9.4 Derivative Formulas

Saturday, May 21, 2022 7:50 PM

① $f(x) = c \Rightarrow f'(x) = 0$

Exp Find $f'(x)$ if $f(x) = \sqrt{707} - \frac{1}{1500} + 2022$

$f'(x) = 0$

② $f(x) = kx \Rightarrow f'(x) = k$

Exp $y = -\sqrt{2}\,x \Rightarrow f'(x) = -\sqrt{2}$

$g(x) = 7 - x \Rightarrow g'(x) = -1$

③ $f(x) = x^{n} \Rightarrow f'(x) = n\,x^{n-1}$, $n \in \mathbb{R}$

Exp Ⓐ $y = x^{15} \Rightarrow y' = 15\,x^{14}$

Ⓑ $h(x) = \frac{1}{x^5} = x^{-5} \Rightarrow h'(x) = -5\,x^{-6} = \frac{-5}{x^6}$

Ⓒ $r(x) = \sqrt[4]{x} = \sqrt[4]{x^1} = x^{\frac{1}{4}}$

$r'(x) = \frac{1}{4}\,x^{\frac{1}{4}-1} = \frac{1}{4}\,x^{\frac{1}{4}-\frac{4}{4}} = \frac{1}{4}\,x^{\frac{-3}{4}} = \frac{1}{4}\,\frac{1}{x^{\frac{3}{4}}}$

$= \frac{1}{4\sqrt[4]{x^3}}$

$\sqrt[3]{x}$

(D) $p = \sqrt[2]{q^3} = q^{\frac{3}{2}} \Rightarrow p' = \frac{3}{2} q^{\frac{3}{2}-1} = \frac{3}{2} q^{\frac{3}{2}-\frac{2}{2}}$

Find $p'(16)$

$= \frac{3}{2} q^{\frac{1}{2}} = \frac{3}{2} \sqrt[2]{q'}$

$p' = \frac{3}{2} \sqrt{q}$

$p'(16) = \frac{3}{2} \sqrt{16} = \frac{3}{\cancel{2}_{1}} (\cancel{4}^{2}) = 6$

(E) $y = \frac{2}{\sqrt[2]{t'}}$ Find $y'(4)$

$y = \frac{2}{t^{\frac{1}{2}}} = 2 t^{-\frac{1}{2}}$

$y' = \cancel{2} \left(-\frac{1}{\cancel{2}}\right) t^{-\frac{1}{2}-1} = -t^{-\frac{1}{2}-\frac{2}{2}} = -t^{\frac{-3}{2}} = -\frac{1}{t^{\frac{3}{2}}}$

$y' = \frac{-1}{\sqrt{t^3}}$

$y'(4) = \frac{-1}{\sqrt{4^3}} = \frac{-1}{\sqrt[2]{\underline{4 \times 4} \times 4}} = \frac{-1}{4\sqrt{4}}$

$= \frac{-1}{4(2)} = \frac{-1}{8}$

(4) $f(x) = K\, g(x) \Rightarrow f'(x) = K\, g'(x)$

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Exp (1) $f(x) = \frac{1}{6} x^{12} \Rightarrow f'(x) = \frac{1}{\cancel{6}} (\cancel{12}^{2}) x^{11} = 2x^{11}$

(2) $h(x) = \frac{7}{\sqrt[3]{x^2}}$ Find $h'(1)$

$$h(x) = \frac{7}{x^{\frac{2}{3}}} = 7\, x^{-\frac{2}{3}}$$

$$h'(x) = 7\left(-\frac{2}{3}\right) x^{-\frac{2}{3}-1} = \frac{-14}{3} x^{-\frac{2}{3}-\frac{3}{3}}$$

$$= \frac{-14}{3} x^{\frac{-5}{3}} = \frac{-14}{3} \frac{1}{x^{\frac{5}{3}}} = \frac{-14}{3} \frac{1}{\sqrt[3]{x^5}}$$

$$h'(1) = \frac{-14}{3\sqrt[3]{1^5}} = \frac{-14}{3}$$

(5) $f(x) = h_1(x) \pm h_2(x)$

$$f'(x) = h_1'(x) \pm h_2'(x)$$

Exp Find (1) $\frac{dp}{dq}$ if $p = \frac{1}{2} q^2 + 3q + 7$

$$\frac{dp}{dq} = p' = (\cancel{\tfrac{1}{2}})(\cancel{2})\, q^1 + 3 = q + 3$$

$$\frac{dP}{dq} = P' = \cancel{\left(\tfrac{1}{2}\right)}\cancel{(2)}\, q + 3 = q + 3$$

