2.3 Business Applications Using Quadratics

- In section 1.6 we show how to find the Break-Even point and the Equilibrium point when the total cost, total revenue, total profit, demand function and supply functions are linear functions.
- In this section we may consider quadratic forms for C(x) or R(x) or demand function or supply function.

Recall that
- the point intersection of the revenue function R(x) and the cost function C(x) is called Break-Even point.
- the point intersection of the demand and supply functions is called the market Equilibrium point
- we find the Break-Even point by setting $C(x) = R(x)$
- we find the market Equilibrium by setting demand = supply

Exp A monopoly market has a company whose total costs are $C(x) = 2000 + 40x + x^2$ and total revenues are $R(x) = 130x$

(1) Find the break-even points

$C(x) = R(x) \Rightarrow 2000 + 40x + x^2 = 130x$

$2000 - 90x + x^2 = 0$

$x^2 - 90x + 2000 = 0$

$(x - 40)(x - 50) = 0$

$ab = 2000$

$a + b = -90$

$a = -40$

$b = -50$

$x - 40 = 0$ or $x - 50 = 0$

$\boxed{x = 40}$ or $\boxed{x = 50}$

(2) Find the cost and revenue at the break-even points

when $x = 40 \Rightarrow C(40) = R(40) = 130(40) = 5{,}200$

$x = 50 \Rightarrow C(50) = R(50) = 130(50) = 6{,}500$

(3) sketch the total cost and the total revenue

$R(x) = 130x$ is line through origin

$C(x) = 2000 + 40x + x^2$ is parabola with

$a = 1 > 0$ opens up

$b = 40$ and $c = 2000$

$x = -\frac{b}{2a} = -\frac{40}{2} = -20$

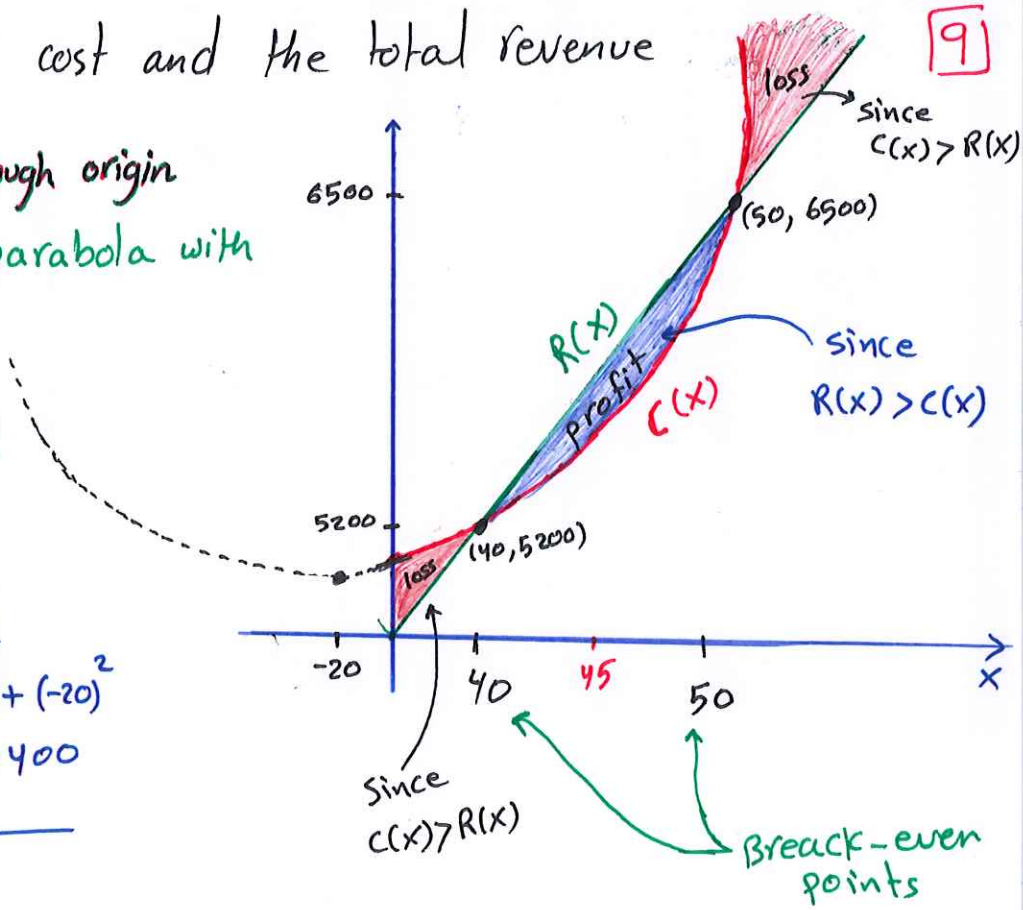
is the axis of symmetry

$f(-20) = C(-20) = 2000 + 40(-20) + (-20)^2$

$= 2000 - 800 + 400$

$= 1600$

$(-20, 1600)$ is the vertex



(4) Identify the regions where the company makes profit and where it has loss

loss region when the production level x is such that

when $0 \le x < 40$ or when $x > 50$

(5) When does the company makes zero profit?

