

صباح الخير
→ 1

نظام خطي
(معادلتين بمجهولين)

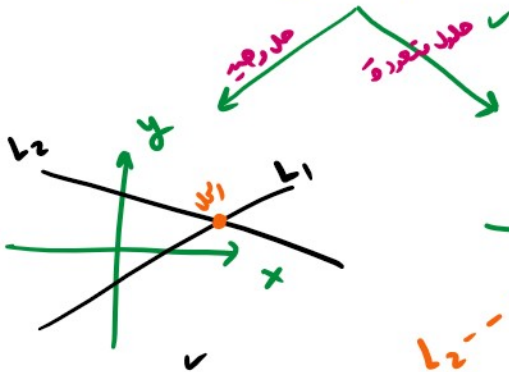
Two methods to solve linear systems

Substitution
Elimination

linear system

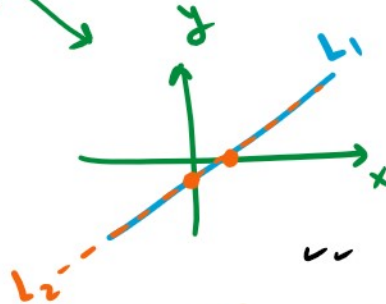
يوجد حل

Consistent



L_1 not parallel L_2
(L_1 intersects L_2)
(only one solution)
($m_1 \neq m_2$)

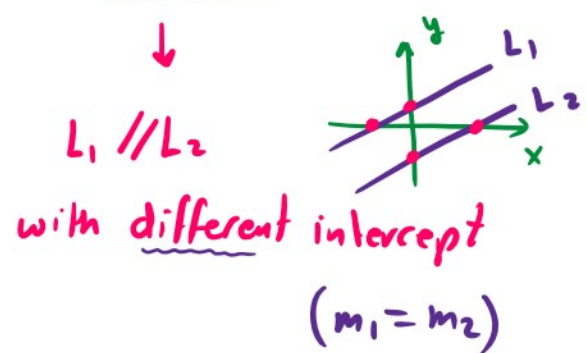
ملا في نقطة واحدة



$L_1 \parallel L_2$
with same intercept
(infinitely many solutions)
($m_1 = m_2$)

لا يوجد حل

inconsistent



$L_1 \parallel L_2$
with different intercept
($m_1 = m_2$)

Exp solve the following linear system using
Substitution method.

$$\begin{aligned} x + y &= 3 \\ x - y &= 1 \end{aligned} \Rightarrow \boxed{x = 1 + y} \Rightarrow \begin{aligned} x + y &= 3 \\ 1 + y + y &= 3 \\ 1 + 2y &= 3 \\ 2y &= 2 \\ y &= 1 \end{aligned}$$

$x = 1 + 1$
 $\boxed{x = 2}$

solution $(x, y) = \underline{(2, 1)}$

• Check:

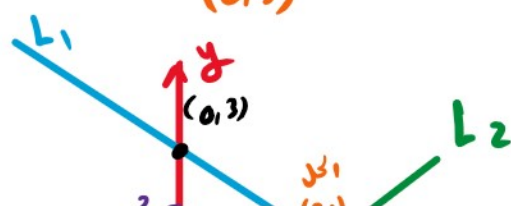
$$\begin{aligned} x + y &\stackrel{?}{=} 3 & \Rightarrow \boxed{2} + \boxed{1} &= 3 \\ x - y &\stackrel{?}{=} 1 & \Rightarrow \boxed{2} - \boxed{1} &= 1 \end{aligned}$$

