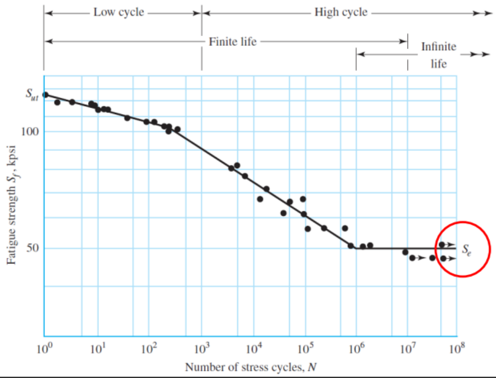
$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \le 200 \text{ kpsi } (1400 \text{ MPa}) \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases}$$



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$$S_e = k_a k_b k_c k_d k_e k_f S'_e \qquad (6\text{–}18)$$

where k_a = surface condition modification factor

k_b = size modification factor

k_c = load modification factor

k_d = temperature modification factor

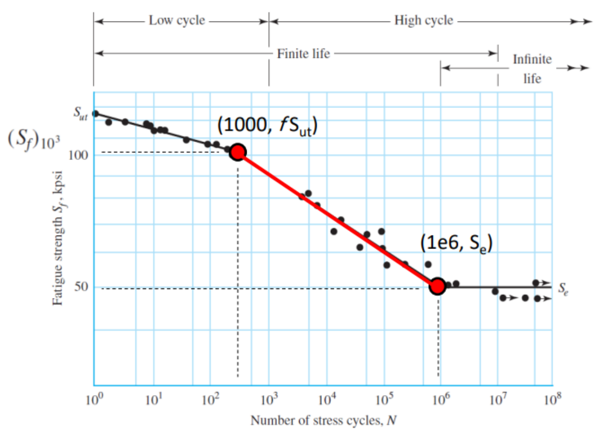
k_e = reliability factor[13]

k_f = miscellaneous-effects modification factor

S'_e = rotary-beam test specimen endurance limit

S_e = endurance limit at the critical location of a machine part in the geometry and condition of use

$$(S_f)_{10^3} = f\, S_{ut}.$$



$$S_f = a\, N^b$$

$$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b}$$

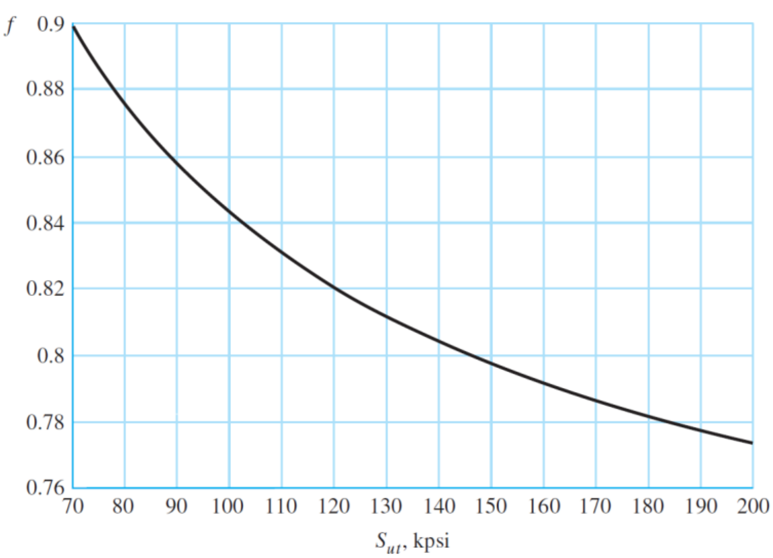
$$a = \frac{(f\, S_{ut})^2}{S_e}$$

$$b = -\frac{1}{3}\log\left(\frac{f\, S_{ut}}{S_e}\right)$$

Surface Factor k_a

$$k_a = aS_{ut}^b$$

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995



Size Factor k_b

- The results for bending and torsion may be expressed as:

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases} \qquad (6\text{–}20)$$

- For axial loading there is no size effect, so

$$k_b = 1 \qquad (6\text{–}21)$$

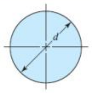
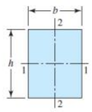
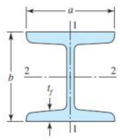
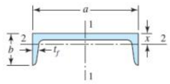
Rotating Components

