Factoring

Def The common monomial factor of a polynomial is a common factor in each term

Exp (Common monomial factor)

Factor the following polynomials by finding the common monomial factor:

① $-3x^2t - 3x + 9xt^2 = 3x(-xt - 1 + 3t^2)$ or

$= -3x(xt + 1 - 3t^2)$

② $8a^2b - 160x + 4bx^2 = 4(2a^2b - 40x + bx^2)$ or

$= -4(-2a^2b + 40x - bx^2)$

Exp (Factoring by grouping)

Factor by grouping:

① $3x - 3y + 7ax - 7ay = 3(x-y) + 7a(x-y)$

$= (x-y)(3+7a)$

② $5y - 20 - x^2y + 4x^2 = 5(y-4) - x^2(y-4)$

$= (y-4)(5-x^2)$

Exp (Factoring Trinomial)

Factor the following expressions as product of binomials:

① $x^2 - 7x + 6 = (x+a)(x+b)$ where $ab = 6$

$a+b = -7$

$= (x-1)(x-6)$

$a = -1, \; b = -6$

② $x^2 - 21x + 20 = (x+a)(x+b)$

$\left.\begin{matrix} ab = 20 \\ a+b = -21 \end{matrix}\right\}$ $= (x-1)(x-20)$

③ $x^2 - 5x - 14 = (x+a)(x+b)$ where $ab = -14$

$= (x+2)(x-7)$ $a+b = -5$

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④ $9x^2 - \underline{\underline{9x}} - 10$

$(9x^2)(-10) = -90x^2$ ⇒ 90

$$\begin{array}{r|l} 2 & 90 \\ 5 & 45 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$90 = 2 \cdot 3 \cdot 3 \cdot 5$

$= 6 \cdot 15$

$= (-15x)(6x)$

since $-15x + 6x = \underline{\underline{-9x}}$

$9x^2 - 9x - 10 = 9x^2 - 15x + 6x - 10$

$= (9x^2 - 15x) + (6x - 10)$

$= 3x(3x-5) + 2(3x-5)$

$= (3x-5)(3x+2)$

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⑤ $9x^2 \underline{\underline{-31x}} + 12$

$(9x^2)(12) = 108x^2$ ⇒ 108

$$\begin{array}{r|l} 2 & 108 \\ 2 & 54 \\ 3 & 27 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$= (-27x)(-4x)$

since $-27x - 4x = \underline{\underline{-31x}}$

$108 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

$= 4 \cdot 27$

$9x^2 - 31x + 12 = 9x^2 - 27x - 4x + 12$

$= 9x(x-3) - 4(x-3)$

$= (x-3)(9x-4)$

(6) $7x^2 - 10x - 8$

$(7x^2)(-8) = -56x^2$ → 56

$= (-14x)(4x)$

2	56
2	28
2	14
7	7
	1

since $-14x + 4x = -10x$

$56 = 2 \cdot 2 \cdot 2 \cdot 7$
$= 4 \cdot 14$

$7x^2 - 10x - 8 = 7x^2 - 14x + 4x - 8$
$= 7x(x-2) + 4(x-2)$
$= (x-2)(7x+4)$

Special Factorizations:

A The perfect-square trinomials

$$x^2 + 2ax + a^2 = (x+a)^2$$
$$x^2 - 2ax + a^2 = (x-a)^2$$

B The difference of two squares

$$x^2 - a^2 = (x+a)(x-a)$$

$(x+a)$ and $(x-a)$ are called conjugates because they differ only in sign

Exp (Special Factorizations)

1. Factor completely the binomial $25x^2 - 36y^2$

$(5x - 6y)(5x + 6y)$

$5x - 6y$ is the conjugate of $5x + 6y$
$5x + 6y$ is the conjugate of $5x - 6y$

so $(5x - 6y)$ and $(5x + 6y)$ are conjugates

[2] Factor completely the trinomial $4x^2+12x+9$

Solution 1 : $4x^2+12x+9 = (2x+3)^2$

since this is perfect square

Solution 2 : $\underline{4x^2} + 12x + \underline{9}$

$(4x^2)(9) = 36\,x^2$

$= (6x)(6x)$

36

$$\begin{array}{c|c} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$36 = 2.2.3.3$

$= 6.6$

since $6x + 6x = \underline{\underline{12x}}$

$$4x^2+12x+9 = 4x^2+6x+6x+9$$
$$= 2x(2x+3)+3(2x+3)$$
$$= (2x+3)(2x+3)$$
$$= (2x+3)^2$$

[3] Factor completely the trinomial $16-40x+25x^2$

This is perfect square since $(4-5x)^2 = 16-40x+25x^2$

[4] Factor completely the binomial $16x^2-25y^2$

$= (4x-5y)(4x+5y)$

Hence, $(4x-5y)$ and $(4x+5y)$ are conjugates

Note : A poly. is factored completely if all possible factorizations have been completed.

Exp $(3x-6)(x+1)$ is not factored completely since $3x-6 = 3(x-2)$

Remark: Most polynomials we have factored are of degree two (quadratic polynomials).

Exp (Polynomials in Quadratic Form)

Factor completely

1. the binomial $x^4 - 25$ Let $a = x^2 \Rightarrow a^2 = x^4$

$a^2 - 25$

$(a-5)(a+5) = (x^2-5)(x^2+5)$

$= (x-\sqrt{5})(x+\sqrt{5})(x^2+5)$

2. $x^4 + 6x^2 + 9$ Let $a = x^2 \Rightarrow a^2 = x^4$

$a^2 + 6a + 9$

$(a+3)^2$ since this trinomial is prefect squar

$(x^2+3)^2$

3. the trinomial $4x^2 - 8x - 60 = 4(x^2 - 2x - 15)$ monomial

$ab = -15$

$= 4(x-5)(x+3)$ $a+b = -2$

$a = -5, b = 3$

4. the trinomial $12x^2 - 36x + 27 = 3(4x^2 - 12x + 9)$ monomial

$= 3(2x-3)^2$ perfect-square

5. the binomial $16x^2 - 64y^2 = 16(x^2 - 4y^2)$ monomial

$= 16(x-2y)(x+2y)$ conjugates

or $16x^2 - 64y^2 = (4x-8y)(4x+8y)$

$= 4(x-2y)\,4(x+2y) = 16(x-2y)(x+2y)$

Factorization with Cubes

Perfect Cube $\Rightarrow a^3 + 3a^2b + 3ab^2 + b^3 = (a+b)^3$

$\Rightarrow a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3$

Difference of two cubes $\Rightarrow a^3 - b^3 = (a-b)(a^2+ab+b^2)$

Sum of two cubes $\Rightarrow a^3 + b^3 = (a+b)(a^2-ab+b^2)$

Exp (Cubes Factorization)

Factor the following polynomials of degree three :

① $8x^3 - 1$ Let $a = 2x$ $\Rightarrow a^3 = 8x^3$

$(a^3 - b^3) = (a-b)(a^2+ab+b^2)$ where $b = 1$

$= (2x-1)(4x^2+2x+1)$

② $x^3 - 12x^2 + 48x - 64$ $a = x$, $b = 4$

$= (a-b)^3 = (x-4)^3$

③ $x^3 + 216$ $a = x$, $b = 6$

$a^3 + b^3 = (a+b)(a^2-ab+b^2)$

$= (x+6)(x^2-6x+36)$