• This system is consistent with one solution

• sketch the two lines

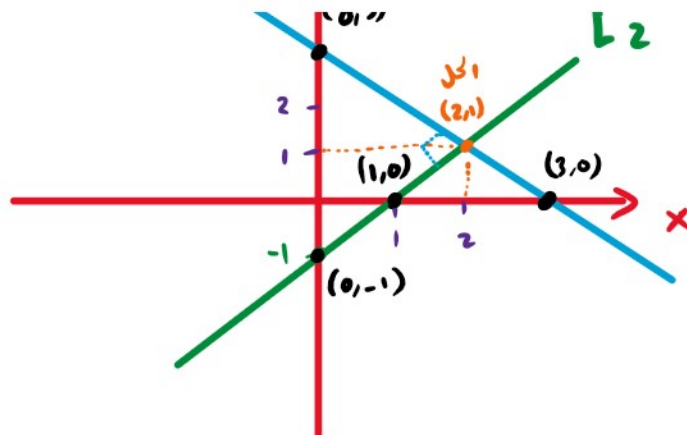
$L_1: x + y = 3 \Rightarrow$ x-intercept $\Rightarrow \boxed{y = 0} \Rightarrow \boxed{x = 3} \Rightarrow (3, 0)$
 $\Rightarrow$ y-intercept $\Rightarrow \boxed{x = 0} \Rightarrow \boxed{y = 3} \Rightarrow (0, 3)$

$L_2: x - y = 1$
 x-intercept $\Rightarrow y = 0 \Rightarrow x = 1 \Rightarrow (1, 0)$



x-intercept $\Rightarrow y = 0 \Rightarrow (1, 0)$

y-intercept $\Rightarrow x = 0 \Rightarrow \boxed{y = -1}$
 $(0, -1)$



• Find m_1 and m_2

$L_1: \underset{-x}{x} + \underset{-x}{y} = 3 \Rightarrow \boxed{y = -x + 3} \Rightarrow m_1 = -1$

$L_2: \underset{-x}{x} - \underset{-x}{y} = 1 \Rightarrow -y = 1 - x \Rightarrow \boxed{y = -1 + x} \Rightarrow m_2 = 1$

• Note that $L_1 \perp L_2$ since $\boxed{m_1 m_2 = -1}$

Exp Solve this linear system using substitution:

$$\begin{aligned} x + y &= 1 \\ 2x + 2y &= 4 \end{aligned} \Rightarrow \boxed{y = 1 - x}$$

كوبنا

$2x + 2y = 4$

$2x + 2(1 - x) = 4$

$2x + 2 - 2x = 4$



$2 = 4$

لا يوجد

• Find m_1 and m_2

$L_1: \cancel{x} + \cancel{y} = 1 \Rightarrow y = 1 - x \Rightarrow \boxed{m_1 = -1}$

$-x$

$$\Rightarrow m_1 = -1$$

$$L_2: 2x + 2y = 4$$

$-2x$

$-2x$

$$\frac{2y}{2} = \frac{4-2x}{2}$$

$$y = 2 - x$$

$$\Rightarrow m_2 = -1$$

$$m_1 = m_2 \Rightarrow L_1 \parallel L_2$$

• sketch L_1 and L_2

$$L_1: x + y = 1$$

$$\Rightarrow \begin{aligned} \text{x-intercept} &\Rightarrow y=0 \Rightarrow x=1 \Rightarrow (1,0) \\ \text{y-intercept} &\Rightarrow x=0 \Rightarrow y=1 \Rightarrow (0,1) \end{aligned}$$