At the break-even points : $x = 40$ and $x = 50$ since $R(x) = C(x)$ at these points

(6) Find the total profit

$P(x) = R(x) - C(x)$

$= 130x - (2000 + 40x + x^2)$

$= 90x - 2000 - x^2$

[7] Graph the profit function

$P(x) = -x^2 + 90x - 2000$

$a = -1 < 0$ opens down

$b = 90$ and $c = -2000$

$x = -\frac{b}{2a} = -\frac{90}{-2} = 45$

is the axis of symmetry

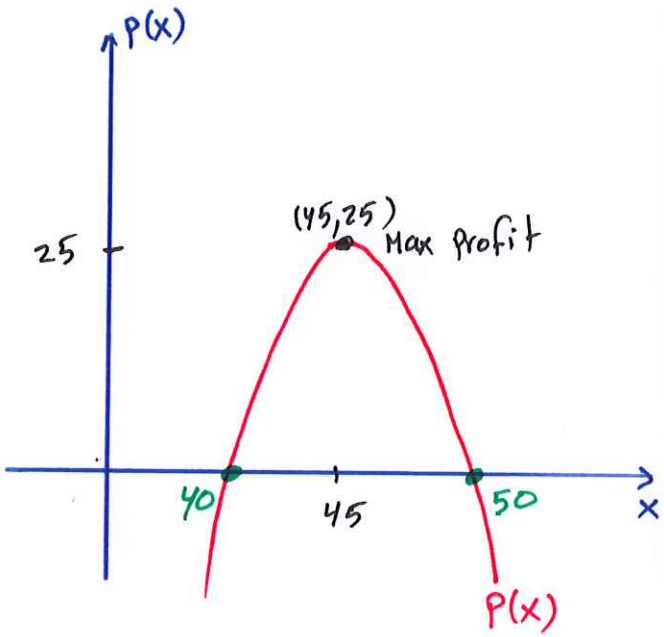
$f(45) = -(45)^2 + 90(45) - 2000$

$= -2,025 + 4,050 - 2000$

$= 25$

(45, 25) is the vertex

which is maximum profit



[8] At what level of production x the company makes maximum profit?

At $x = 45$ the company makes max profit $P(45) = 25$

[9] what does x-intercept for the profit function mean?

$x = 40$ and $x = 50$ are the break-even points

which make the profit zero

[10] For what values of x the profit is positive

$40 < x < 50$

[11] For what values of x the profit is negative?

$0 \leq x < 40$ and $x > 50$

[13] Describe the average rate of change of the profit before and closer and after $x = 45$

Befor $x = 45$ it is positive/closer to $x = 45$ it is zero/after $x = 45$ it is negative

Exp If the supply and demand functions for a commodity are given by $p - q = 10$ and $q(2p - 10) = 2100$

(1) what is the Equilibrium quantity and price

$p = q + 10 \Rightarrow q[2(q+10) - 10] = 2100$

$q[2q + 20 - 10] = 2100$

$q[2q + 10] = 2100$

$2q^2 + 10q = 2100$

$q^2 + 5q = 1050$

$q^2 + 5q - 1050 = 0$

$(q - 30)(q + 35) = 0$

Either $q - 30 = 0$ or $q + 35 = 0$

$q^* = 30$

Equilibrium quantity

$q = -35$ rejected since q must be non negative

$ab = -1050$

$a + b = 5$

$a = -30$

$b = 35$

$p = 30 + 10$

$p^* = 40$

Equilibrium Price

(2) Draw the demand and supply functions

$p = q + 10$ is the supply function

when $q = 0 \Rightarrow p = 10$

$q^* = 30 \Rightarrow p^* = 40$

$q(2p - 10) = 2100$

$2p - 10 = \frac{2100}{q}$

$p - 5 = \frac{1050}{q}$

$p = 5 + \frac{1050}{q}$ is the demand function

q	1	30	70	150	350	1050
p	1055	40	20	12	8	6