1/13/2021

Size Factor k_b

Table 6–3

$A_{0.95\sigma}$ Areas of Common Nonrotating Structural Shapes

	$A_{0.95\sigma} = 0.01046d^2$ $d_e = 0.370d$
	$A_{0.95\sigma} = 0.05hb$ $d_e = 0.808\sqrt{hb}$
	$A_{0.95\sigma} = \begin{cases} 0.10at_f & \text{axis 1-1} \\ 0.05ba \quad t_f > 0.025a & \text{axis 2-2} \end{cases}$
	$A_{0.95\sigma} = \begin{cases} 0.05ab & \text{axis 1-1} \\ 0.052xa + 0.1t_f(b - x) & \text{axis 2-2} \end{cases}$

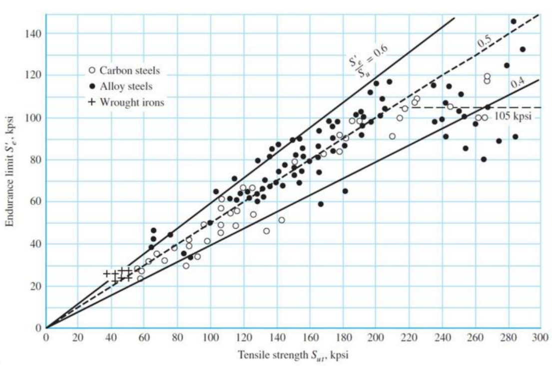
$$d_e = 0.370d$$

Load Factor k_c

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases} \qquad (6\text{–}26)$$

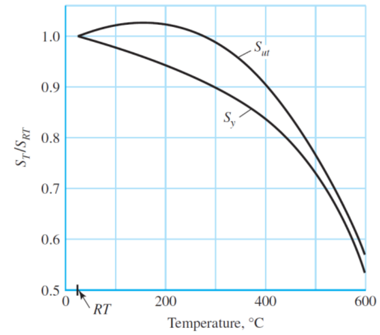
When torsion is combined with other loading, such as bending, set $k_c = 1$, and the combined loading is managed by using the effective von Mises stress as described in Sec. 6-14

$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \le 200 \text{ kpsi (1400 MPa)} \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases} \qquad (6\text{–}8)$$



Temperature Factor k_d

$$k_d = \frac{S_T}{S_{RT}}$$



A fourth-order polynomial curve fit to the data underlying Fig. 2–9 gives

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4 \qquad (6\text{–}27)$$

where $70 \le T_F \le 1000°F$.

Temperature, °C	S_T/S_{RT}	Temperature, °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

Reliability Factor k_e

$$k_e = 1 - 0.08\, z_a$$

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

Stress Concentration factors:

- This index at stress concentration is referred as the stress concentration factor K_t or K_{ts}

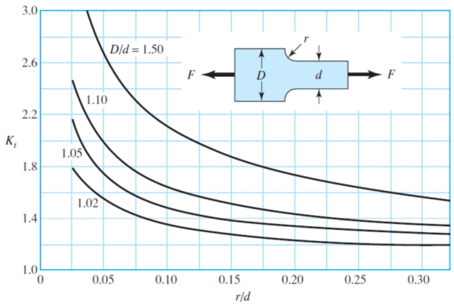
$$K_t = \frac{\sigma_{max}}{\sigma_0} \qquad K_{ts} = \frac{\tau_{max}}{\tau_0}$$

- The stress concentration factors K_t or K_{ts} are taken from experiments

Stress Concentration Factor k_t

Figure A–15–5

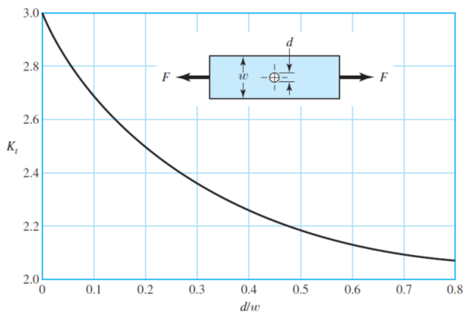
Rectangular filleted bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.



