

Polar point $(r, \theta) \Rightarrow$ Same point $1) (r, \theta + 2\pi m)$ $m = 0, \pm 1, \pm 2, \dots$
 $2) (-r, \theta + \pi + 2\pi m) = (-r, \theta + (2m+1)\pi)$

- Q1
- a) $(3, 0) \Rightarrow$ c) $(-3, \pi) \Rightarrow (-r, \theta + \pi) + 2\pi m = (-r, \theta + \pi) = (-3, \pi) \checkmark$
 b) $(-3, 0) \Rightarrow$ g) $(-3, 2\pi) \Rightarrow (-r, \theta + 2\pi m) = (3, 2\pi) \checkmark$
 c) $(2, 2\pi/3) \Rightarrow$ i) $(r, \theta + 2\pi m) = (2, 2\pi/3 + 2\pi) = (2, 2\pi/3) \times$
 $2) (-r, \theta + (2m+1)\pi) = (-2, 2\pi/3 + \pi) = (-2, 2\pi/3 + 3\pi/3) = (-2, 5\pi/3) \Rightarrow$ h) $\therefore c \Rightarrow h \checkmark$
 d) $(2, 7\pi/3) \Rightarrow$ f) $(2, \pi/3) \Rightarrow (r, \theta + 2\pi m) = (2, \pi/3 + 2\pi) = (2, \pi/3) \checkmark$

- Q6
- a) $(\sqrt{2}, \pi/4) \Rightarrow$ 1) $x = r \cos \theta = \sqrt{2} \cos(\pi/4) = \sqrt{2} \cdot \frac{1}{\sqrt{2}} = 1$
 2) $y = r \sin \theta = \sqrt{2} \sin(\pi/4) = \sqrt{2} \cdot \frac{1}{\sqrt{2}} = 1$
 $(1, 1) \Rightarrow$ Cartesian point

- b) $(1, 0) \Rightarrow$ 1) $x = r \cos \theta = 1 \cos 0 = 1$
 2) $y = r \sin \theta = 1 \sin 0 = 0$
 $(1, 0)$

- c) $(0, \pi/2) \Rightarrow$ 1) $x = r \cos \theta = 0 \cos(\pi/2) = 0$
 2) $y = r \sin \theta = 0 \sin(\pi/2) = 0$
 $(0, 0)$

- d) $(-\sqrt{2}, \pi/4) \Rightarrow x = r \cos \theta = -\sqrt{2} \cos(\pi/4) = -\sqrt{2} \cdot \frac{1}{\sqrt{2}} = -1$
 $y = r \sin \theta = -\sqrt{2} \sin(\pi/4) = -\sqrt{2} \cdot \frac{1}{\sqrt{2}} = -1$
 $(-1, -1)$

- e) $(-3, 5\pi/6) \Rightarrow x = r \cos \theta = -3 \cos(5\pi/6) = -3 \cdot \left(-\frac{\sqrt{3}}{2}\right) = \frac{3\sqrt{3}}{2}$
 $y = r \sin \theta = -3 \sin(5\pi/6) = -3 \cdot \frac{1}{2} = -\frac{3}{2}$
 $\left(\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)$

① $\begin{array}{r} 180 \\ \times 6 \\ \hline 1080 \end{array}$ $\rightarrow \frac{180}{6} = 30$ $\therefore \frac{5\pi}{6} = 150^\circ$
 الـ $5\pi/6$ يعني الـ 150°
 الـ $\cos$ الـ 30°

- f) $(5, \tan^{-1}(4/3)) \Rightarrow x = r \cos \theta = 5 \cos(\tan^{-1}(4/3)) = 3$
 $y = r \sin \theta = 5 \sin(\tan^{-1}(4/3)) = 4$
 $(3, 4)$

- g) $(-1, 7\pi) \Rightarrow x = r \cos \theta = -1 \cos(7\pi) = -1 \cdot (-1) = 1$ $\times \cos \pi = \cos(m\pi) \quad m = 1, 3, 5, 7, \dots$
 $y = r \sin \theta = -1 \sin(7\pi) = -1 \cdot 0 = 0$ $\times \cos 0 = \cos(n\pi) \quad n = 0, 2, 4, \dots$
 $(1, 0)$

- h) $(2\sqrt{3}, 2\pi/3) \Rightarrow x = r \cos \theta = 2\sqrt{3} \cos(2\pi/3) = 2\sqrt{3} \cdot \left(-\frac{1}{2}\right) = -\sqrt{3}$ $2 \times 180 = 360$
 $y = r \sin \theta = 2\sqrt{3} \sin(2\pi/3) = 2\sqrt{3} \cdot \frac{\sqrt{3}}{2} = 3$ $\frac{360}{3} = 120$
 $(-\sqrt{3}, 3)$ $\rightarrow$ الـ 120°
 الـ $\sin$ الـ 60°
 الـ $\cos$ الـ 60°

Q7 $0 \leq \theta < 2\pi, r \geq 0$

- a) $(1, \pi/4) \Rightarrow x^2 + y^2 = r^2 \Rightarrow 1 + 1 = r^2 \Rightarrow r = \sqrt{2}$
 $x = r \cos \theta \Rightarrow 1 = \sqrt{2} \cos \theta \Rightarrow \cos \theta = \frac{1}{\sqrt{2}}$
 $y = r \sin \theta \Rightarrow 1 = \sqrt{2} \sin \theta \Rightarrow \sin \theta = \frac{1}{\sqrt{2}}$
 $(\sqrt{2}, \pi/4)$

b) $(-3, 0) \Rightarrow x^2 + y^2 = r^2 \Rightarrow 9 = r^2 \Rightarrow r = \pm 3 \Rightarrow r > 0 \Rightarrow r = 3$

