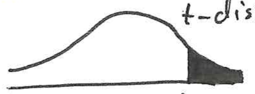
9.4 Hypothesis Testing about the population Mean (μ) when σ is Unknown.

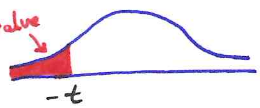
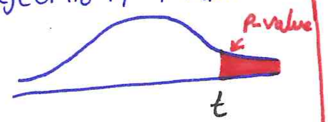
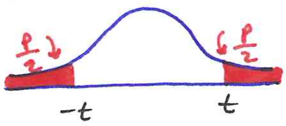
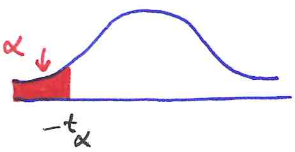
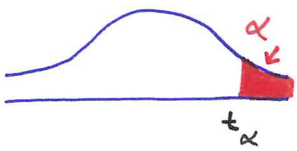
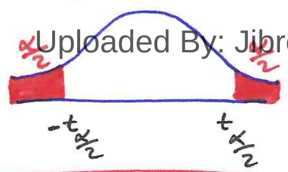
* When σ is Known, the sampling distribution of the test statistic z has a standard normal distribution. see pages 581 - 582

* when σ is unknown, the sampling distribution of the test statistic t has a t distribution see pages 583 - 585

⇒ The test statistic for hypothesis tests about the population mean μ when σ is unknown is

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$



	lower Tail Test	Upper Tail Test	Two Tailed Test
Hypothesis	$H_0: \mu \geq \mu_0$ $H_a: \mu < \mu_0$	$H_0: \mu \leq \mu_0$ $H_a: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_a: \mu \neq \mu_0$
Test statistic	$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$	$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$	$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$
Rejection Rule using p-value approach	Reject H_0 if P-value $\leq \alpha$ 	Reject H_0 if P-value $\leq \alpha$ 	Reject H_0 if P-value $\leq \alpha$ 
Rejection Rule using critical value approach	Reject H_0 if $t \leq -t_\alpha$ 	Reject H_0 if $t \geq t_\alpha$ 	Reject H_0 if $t \leq -t_{\alpha/2}$ or $t \geq t_{\alpha/2}$ 

* when the population is normally distributed, the hypothesis tests provide exact results
* " " " " not " " " , " " " " " approximation.
* If $n \geq 30$, then the hypothesis tests provid a good results.
* If the population is approximately normal, then small sample sizes ($n < 15$) will provid acceptable results.
* If the population is highly skewed or contains outliers, then $n \geq 50$ is recommended.

Example (Q 23 page 357) Consider the following hypothesis $H_0: \mu \le 12$
$H_a: \mu > 12$

A sample of 25 provided a sample mean 14 and a sample standard deviation $s = 4.32$.

Upper Tail Test

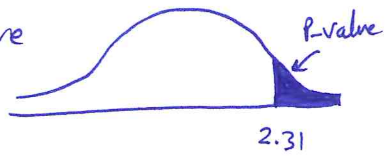
[a] Compute the value of test statistics. $\bar{x} = 14$, $\mu_0 = 12$
$s = 4.32$, $n = 25$, $d.f = 24$

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{14 - 12}{\frac{4.32}{\sqrt{25}}} = \frac{2}{0.864} = 2.31$$

[b] Compute the range for the p-value (use table of t-distribution)

$d.f = 24 \Rightarrow$ from the t table, we have

p is between 0.01 and 0.025

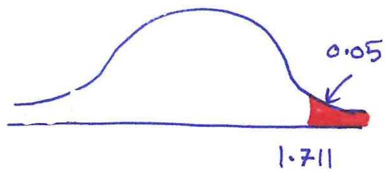


[c] At $\alpha = 0.05$, what is your conclusion?

Reject H_0 since p-value $\le \alpha = 0.05$

[d] what is the rejection rule using the critical value? what is your conclusion.

Reject H_0 if $t \ge t_\alpha = t_{0.05} = 1.711$



From the t table, we have $t_{0.05} = 1.711$

since $2.31 \ge 1.711$, so we reject H_0.