Stress Concentration Factor k_t

Figure A–15–1

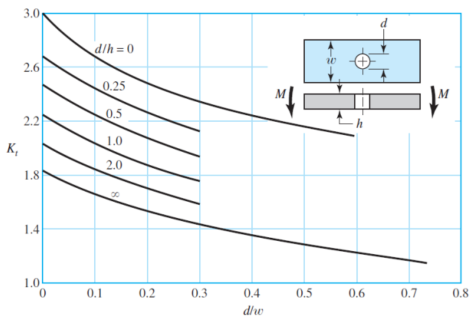
Bar in tension or simple compression with a transverse hole. $\sigma_0 = F/A$, where $A = (w - d)t$ and t is the thickness.



Stress Concentration Factor k_t

Figure A–15–2

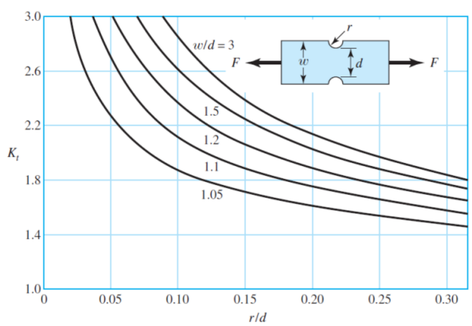
Rectangular bar with a transverse hole in bending. $\sigma_0 = Mc/I$, where $I = (w - d)h^3/12$.



Stress Concentration Factor k_t

Figure A–15–3

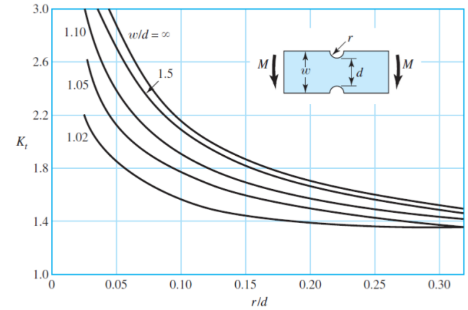
Notched rectangular bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.



Stress Concentration Factor k_t

Figure A–15–4

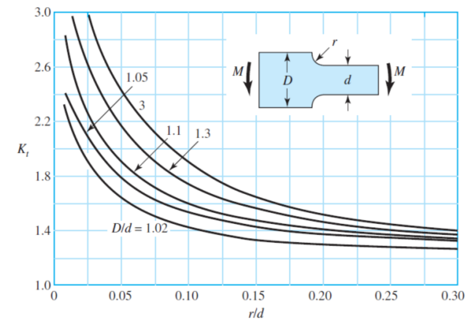
Notched rectangular bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$, $I = td^3/12$, and t is the thickness.



Stress Concentration Factor k_t

Figure A–15–6

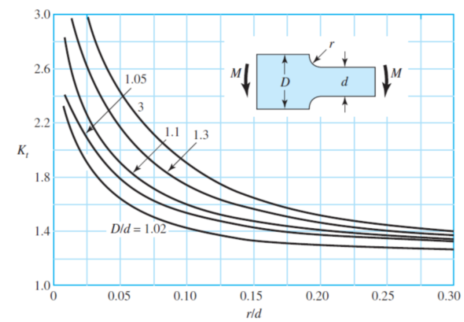
Rectangular filleted bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$, $I = td^3/12$, t is the thickness.



Stress Concentration Factor k_t

Figure A–15–6

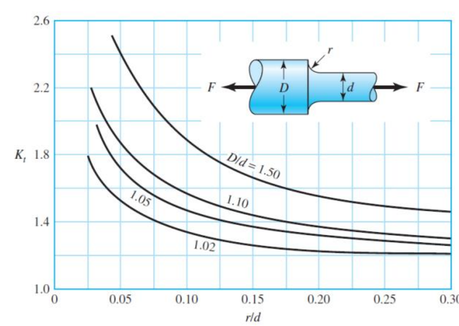
Rectangular filleted bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$, $I = td^3/12$, t is the thickness.