$x = r \cos \theta \Rightarrow -3 = 3 \cos \theta \Rightarrow \cos \theta = -1 \Rightarrow \theta = \pi$
 $y = r \sin \theta \Rightarrow 0 = 3 \sin \theta \Rightarrow \sin \theta = 0$
 $(3, \pi)$

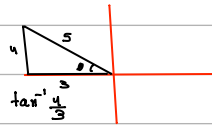
c) $(\sqrt{3}, -1) \Rightarrow x^2 + y^2 = r^2 \Rightarrow 3 + 1 = r^2 \Rightarrow r = \pm 2 \Rightarrow r = 2$

$x = r \cos \theta \Rightarrow \sqrt{3} = 2 \cos \theta \Rightarrow \cos \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}$
 $y = r \sin \theta \Rightarrow -1 = 2 \sin \theta \Rightarrow \sin \theta = -\frac{1}{2} \Rightarrow \theta = \frac{7\pi}{6}$
 $(2, \frac{7\pi}{6})$

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 $\frac{180}{\pi} x \leftarrow$
 $\frac{\pi}{180} \leftarrow$

d) $(-3, 4) \Rightarrow x^2 + y^2 = r^2 \Rightarrow 9 + 16 = r^2 \Rightarrow r = 5$

$x = r \cos \theta \Rightarrow -3 = 5 \cos \theta \Rightarrow \cos \theta = -\frac{3}{5}$
 $y = r \sin \theta \Rightarrow 4 = 5 \sin \theta \Rightarrow \sin \theta = \frac{4}{5}$

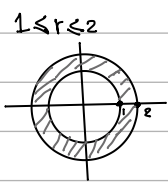


$\theta = \pi - \tan^{-1}(\frac{4}{3})$

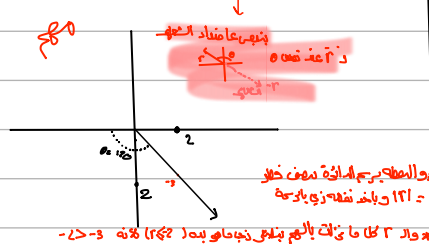
$(5, \pi - \tan^{-1}(\frac{4}{3}))$

باعد من موجب و موجب (2, 1) و (1, 2)
 دمن لاله و سلب (1, -2) و (-1, 1)

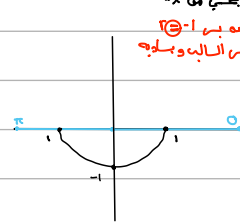
Q4



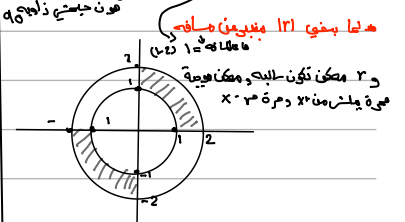
Q16 $\theta = 2\pi/3, r \leq -2$



Q22 $0 \leq \theta \leq \pi, r = -1$



Q26 $0 \leq \theta \leq \pi/2, 1 \leq r \leq 2$



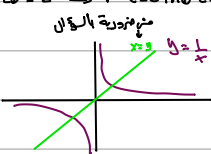
Q32

$r = -3 \sec \theta \Rightarrow r = -\frac{3}{\cos \theta} \Rightarrow r \cos \theta = -3 \Rightarrow x = -3$ vertical line $\Rightarrow (x, y) = (-3, 0)$



Q38

$r^2 \sin 2\theta = 2 \Rightarrow r^2 (2 \sin \theta \cos \theta) = 2 \Rightarrow r^2 \sin \theta \cos \theta = 1 \Rightarrow (r \sin \theta)(r \cos \theta) = 1 \Rightarrow yx = 1 \Rightarrow$ hyperbola with focal axis $x = y$
 $\Rightarrow y = \frac{1}{x}$



Q42

$r \sin \theta = \ln r + \ln \cos \theta \Rightarrow r \sin \theta = \ln(r \cos \theta) \Rightarrow y = \ln x$



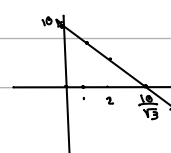
Q52

$r \sin(\frac{2\pi}{3} - \theta) = 5 \Rightarrow \sin(A+B) = \sin A \cos B + \sin B \cos A$

$r [\sin \frac{2\pi}{3} \cos \theta - \sin \theta \cos \frac{2\pi}{3}] = 5$

$r [\frac{\sqrt{3}}{2} \cos \theta - (-\frac{1}{2}) \sin \theta] = 5$

$\frac{\sqrt{3}}{2} r \cos \theta + \frac{1}{2} r \sin \theta = 5 \Rightarrow \frac{1}{2} [\sqrt{3} r \cos \theta + r \sin \theta] = 5 \Rightarrow \sqrt{3} r \cos \theta + r \sin \theta = 10 \Rightarrow r \sin \theta = 10 - \sqrt{3} r \cos \theta \Rightarrow y = 10 - \sqrt{3} x$



x	y
0	10
1	10 - sqrt(3)
2	10 - 2sqrt(3)

Q58 $x^2 - y^2 = 1 \Rightarrow (r \cos \theta)^2 - (r \sin \theta)^2 = 1 \Rightarrow r^2 \cos^2 \theta - r^2 \sin^2 \theta = 1 \Rightarrow r^2 [\overset{\text{double}}{\cos^2 \theta - \sin^2 \theta}] = 1 \Rightarrow r \cos 2\theta = 1$

Q62 $x^2 + xy + y^2 = 1 \Rightarrow \overset{r}{x^2 + y^2} + xy = 1 \Rightarrow r^2 + r^2 \cos \theta \sin \theta = 1 \Rightarrow r^2 [1 + \cos \theta \sin \theta] = 1$