrow
 $a=2$

$$\begin{aligned} 4x - x^2 &= -[x^2 - 4x] \\ &= -[(x-2)^2 + \boxed{-4}] \\ &= 4 - (x-2)^2 \end{aligned}$$

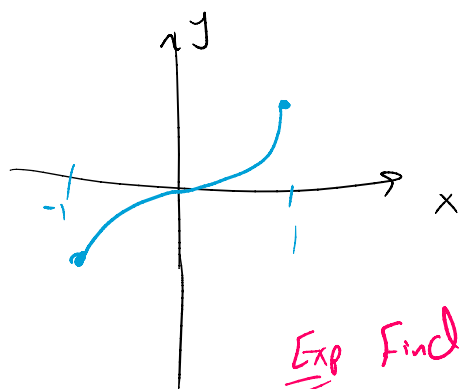
$$\begin{aligned} u &= x-2 \\ du &= dx \end{aligned}$$

$$\begin{aligned} \int \frac{du}{\sqrt{2^2 - u^2}} &= \sin^{-1} \frac{u}{a} + C \\ &= \sin^{-1} \left(\frac{x-2}{2} \right) + C \end{aligned}$$

$$= \sin\left(\frac{1}{2}\right)$$



$$\sin^{-1}x \quad -1 \leq x \leq 1$$



$$\sin^{-1}(\ln x) \quad -1 \leq \ln x \leq 1$$

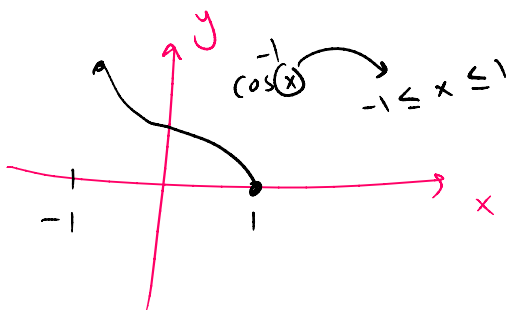
$$-1 \leq \ln x \leq 1$$

$$\frac{1}{e} \leq x \leq e$$

$$\frac{1}{e} \leq x \leq e$$

Ex Find D of $\cos^{-1}(\ln(x+2))$

Domain



$$-1 \leq \ln(x+2) \leq 1$$

$$\frac{1}{e} \leq \ln(x+2) \leq e$$

$$\frac{1}{e} \leq x+2 \leq e$$

$$\frac{1}{e} - 2 \leq x \leq e - 2$$