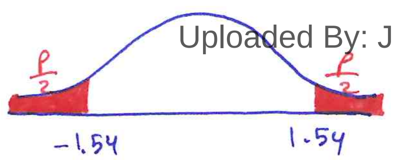
Example (Q 24 page 357) Consider the following hypothesis $H_0: \mu = 18$
$H_a: \mu \ne 18$

A sample of 48 provided a sample mean $\bar{x} = 17$ and a sample standard deviation $s = 4.5$

Tow Tailed Test

[a] Compute the value of the test statistic. $n = 48$, $\bar{x} = 17$, $\mu_0 = 18$, $s = 4.5$

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{17 - 18}{\frac{4.5}{\sqrt{48}}} = \frac{-1}{0.65} = -1.54$$



[b] Use the t distribution table to compute a range for p-value?

From the t-table, we have $\frac{p}{2}$ is between 0.05 and 0.10
$\Rightarrow$ p is between 0.10 and 0.20

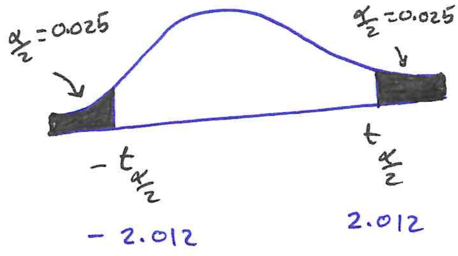
[c] At $\alpha = 0.05$, what is your conclusion?

Do not reject H_0 since p-value $> \alpha = 0.05$

[d] what is the rejection rule using the critical value? what is your conclusion?

$\alpha = 0.05$, $d.f = 47$

From the table, we have $t_{\frac{\alpha}{2}} = t_{0.025} = 2.012$



- Reject Ho if $t \leq -t_{\frac{\alpha}{2}} = -2.012$ or
  $t \geq t_{\frac{\alpha}{2}} = 2.012$

- since $t = -1.54 > -2.012$, we do not reject Ho.

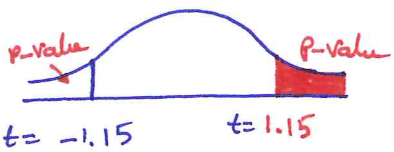
Example (Q 25 page 357) Consider the following hypothesis test $H_0: \mu \geq 45$
$H_a: \mu < 45$

A sample of 36 is used. Identify the p-value and state your conclusion for the following sample results: (Use $\alpha = 0.01$) lower Tail Test

$\boxed{a}$ $\bar{x} = 44$ and $s = 5.2$

$n = 36$, $d.f = 35$, $\mu_0 = 45$

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{44 - 45}{\frac{5.2}{\sqrt{36}}} = \frac{-1}{0.87} = -1.15$$



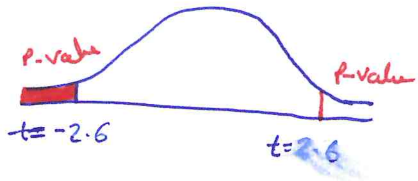
From the t-table we have p is between 0.10 and 0.20

$p \approx \frac{0.1 + 0.2}{2} = 0.15$

Don't reject Ho since p-value $= 0.15 > 0.01$

$\boxed{b}$ $\bar{x} = 43$ and $s = 4.6$

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{43 - 45}{\frac{4.6}{\sqrt{36}}} = \frac{-2}{0.77} = -2.6$$



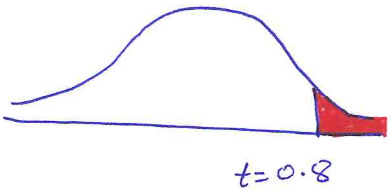
From the t-table, we have p is between 0.005 and 0.01

$p \approx 0.0075$

reject Ho since p-value $= 0.0075 < 0.01$

$\boxed{c}$ $\bar{x} = 46$ and $s = 5$

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{46 - 45}{\frac{5}{\sqrt{36}}} = \frac{1}{1.25} = 0.8$$



From the t-table, we have p is between 0.20 and more

Don't reject Ho since p-value > 0.01
