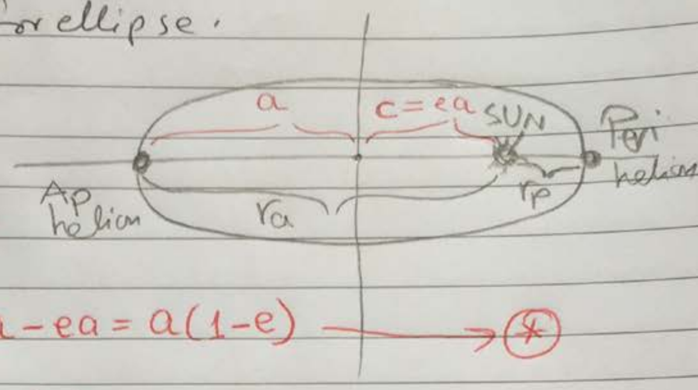
Ch. 6

Exercise 6.1: Eq. (6.11): $\frac{1}{2}V^2 - \frac{\mu}{r} = h$

wher $h = -\frac{\mu}{2a}$ for ellipse.

from the ellipse:



when the planet is at Perihelion:

$$r_p = a - c = a - ea = a(1-e) \longrightarrow (*)$$

when the planet is at Aphelion:

$$r_a = a + c = a + ea = a(1+e) \longrightarrow (**)$$

using Eq. (6.11) $\Rightarrow$ solve for V

$$V = \sqrt{2\mu\left(\frac{1}{r} - \frac{1}{2a}\right)}$$

Now: at Perihelion: $V_p = \sqrt{2\mu\left(\frac{1}{r_p} - \frac{1}{2a}\right)}$

at Aphelion: $V_a = \sqrt{2\mu\left(\frac{1}{r_a} - \frac{1}{2a}\right)}$

We substitute for r_p and r_a from (*) & (**)

Then find the ratio:

$$\frac{V_a}{V_p} = \sqrt{\frac{\cancel{2\mu}\left(\frac{1}{a(1-e)} - \frac{1}{2a}\right)}{\cancel{2\mu}\left(\frac{1}{a(1+e)} - \frac{1}{2a}\right)}} = \sqrt{\frac{(1-e)^2}{(1+e)^2}} \Rightarrow$$

$$\boxed{\frac{V_a}{V_p} = \frac{1-e}{1+e}}$$

for Earth's orbit around the SUN
$e = 0.0167$

$$\Rightarrow \left.\frac{V_a}{V_b}\right|_{\oplus} = \frac{1-0.0167}{1+0.0167} = \underline{\underline{0.967}}\,.$$

Ex. 6.2 **Eros:** $r_p = 1.1084$ AU
$r_a = 1.8078$ AU

Find velocity of Eros when r = mean distance of Mars = 1.5207 AU

For Eros: $r_p + r_a = a(1-e) + a(1+e)$

$$r_p + r_a = 2a$$

$$\Rightarrow a = \frac{r_p + r_a}{2} = \frac{1.1084 + 1.8078}{2}$$

$$\boxed{a = 1.4581 \text{ AU}}$$ for Eros

We use Eq. (6.11): "see Exercise 6.1"

$$V = \sqrt{2\mu\left(\frac{1}{r} - \frac{1}{2a}\right)} \quad , \quad \mu = G(M_\odot + \underbrace{M_{Eros}}_{\approx 0 \text{ relative to } M_\odot})$$

$$\mu = GM_\odot$$

we use SI units.

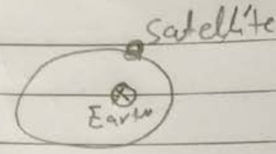
$$V = \sqrt{2 * 6.673\times10^{-11} * 1.989\times10^{30}\left(\frac{1}{1.5207} - \frac{1}{2*1.4581}\right) * \frac{1}{1.496\times10^{11}}}$$

$$\boxed{V = 2.36\times10^{4}\ \text{m/s} = 23.6\ \text{km/s}.}$$

Ex 6.3 geostationary satellite to remain always over the same point of the Equator of Earth $\Rightarrow$

Period of Satellite = Period of Earth rotation
= 1 Sidereal day

We use Kepler's 3rd Law



$$p^2 = \frac{4\pi^2}{G(m_{\oplus} + \underbrace{m_{sat}}_{\approx 0})} \, \underbrace{a^3}_{sat.}$$

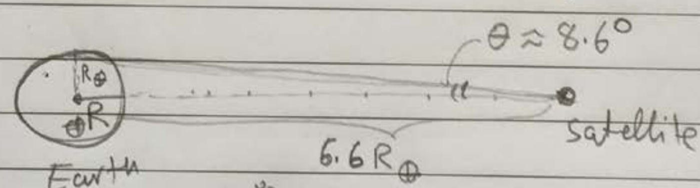
$m_{sat} << m_{\oplus}$

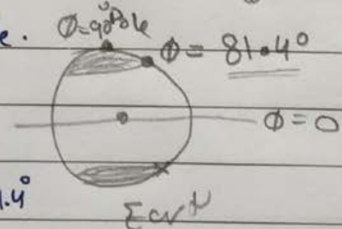
SI units

$$(1 * 24 * 3600)^2 = \frac{4\pi^2}{6.673\times10^{-11} * 5.974\times10^{24}} \, a^3$$

$\Rightarrow \boxed{a = 42.2\times10^{6}\,m}$ This is $\approx 6.6\,R_{\oplus}$

$\Rightarrow$ Areas within 8.6° from the Poles can NOT be seen by the Satellite.





surface Area at Latitude = 81.4°: $R\big|_{\phi=81.4°} = R_{\oplus} \cos 81.4°$

$= 9.5\times10^{5}\,m$

$\text{Area} = 4\pi R^2\big|_{81.4°} = 1.14\times10^{13}\,m^2$

Ratio $\dfrac{\text{Area}\big|_{81.4°}}{\text{total Area}} = \dfrac{1.14\times10^{13}}{5.1\times10^{14}}$

≈ 0.022

2.2%