Stress Concentration Factor k_t

Figure A–15–7

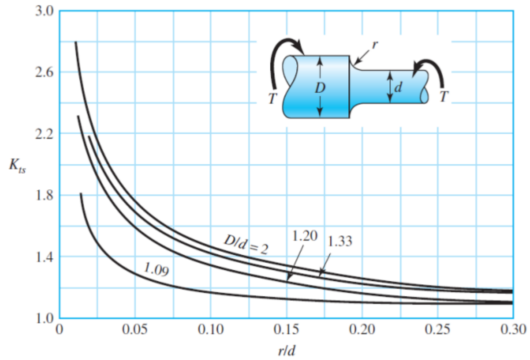
Round shaft with shoulder fillet in tension. $\sigma_0 = F/A$, where $A = \pi d^2/4$.



Stress Concentration Factor k_{ts}

Figure A–15–8

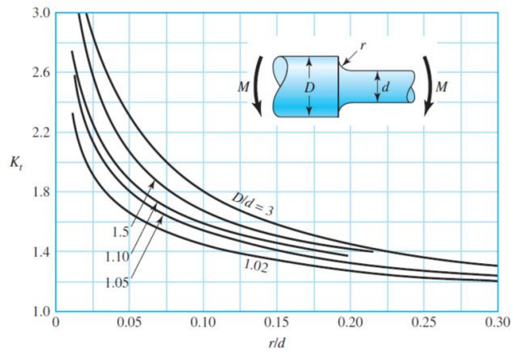
Round shaft with shoulder fillet in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.



Stress Concentration Factor k_t

Figure A–15–9

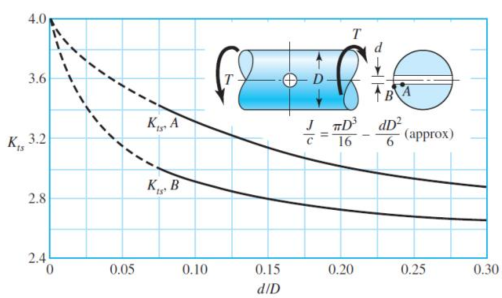
Round shaft with shoulder fillet in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.



Stress Concentration Factor k_{ts}

Figure A–15–10

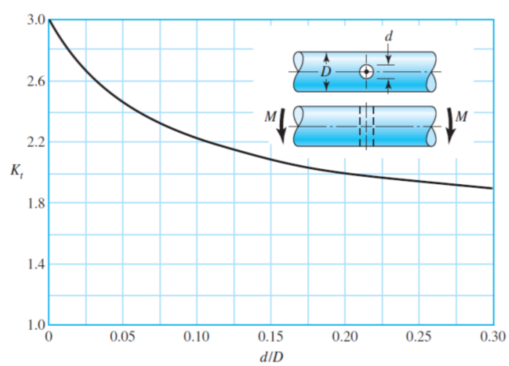
Round shaft in torsion with transverse hole.



Stress Concentration Factor k_t

Figure A–15–11

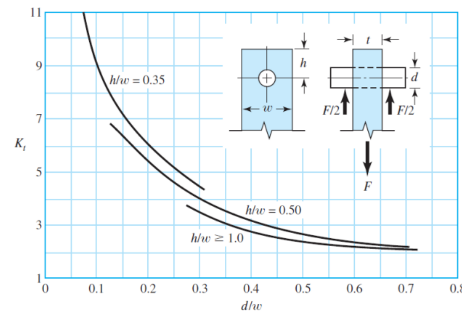
Round shaft in bending with a transverse hole. $\sigma_0 = M/[(\pi D^3/32) - (dD^2/6)]$, approximately.



Stress Concentration Factor k_t

Figure A–15–12

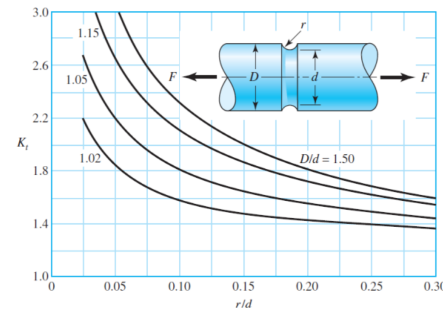
Plate loaded in tension by a pin through a hole. $\sigma_0 = F/A$, where $A = (w - d)t$. When clearance exists, increase K_t 35 to 50 percent. *(M. M. Frocht and H. N. Hill, "Stress-Concentration Factors around a Central Circular Hole in a Plate Loaded through a Pin in Hole," J. Appl. Mechanics, vol. 7, no. 1, March 1940, p. A-5.)*



Stress Concentration Factor k_t

Figure A–15–13

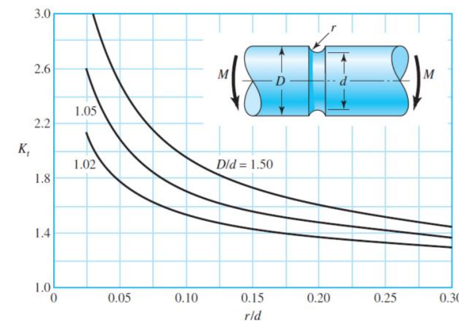
Grooved round bar in tension. $\sigma_0 = F/A$, where $A = \pi d^2/4$.



