

# MOTION IN A CIRCLE

Ch.3

الحركة  
الدورانية

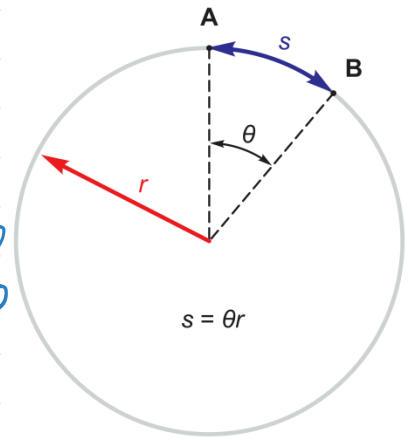
\*  $\theta = \frac{s}{r}$

$\theta = 2\pi$  ←  $s = 2\pi r$    
 ←  $s$  بال rad   
 ←  $\theta$  بال rad

$x \rightarrow \theta$

$v \rightarrow \omega$

$a \rightarrow a_c$



\*  $\omega = \frac{v}{r}$

$\omega = \frac{d\theta}{dt}$    
 (+) عكس عقارب   
 (-) مع عقارب

\*  $f = \frac{\text{عدد الدورات}}{t} \Rightarrow \omega = 2\pi f$    
 ← عند الدورة الكاملة

التردد ← Hz أو  $s^{-1}$

$\pi = 3.14$

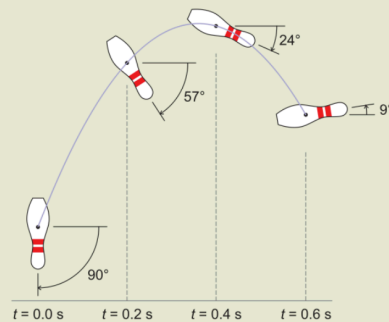
\*  $\frac{\theta^\circ}{360^\circ} * 2\pi$  ← عشان تحول من درجة ← راديان

## Example 3.1 Angular displacement and angular velocity

### Problem:

A juggler throws a bowling pin as shown in Figure 3.2.

- What is the angular velocity of the pin?
- What will the angular displacement of the pin be at a time of 0.9 s?



$\Delta\theta = \omega t$

①  $\omega = \frac{d\theta}{dt} \rightarrow \frac{-33^\circ}{0.2} * \frac{2\pi}{360} = \underline{2.587 \frac{\text{rad}}{\text{s}}}$  مع عقارب الساعة

②  $\theta_f - \theta_i = \omega t + \frac{1}{2} \alpha t^2$    
  $0 = \alpha$    
 مستقيم لا يتسارع

$= \frac{90}{360} * 2\pi + -2.587 * 0.9$

$1.57 - 2.58 = -1.013 \text{ rad}$

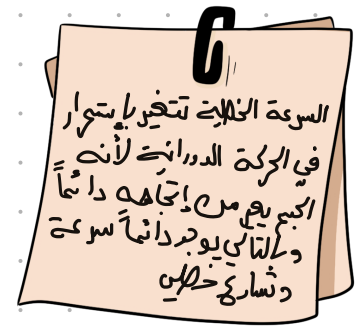
مع عقارب الساعة



\*  $v = \frac{2\pi}{T}$  where  $T \rightarrow$  (دورة)

**Key concept:**

If an object is travelling in a circle at constant speed, its instantaneous acceleration is always pointed exactly toward the centre of the circle.