$$L_2: 2x + 2y = 4$$

$$\text{x-intercept} \Rightarrow y=0$$

$$2x = 4$$

$$(2,0)$$

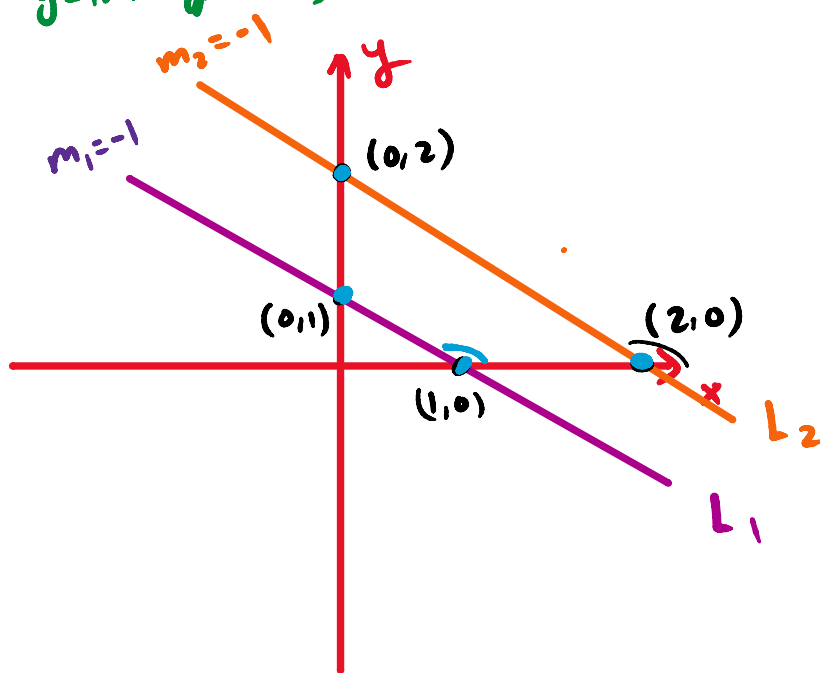
$$x=2$$

$$\text{y-intercept} \Rightarrow x=0$$

$$2y = 4$$

$$(0,2)$$

$$y=2$$



This system is inconsistent



$$L_1 \parallel L_2 \checkmark$$

with different intercepts

$$(m_1 \neq m_2) \checkmark$$

Exp solve this system $-x + y = 1$ $\leftarrow \boxed{x = -2}$
 using **Elimination**
 $-2x + 2y = 2$

• Find m_1 and m_2

$L_1: -x + y = 1$
 $\begin{array}{r} -x + y = 1 \\ +x \quad +x \\ \hline y = 1 + x \end{array} \Rightarrow m_1 = 1$

$L_2: -2x + 2y = 2$
 $\begin{array}{r} -2x + 2y = 2 \\ +2x \quad +2x \\ \hline 2y = 2 + 2x \\ \frac{2y}{2} = \frac{2}{2} + \frac{2x}{2} \\ y = 1 + x \end{array} \Rightarrow m_2 = 1$
 $m_1 = m_2 \Rightarrow L_1 \parallel L_2$

$$\begin{array}{r} \cancel{2x} - \cancel{2y} = -2 \\ -\cancel{2x} + \cancel{2y} = 2 \\ \hline \end{array} \quad +$$

$$\begin{array}{r} 1=1 \\ 2=2 \\ -2=-2 \end{array}$$

$\odot = \odot$
 حل متعدد
 infinitely many solution

$\Downarrow$
 $L_1 \parallel L_2$
 $\Downarrow$
 $m_1 = m_2$

Consistent system
 with same intercepts

• sketch L_1 and L_2

$L_1: -x + y = 1 \Rightarrow x\text{-intercept} \Rightarrow y = 0 \Rightarrow -x = +1 \Rightarrow \boxed{x = -1}$
 $(-1, 0)$