Stress Concentration Factor k_t

Figure A–15–14

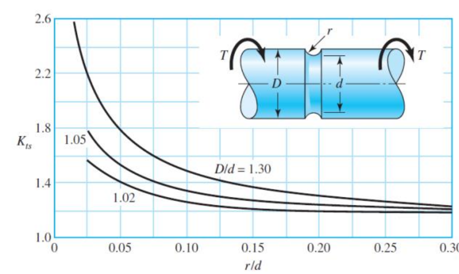
Grooved round bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.



Stress Concentration Factor k_{ts}

Figure A–15–15

Grooved round bar in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.



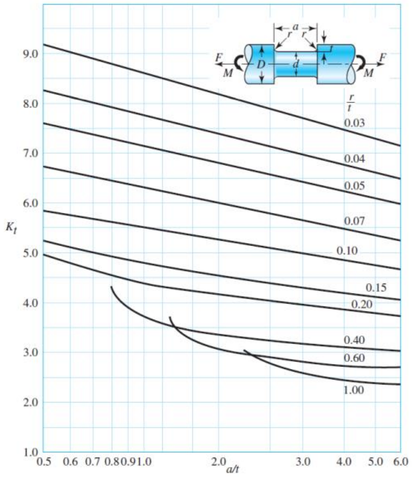
Stress Concentration Factor k_t

Figure A–15–16

Round shaft with flat-bottom groove in bending and/or tension.

$\sigma_0 = \frac{4F}{\pi d^2} + \frac{32M}{\pi d^3}$

Source: W. D. Pilkey and D. F. Pilkey, *Peterson's Stress-Concentration Factors,* 3rd ed. John Wiley & Sons, Hoboken, NJ, 2008, p. 115.



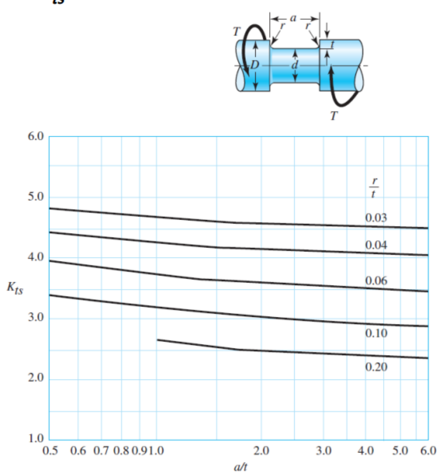
Stress Concentration Factor k_{ts}

Figure A–15–17

Round shaft with flat-bottom groove in torsion.

$\tau_0 = \frac{16T}{\pi d^3}$

Source: W. D. Pilkey and D. F. Pilkey, *Peterson's Stress-Concentration Factors,* 3rd ed. John Wiley & Sons, Hoboken, NJ, 2008, p. 133

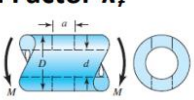


Stress Concentration Factor k_t

Table A–16

Approximate Stress-Concentration Factor K_t of a Round Bar or Tube with a Transverse Round Hole and Loaded in Bending

Source: R. E. Peterson, *Stress-Concentration Factors,* Wiley, New York, 1974, pp. 146, 235.