centripetal acceleration. التسارع يكون نحو المحاور

\*  $a_c = \frac{v^2}{r}$  تسارع مركزي

\*  $F_c = ma_c \rightarrow \frac{mv^2}{r}$

$\alpha = \frac{a}{r}$  تسارع دائري  
تسارع يكون حول محور الدائرة

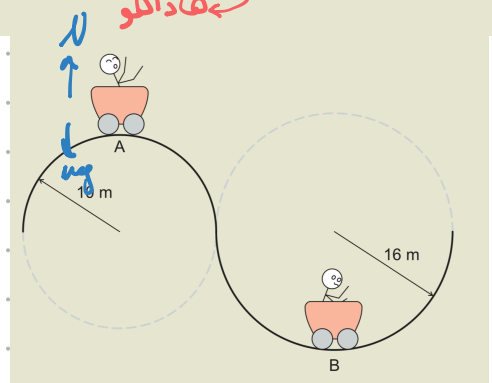
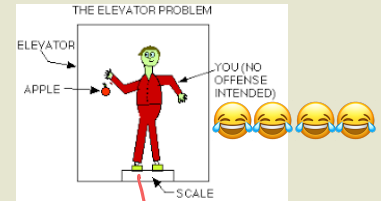
المقوة إلى بتحدث لما تكون

للسيارة بتدور ← الإمتكالك  
قمر والأرض ← جنب عام  
والجبل ← قوة شد

**Example 3.2 Angular velocity and centripetal force**

**Problem:** A 45 kg child is riding the rollercoaster at an amusement park. A portion of the track is made up of two semicircular sections as shown in Figure 3.4, one with a radius of 10 m and the other with a radius of 16 m. The rollercoaster travels along this section of track at a constant speed of 7 m s<sup>-1</sup>.

- In what direction is the child accelerating at point A?
- In what direction is the child accelerating at point B?
- If the child was on bathroom scales, what would they read (in kg) at point A?
- If the child was on bathroom scales, what would they read (in kg) at point B?



قمره سرعة مركزي ← دايماً نحو المركز  
ب) = = = =

ج) به حسب  $\frac{v}{r}$

$\Sigma F = N - mg$

$\frac{mv^2}{r} = N - mg \rightarrow$

$N = 230 \text{ N}$

$m = 23 \text{ kg}$

3.1 (a) Convert the following values from radians to degrees: (i)  $\frac{\pi}{6}$

(ii)  $\frac{\pi}{4}$  (iii)  $\frac{\pi}{2}$  (iv) 0.1 (v)  $\frac{3\pi}{4}$

(b) Convert the following values from degrees to radians: (i)  $1^\circ$

(ii)  $45^\circ$  (iii)  $60^\circ$  (iv)  $180^\circ$  (v)  $360^\circ$  Answer: (a) (i)  $30^\circ$  (ii)  $45^\circ$  (iii)

$90^\circ$  (iv)  $5.7^\circ$  (v)  $135^\circ$

(b) (i)  $\frac{\pi}{180}$  or 0.018 radians (ii)  $\frac{\pi}{4}$  or 0.79 radians (iii)  $\frac{\pi}{3}$  or 1.05 radians (iv)  $\pi$  or 3.14 radians (v)  $2\pi$  or 6.28 radians.

$$a) \text{ rad} \rightarrow \text{deg} \Rightarrow \frac{\text{rad}}{2\pi} * 360^\circ$$

$$i) \frac{\pi}{6} * \frac{360}{2\pi} = 30^\circ$$

$$ii) \frac{\pi}{4} * \frac{360}{2\pi} = 45^\circ$$

$$iii) \frac{\pi}{2} * \frac{360}{2\pi} = 90^\circ$$

$$iv) 0.1 * \frac{360}{2\pi} = 5.73^\circ$$

$$v) \frac{3\pi}{4} * \frac{360}{2\pi} = 135^\circ$$

$$b) \text{ deg} \rightarrow \text{rad}$$

$$i) 1^\circ * \frac{2\pi}{360} = 0.017 \text{ rad}$$

$$ii) 45^\circ * \frac{2\pi}{360} = 0.785 \text{ rad}$$

$$iii) 60^\circ * \frac{2\pi}{360} = 1.046 \text{ rad}$$

$$iv) 180 * \frac{2\pi}{360} = 3.14 \text{ rad}$$

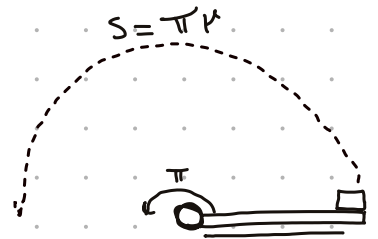
$$v) 360 * \frac{2\pi}{360} = 2.09 \text{ rad}$$