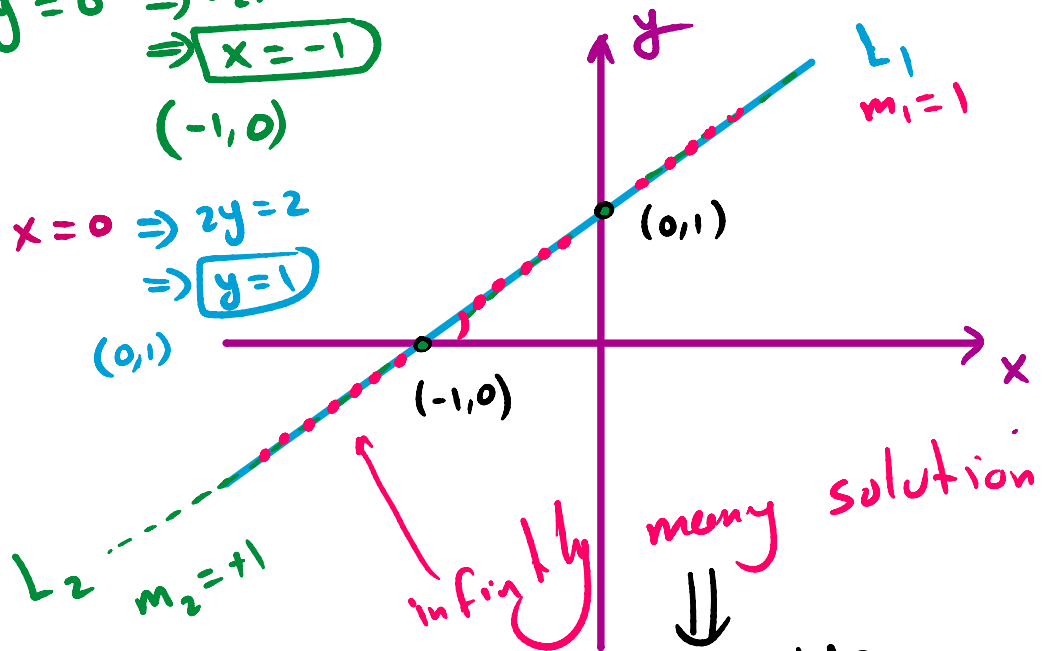
$\Rightarrow y\text{-intercept} \Rightarrow x = 0 \Rightarrow y = 1 \Rightarrow (0, 1)$

$L_2: -2x + 2y = 2$

$$L_2: -2x + 2y = 2$$

$$\begin{aligned} \text{x-intercept} \Rightarrow y=0 &\Rightarrow -2x=2 \\ &\Rightarrow \boxed{x=-1} \\ &(-1, 0) \end{aligned}$$

$$\begin{aligned} \text{y-intercept} \Rightarrow x=0 &\Rightarrow 2y=2 \\ &\Rightarrow \boxed{y=1} \end{aligned}$$



many solution

↓

the solution

$$- \infty \quad (-1, 0) \checkmark \checkmark$$

$$- \infty \quad (0, 1) \checkmark \checkmark$$

$$(5, 6) \checkmark \checkmark$$

$$(4, 5) \checkmark \checkmark$$

$$(-10, -9) \checkmark \checkmark$$

$$(99, 100) \checkmark \checkmark$$

$$-x + y = 1$$

$$x=5 \Rightarrow \begin{array}{r} -5 + y = 1 \\ \quad \quad +5 \end{array} \Rightarrow y=6$$

$$(x=99 \Rightarrow -x + y = 1$$

$$\begin{array}{r} -99 + y = 1 \\ \quad \quad +99 \end{array} \Rightarrow y=100$$

Exp Use Elimination

$$2x + 5y = 24$$

multiply by 3

$$2x + 5y = 24$$

$$-6x + 2y = 30$$

multiplying by -

$$\cancel{6x} + 15y = 72$$

$$\cancel{-6x} + 2y = 30$$

+

$$\frac{17y}{17} = \frac{102}{17}$$

$$y = 6$$

نكون في أي معادلة

$$2x + 5y = 24$$

$$2x + 5(6) = 24$$

$$2x + \cancel{30} = 24$$

$$\frac{2x}{2} = \frac{-6}{2}$$

$$x = -3$$

$$y = 6$$

$(-3, 6)$ one sol.

only one solution

Consistent with only one solution
 $\Rightarrow L_1$ not parallel L_2
(L_1 intersects L_2 in $(-3, 6)$)