The nominal bending stress is $\sigma_0 = M/Z_{net}$ where Z_{net} is a reduced value of the section modulus and is defined by

$$Z_{net} = \frac{\pi A}{32D}(D^4 - d^4)$$

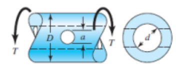
Values of A are listed in the table. Use $d = 0$ for a solid bar

	d/D					
	0.9		0.6		0	
a/D	A	K_t	A	K_t	A	K_t
0.050	0.92	2.63	0.91	2.55	0.88	2.42
0.075	0.89	2.55	0.88	2.43	0.86	2.35
0.10	0.86	2.49	0.85	2.36	0.83	2.27
0.125	0.82	2.41	0.82	2.32	0.80	2.20
0.15	0.79	2.39	0.79	2.29	0.76	2.15
0.175	0.76	2.38	0.75	2.26	0.72	2.10
0.20	0.73	2.39	0.72	2.23	0.68	2.07
0.225	0.69	2.40	0.68	2.21	0.65	2.04
0.25	0.67	2.42	0.64	2.18	0.61	2.00
0.275	0.66	2.48	0.61	2.16	0.58	1.97
0.30	0.64	2.52	0.58	2.14	0.54	1.94

Stress Concentration Factor k_{ts}

Table A–16 *(Continued)*

Approximate Stress-Concentration Factors K_{ts} for a Round Bar or Tube Having a Transverse Round Hole and Loaded in Torsion *Source:* R. E. Peterson, *Stress-Concentration Factors,* Wiley, New York, 1974, pp. 148, 244.



The maximum stress occurs on the inside of the hole, slightly below the shaft surface. The nominal shear stress is $\tau_0 = TD/2J_{net}$, where J_{net} is a reduced value of the second polar moment of area and is defined by

$$J_{net} = \frac{\pi A(D^4 - d^4)}{32}$$

Values of A are listed in the table. Use $d = 0$ for a solid bar.

	d/D									
	0.9		0.8		0.6		0.4		0	
a/D	A	K_{ts}	A	K_{ts}	A	K_{ts}	A	K_{ts}	A	K_{ts}
0.05	0.96	1.78							0.95	1.77
0.075	0.95	1.82							0.93	1.71
0.10	0.94	1.76	0.93	1.74	0.92	1.72	0.92	1.70	0.92	1.68
0.125	0.91	1.76	0.91	1.74	0.90	1.70	0.90	1.67	0.89	1.64
0.15	0.90	1.77	0.89	1.75	0.87	1.69	0.87	1.65	0.87	1.62
0.175	0.89	1.81	0.88	1.76	0.87	1.69	0.86	1.64	0.85	1.60
0.20	0.88	1.96	0.86	1.79	0.85	1.70	0.84	1.63	0.83	1.58
0.25	0.87	2.00	0.82	1.86	0.81	1.72	0.80	1.63	0.79	1.54
0.30	0.80	2.18	0.78	1.97	0.77	1.76	0.75	1.63	0.74	1.51
0.35	0.77	2.41	0.75	2.09	0.72	1.81	0.69	1.63	0.68	1.47
0.40	0.72	2.67	0.71	2.25	0.68	1.89	0.64	1.63	0.63	1.44

3/13/2021

Stress Concentration factors:

- This index at stress concentration is referred as the stress concentration factor **K_t** or **K_{ts}**

$$K_t = \frac{\sigma_{max}}{\sigma_0} \qquad K_{ts} = \frac{\tau_{max}}{\tau_0}$$

- The stress concentration factors ***K_t*** or ***K_{ts}*** are taken from experiments

- The Fatigue stress concentration factors ***K_f*** or ***K_{fs}***

$$K_f = \frac{\text{maximum stress in notched specimen}}{\text{stress in notch-free specimen}}$$

Notch sensitivity q is defined by the equation

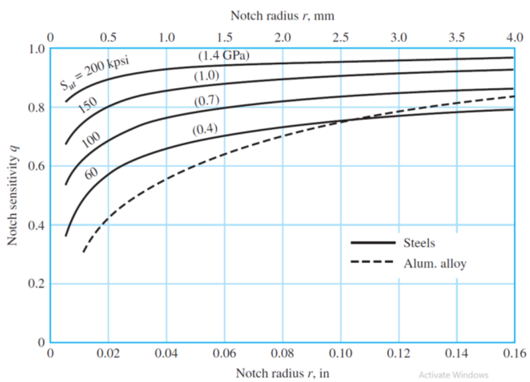
$$q = \frac{K_f - 1}{K_t - 1} \qquad \text{or} \qquad q_{shear} = \frac{K_{fs} - 1}{K_{ts} - 1}$$