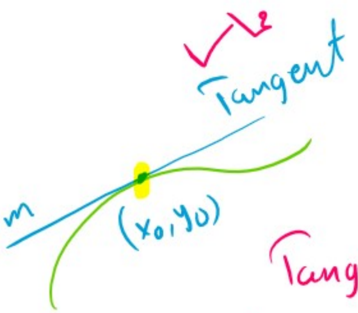
$$② \quad P'(2) = \boxed{2} + 3 = 5$$

Exp Find ① y' if $y = 12x^3 - 5x^2 + 3x - 1$

$$y' = 12(3)x^2 - 5(2)x^1 + 3 - 0$$

$$y' = 36x^2 - 10x + 3$$

② tangent at $x_0 = 1$



$$(x_0, y_0) = (1, y(1)) = (1, 9)$$

$$y_0 = y(1) = 12\boxed{1}^3 - 5\boxed{1}^2 + 3\boxed{1} - 1$$

$$= 12 - 5 + 3 - 1$$

$$= 9$$

$$m = y'(x_0) = y'(1) = 36\boxed{1}^2 - 10\boxed{1} + 3$$

$$= 36 - 10 + 3$$

$$= 29$$

Tangent line

$$y - y_0 = m(x - x_0)$$

$$y - \boxed{9} = 29(x - 1)$$

$$y - \cancel{9} = 29x - 29$$

$$\cancel{+9} \qquad\qquad +9$$

$$\boxed{y = 29x - 20}$$

$$y(1) = 29\,(1) - 20 = 29 - 20 = 9$$

Exp Find g' if $g(x) = \sqrt{x} - \frac{1}{2} + 4x^3$

Exp Find g' if $g(x) = \sqrt[2]{x^1} - \frac{1}{x^2} + 4x^3$

$g(x) = x^{\frac{1}{2}} - x^{-2} + 4x^3$

$g'(x) = \frac{1}{2} x^{\frac{1}{2}-1} - (-2) x^{-2-1} + 4(3) x^2$

$= \frac{1}{2} x^{\frac{1}{2}-\frac{2}{2}} + 2x^{-3} + 12x^2$

$= \frac{1}{2} x^{\frac{-1}{2}} + \frac{2}{x^3} + 12x^2$

$= \frac{1}{2} \frac{1}{x^{\frac{1}{2}}} + \frac{2}{x^3} + 12x^2$

$g'(x) = \frac{1}{2\sqrt[2]{x^1}} + \frac{2}{x^3} + 12x^2$

Exp Find slope of $f(x) = x^2 - x$ at $x = 2$

$f'(x) = 2x - 1$

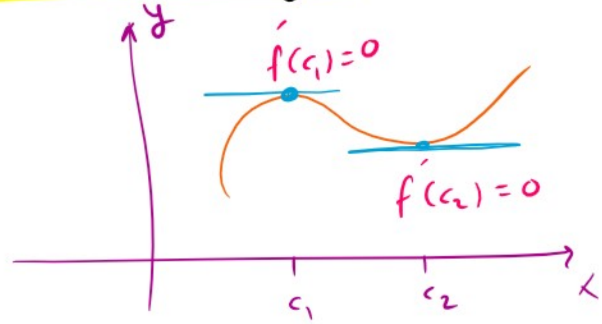
slope $= m = f'(2)$

$= 2\boxed{2} - 1$

$= 4 - 1$

$= 3$

Exp Find points on $f(x) = \frac{x^3}{3} + \frac{x^2}{2} - 2x + 3$ where f has horizontal tangent



$f'(x) = 0$

$\frac{1}{\cancel{3}}(\cancel{3})x^2 + \frac{1}{\cancel{2}}(\cancel{2})x - 2 = 0$

$x^2 + x - 2 = 0$

$(x + a)(x + b) = 0$

$ab = -2$

$a + b = 1$

$2, -1$

$(x+2)(x-1) = 0$

$x + 2 = 0 \Rightarrow x = -2$

or

$x - 1 = 0 \Rightarrow x = 1$

Points $(1, f(1)), (-2, f(-2))$

$f(1) \to \frac{5}{6}$ $\quad f(-2) \to \frac{19}{3}$

$f(1) = \frac{\boxed{1}^3}{3} - \frac{\boxed{1}^2}{2} - 2\boxed{1} + 3$

$= \frac{1}{3} - \frac{1}{2} - 2 + 3$

same thing

⋮

$$= \frac{1}{3} - \frac{1}{2} - 2 + 3$$

$$= \frac{2}{6} - \frac{3}{6} + 1$$

$$= \frac{2-3}{6} + \frac{6}{6}$$

$$= \frac{-1+6}{6}$$

$$= \frac{5}{6}$$

$$f(-2) = \frac{\boxed{-2}^3}{3} + \frac{\boxed{-2}^2}{2} - 2\boxed{-2} + 3$$

$$= \frac{-8}{3} + \frac{4}{2} + 4 + 3$$

$$= \frac{-16}{6} + \frac{12}{6} + 7$$

$$= \frac{-4}{6} + 7$$

$$= \frac{-2}{3} + \frac{7^{\times 3}}{1 \times 3}$$

$$= \frac{19}{3}$$