**3.2** In a particular rear-end car collision the driver's head rotates  $45^\circ$  backward before being stopped by the headrest. What is the average angular velocity of the driver's head if the duration of the collision was  $0.1 \text{ s}$ ? Answer:  $\omega_{av} = 7.9 \text{ radians s}^{-1}$

$$\theta = 45^\circ \rightarrow \frac{45}{360} \times 2\pi = 0.785 \text{ rad}$$

$$\omega = \frac{\Delta\theta}{\Delta t} \rightarrow \frac{0.785}{0.1} = 7.85 \text{ rad/s}$$

**3.3** When an athlete throws a javelin her forearm snaps through an angle of approximately  $\pi$  radians in  $0.20 \text{ s}$ . The athlete's hand moves with approximately constant speed, the length of her forearm is  $45 \text{ cm}$ , and the combined mass of her forearm and javelin is  $2.0 \text{ kg}$ . Assuming that the system is well approximated by a mass of  $2.0 \text{ kg}$  located  $45 \text{ cm}$  from the pivot, what force do the ligaments holding the forearm to the elbow need to exert? Answer:  $F = 220 \text{ N}$



$$\theta = 3.14 \text{ rad}$$

$$M = 2 \text{ kg}$$

$$\Delta t = 0.2 \text{ s}$$

$$r = 45 \text{ cm}$$

$$= 0.45 \text{ m}$$

لأننا عند نهاية  
pivot ما يباذ  $\theta$   
بماذ

$$F_c = M a_c \rightarrow F = \frac{M v^2}{r} = \frac{2 \times \left(\frac{3.14 \times 0.45}{0.2}\right)^2}{0.45} = 220 \text{ N}$$

**3.4 (a)** A  $3800 \text{ kg}$  car travels round an unbanked corner (i.e. a horizontal road) at the recommended speed of  $65 \text{ km h}^{-1}$ . The radius of curvature is  $80 \text{ m}$ . What is the force that the road exerts on the car to keep it in motion around the corner?

(b) What force would the road need to exert if the car was travelling at  $100 \text{ km h}^{-1}$ ? Answer: (a)  $F = 16 \times 10^3 \text{ N}$  (b)  $F = 37 \times 10^3 \text{ N}$

$$A) M = 3800 \text{ kg} \quad v = \frac{65 \text{ km}}{\text{h}} \quad r = 80 \text{ m}$$

$$F = \frac{M v^2}{r} \rightarrow \frac{3800 \times \left(\frac{65 \times 1000}{3600}\right)^2}{80} = 15.4 \times 10^3 \text{ N}$$

$$b) F = \frac{M v^2}{r} \rightarrow \frac{3800 \times \left(\frac{100 \times 1000}{3600}\right)^2}{80} = 36.65 \times 10^3 \text{ N}$$

3.5) A car is traveling around a circular race track at  $180 \text{ km h}^{-1}$ . If a single lap of the track is  $2.4 \text{ km}$  long, what is the angular velocity of the car (in  $\text{rad s}^{-1}$ )? Answer:  $0.13 \text{ rad s}^{-1}$

3.6 If the car in problem 3.5 weighs  $2500 \text{ kg}$ , what is the centripetal force acting on the car as it travels around the track?