$$(\sigma_{rev})_{max} = K_f(\sigma_{rev})_{nom}$$

Notch Sensitivity q

Figure 6–20

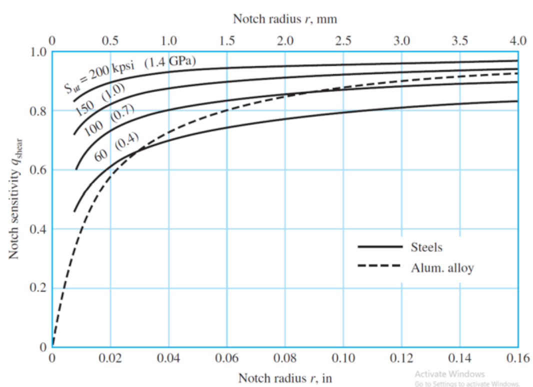
Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of q corresponding to the r = 0.16-in (4-mm) ordinate. *(From George Sines and J. L. Waisman (eds.),* Metal Fatigue, *McGraw-Hill, New York. Copyright © 1969 by The McGraw-Hill Companies, Inc. Reprinted by permission.)*



Notch Sensitivity q_s

Figure 6–21

Notch-sensitivity curves for materials in reversed torsion. For larger notch radii, use the values of q_{shear} corresponding to r = 0.16 in (4 mm).



Notch Sensitivity

Figure 6–20 has as its basis the *Neuber equation*, which is given by

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} \qquad (6\text{–}33)$$

where $\sqrt{a}$ is defined as the *Neuber constant* and is a material constant. Equating Eqs. (6–31) and (6–33) yields the notch sensitivity equation

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} \qquad (6\text{–}34)$$

correlating with Figs. 6–20 and 6–21 as

Bending or axial: $\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$ (6–35a)

Torsion: $\sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$ (6–35b)

Mechanical Properties of Three Non-Steel Metals

(a) Typical Properties of Gray Cast Iron

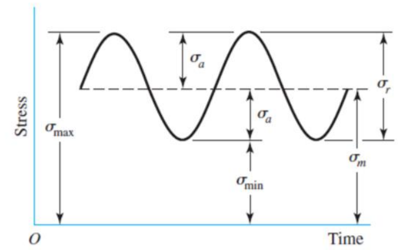
ASTM Number	Tensile Strength S_{ut}, kpsi	Compressive Strength S_{uc}, kpsi	Shear Modulus of Rupture S_{su}, kpsi	Modulus of Elasticity, Mpsi Tension†	Modulus of Elasticity, Mpsi Torsion	Endurance Limit* S_e, kpsi	Brinell Hardness H_B	Fatigue Stress-Concentration Factor K_f
20	22	83	26	9.6–14	3.9–5.6	10	156	1.00
25	26	97	32	11.5–14.8	4.6–6.0	11.5	174	1.05
30	31	109	40	13–16.4	5.2–6.6	14	201	1.10
35	36.5	124	48.5	14.5–17.2	5.8–6.9	16	212	1.15
40	42.5	140	57	16–20	6.4–7.8	18.5	235	1.25
50	52.5	164	73	18.8–22.8	7.2–8.0	21.5	262	1.35
60	62.5	187.5	88.5	20.4–23.5	7.8–8.5	24.5	302	1.50

$$F_m = \frac{F_{max} + F_{min}}{2} \qquad F_a = \left|\frac{F_{max} - F_{min}}{2}\right|$$

σ_{min} = minimum stress

σ_{max} = maximum stress

σ_a = amplitude component



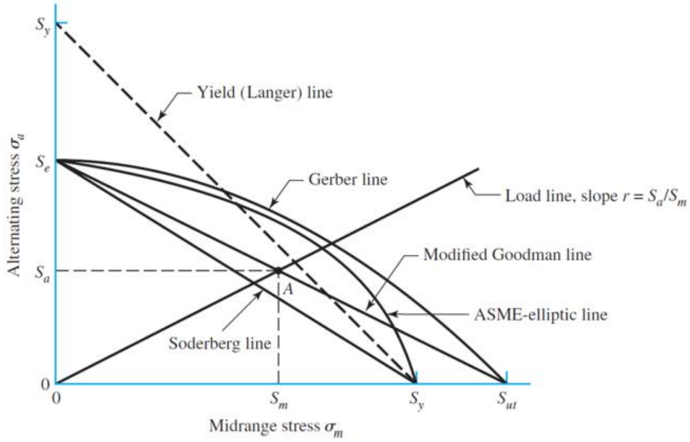
$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_a = \left|\frac{\sigma_{max} - \sigma_{min}}{2}\right|$$

