Exercises:

Q22: If a Random variable X has a poisson distribution such that $Pr(X=1) = Pr(X=2)$, Find $Pr(X=4)$.

$\rightarrow Pr(X=1) = \frac{m^1 e^{-m}}{1!} = m e^{-m}$.

$\rightarrow Pr(X=2) = \frac{m^2 e^{-m}}{2!} = \frac{m^2 e^{-m}}{2}$

But $\frac{m e^{-m}}{1} \neq \frac{m^2 e^{-m}}{2}$

$m^2 e^{-m} - 2me^{-m} = 0$

$me^{-m} (m-2) = 0$ $\quad\quad m \neq 0, \; m > 0$

$m - 2 = 0$ $\quad\quad e^{-m} \neq 0$

$m = 2$.

so $Pr(X=4) = \frac{2^4 e^{-2}}{4!} = \frac{16 e^{-2}}{24} = 0.09$.

Q23: The m.g.f of a r.v X is $e^{4(e^t-1)}$, show that $Pr(\mu - 2\sigma < X < \mu + 2\sigma) = 0.931$.

$X \sim$ Poison, $M(t) = e^{m(e^t-1)}$

so $m = 4$.

and we can find $\mu = m = 4$

and $\sigma = \sqrt{4} = 2$

$\rightarrow$

So $Pr(\mu - 2\sigma < X < \mu + 2\sigma) = Pr(0 < X < 8)$

$= Pr(X < 8) - Pr(X \leq 0)$

$= Pr(X \leq 7) - Pr(X \leq 0)$

50% By table $= 0.949 - 0.018$

$= 0.931$

Q25: let the p.d.f $f(x)$ be positive on and only on the nonnegative integers, given that $f(x) = \frac{4}{x} f(x-1)$, $x = 1, 2, \ldots$. Find $f(x)$.

$f(1) = 4 f(0)$, $f(2) = \frac{4^2}{2!} f(0)$, $f(3) = \frac{4^3}{3!} f(0)$, ...

$\Rightarrow f(x) = \frac{4^x}{x!} f(0)$.

Now $\sum_{x=0}^{\infty} f(x) = 1$ since it p.d.f

$\sum_{x=0}^{\infty} \frac{4^x}{x!} f(0) = 1$

$f(0) \sum_{x=0}^{\infty} \frac{4^x}{x!} = 1$ (we know : $\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$)

$f(0)\, e^4 = 1 \rightarrow f(0) = e^{-4}$

$\Rightarrow f(x) = \frac{4^x}{x!} e^{-4}$.

$\mu = 100$, $\sigma^2 = 100 \rightarrow \sigma = 10$

Q26: let X have a poisson distribution with $\mu = 100 \overset{Var}{=}$. use chebyshev's inequality to determine a lower bound for $Pr(75 < X < 125)$.

$Pr(|X-\mu| \geq k\sigma) \leq \frac{1}{k^2}$ " chebyshev's inequality "

$Pr(75 < X < 125) = Pr(|X-100| < 25)$

$= 1 - Pr(|X-100| \geq 25)$

$\geq 1 - \frac{100}{25^2}$

≥ 0.84

Q31: let X have a poisson distribution. if $Pr(X=1) = Pr(X=3)$. Find the mode of the distribution.

$\rightarrow Pr(X=1) = \frac{m e^{-m}}{1!} = m e^{-m}$.

$\rightarrow Pr(X=3) = \frac{m^3 e^{-m}}{3!} = \frac{m^3 e^{-m}}{6}$

But $m e^{-m} = \frac{m^3 e^{-m}}{6}$

$\Rightarrow m^3 e^{-m} - 6 m e^{-m} = 0$

$\Rightarrow m e^{-m} (m^2 - 6) = 0$ $m \neq 0$, $e^{-m} \neq 0$.

$\Rightarrow m^2 - 6 = 0$

$\Rightarrow m = \sqrt{6} = 2.44$ Not integer.

So mode = 2

Done