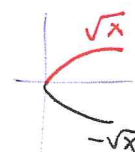


3.7 Implicit Differentiation

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Example: Find $\frac{dy}{dx}$ if $y^2 = x$

$$2y y' = 1 \Rightarrow y' = \frac{1}{2y}$$

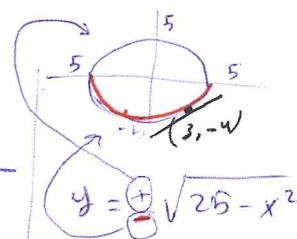


Note that $y = \pm \sqrt{x} \Rightarrow y'_1 = \frac{1}{2\sqrt{x}}$ for $y_1 = \sqrt{x}$
 $y'_2 = -\frac{1}{2\sqrt{x}}$ for $y_2 = -\sqrt{x}$

Example: Find the slope of the circle $x^2 + y^2 = 25$ at the point $(3, -4)$.

$$\text{slope: } 2x + 2y y' = 0 \Rightarrow y' = \frac{-x}{y}$$

$$\text{slope at } (3, -4) \text{ is } y' = \frac{-3}{-4} = \frac{3}{4}$$



Example: Find $\frac{dy}{dx}$ for $y^2 = x^2 + \sin xy$

$$2y y' = 2x + \cos xy (x y' + y)$$

$$2y y' - x y' \cos xy = 2x + y \cos xy$$

$$y' [2y - x \cos xy] = 2x + y \cos xy$$

$$y' = \frac{2x + y \cos xy}{2y - x \cos xy}$$

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Example: Find $\frac{d^2y}{dx^2}$ for $2x^3 - 3y^2 = 8$

$$6x^2 - 6y y' = 0 \Rightarrow y' = \frac{x^2}{y}, \quad y \neq 0$$

$$y'' = \frac{2xy - x^2 y'}{y^2} = \frac{2x}{y} - \frac{x^2}{y^2} y' = \frac{2x}{y} - \frac{x^4}{y^3}, \quad y \neq 0$$

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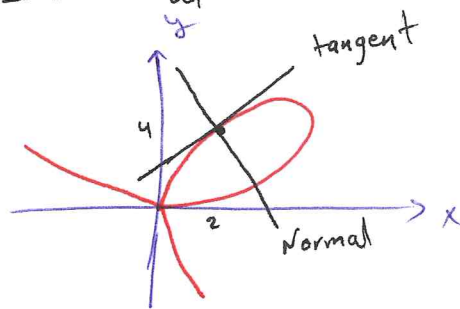
Example: Find the tangent and the normal to the curve $x^3 + y^3 - 9xy = 0$ at the point $(2, 4)$

$$3x^2 + 3y^2 y' - 9xy' - 9y = 0$$

$$y'(3y^2 - 9x) = 9y - 3x^2$$

$$y' = \frac{3y - x}{y^2 - 3x}$$

$$y' \Big|_{2,4} = \frac{3(4) - (2)}{(4)^2 - 3(2)} = \frac{12 - 2}{16 - 6} = \frac{10}{10} = 1$$



• The tangent line at $(2, 4)$ with slope 1 is

$$y = 1(x - 2) + 4 = x + 2$$

• The normal line at $(2, 4)$ is the line perpendicular to the tangent line at $(2, 4)$

$$y = -1(x - 2) + 4 = -x + 6$$

$$y = -x + 6$$