Answer:  $16 \times 10^3 \text{ N}$

Single lap  $\rightarrow$  دورة كاملة

$$3.5) v = 180 \text{ km/h} \quad r = \frac{2.4 \text{ km}}{2\pi} \quad 2\pi r = \text{المدار}$$

$$\omega = \frac{v}{r} \rightarrow \frac{180 \times 1000}{3600} \div \frac{2.4 \times 1000}{2\pi} = 0.13 \text{ rad/s}$$

$$3.6) F = m \frac{v^2}{r} \rightarrow \frac{2500 \times \left( \frac{180 \times 1000}{3600} \right)^2}{\frac{2.4 \times 1000}{2\pi}} = 16 \times 10^3 \text{ N}$$

3.7 An adventurous ant finds herself at the end of a fan blade when it is switched on. It is a high speed fan with blades measuring  $0.20 \text{ m}$  long. If she has a mass of  $0.20 \text{ g}$  and can hold on to the fan blade with a maximum force of  $0.0124 \text{ N}$ . What is the maximum number of *revolutions per minute* the fan can run at before she will be flung off? Answer:  $154 \text{ rpm}$

$$r = 0.2 \text{ m} \quad m = 0.2 \times 10^{-3} \quad F = 0.0124$$

$$\frac{\text{rev}}{\text{min}} \quad F = \frac{m(\omega r)^2}{r}$$

$$0.0124 = 0.2 \times 10^{-3} (\omega)^2 0.2$$

$$\omega = 17.60 \text{ rad/s}$$

$$\frac{17.60}{2\pi} \times 60 = 168.1 \text{ rpm}$$

3.8 An 8.0 m radius merry-go-round completes one revolution every 7.0 s.

- What is the angular velocity of the merry-go-round?
- With what speed are children moving when they ride on the merry-go-round?
- What is the centripetal *acceleration* these children feel when riding on the merry-go-round?
- What is the average acceleration of each child over the course of half a revolution of the merry-go-round?
- What is the average acceleration of each child over the course of a full revolution turn of the merry-go-round?

Answer: (a)  $0.90 \text{ rad s}^{-1}$  (b)  $7.2 \text{ m s}^{-1}$  (c)  $6.5 \text{ m s}^{-2}$  (d)  $4.1 \text{ m s}^{-2}$  (e)  $0 \text{ m s}^{-2}$

a)  $r = 8 \text{ m}$      $t = 7 \text{ s}$      $\omega = \frac{v}{r}$   
 $s = 2\pi r \rightarrow \omega = \frac{2\pi r}{t} \rightarrow \frac{2\pi}{7} = 0.89 \text{ rad/s}$

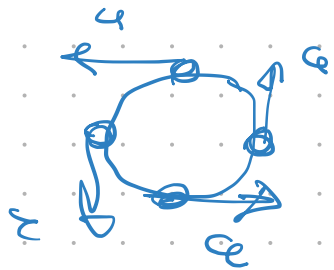
$\vec{a} = \frac{\vec{v} - (-\vec{v})}{\frac{T}{2}} = \frac{2\vec{v}}{\frac{T}{2}} = \frac{4\vec{v}}{T}$   
 $\frac{2\pi}{7} = 0.89 \text{ rad/s}$

b)  $v = \omega r \rightarrow 0.89 \times 8 = 7.17 \text{ m/s}$

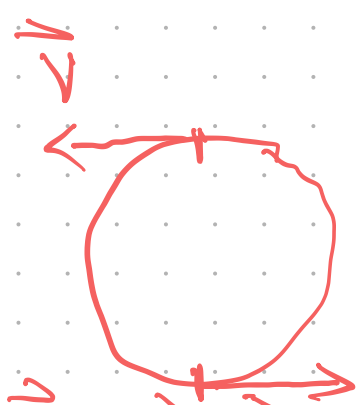
c)  $a_c = \frac{v^2}{r} = \frac{(7.17)^2}{8} = 6.42 \text{ m/s}^2$

d) ?

$\frac{v_f - v_i}{t} = 0$



$v = \frac{2\pi r}{T}$



مقدار التسارع المتوسط = 0

المتوسط يساوي صفر  
 لأن الاتجاه يتغير