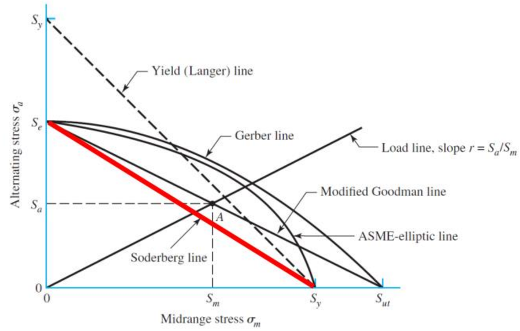
(b) Mechanical Properties of Some Aluminum Alloys

Aluminum Association Number	Temper	Strength: Yield, S_y, MPa (kpsi)	Strength: Tensile, S_u, MPa (kpsi)	Strength: Fatigue, S_f, MPa (kpsi)	Elongation in 2 in, %	Brinell Hardness H_B
Wrought:						
2017	O	70 (10)	179 (26)	90 (13)	22	45
2024	O	76 (11)	186 (27)	90 (13)	22	47
	T3	345 (50)	482 (70)	138 (20)	16	120
3003	H12	117 (17)	131 (19)	55 (8)	20	35
	H16	165 (24)	179 (26)	65 (9.5)	14	47
3004	H34	186 (27)	234 (34)	103 (15)	12	63
	H38	234 (34)	276 (40)	110 (16)	6	77
5052	H32	186 (27)	234 (34)	117 (17)	18	62
	H36	234 (34)	269 (39)	124 (18)	10	74
Cast:						
319.0*	T6	165 (24)	248 (36)	69 (10)	2.0	80
333.0†	T5	172 (25)	234 (34)	83 (12)	1.0	100
	T6	207 (30)	289 (42)	103 (15)	1.5	105
335.0*	T6	172 (25)	241 (35)	62 (9)	3.0	80
	T7	248 (36)	262 (38)	62 (9)	0.5	85

1 UNS No.	2 SAE and/or AISI No.	3 Processing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)	6 Elongation in 2 in, %	7 Reduction in Area, %	8 Brinell Hardness
G10060	1006	HR	300 (43)	170 (24)	30	55	86
		CD	330 (48)	280 (41)	20	45	95
G10100	1010	HR	320 (47)	180 (26)	28	50	95
		CD	370 (53)	300 (44)	20	40	105
G10150	1015	HR	340 (50)	190 (27.5)	28	50	101
		CD	390 (56)	320 (47)	18	40	111
G10180	1018	HR	400 (58)	220 (32)	25	50	116
		CD	440 (64)	370 (54)	15	40	126
G10200	1020	HR	380 (55)	210 (30)	25	50	111
		CD	470 (68)	390 (57)	15	40	131
G10300	1030	HR	470 (68)	260 (37.5)	20	42	137
		CD	520 (76)	440 (64)	12	35	149
G10350	1035	HR	500 (72)	270 (39.5)	18	40	143
		CD	550 (80)	460 (67)	12	35	163
G10400	1040	HR	520 (76)	290 (42)	18	40	149
		CD	590 (85)	490 (71)	12	35	170
G10450	1045	HR	570 (82)	310 (45)	16	40	163
		CD	630 (91)	530 (77)	12	35	179
G10500	1050	HR	620 (90)	340 (49.5)	15	35	179
		CD	690 (100)	580 (84)	10	30	197

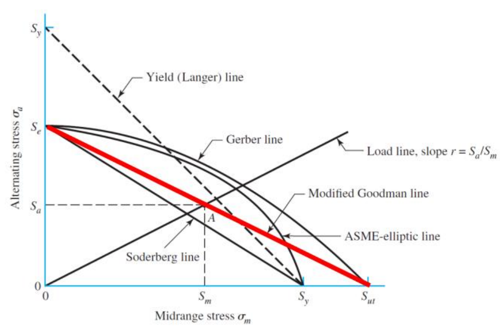


Soderberg Line



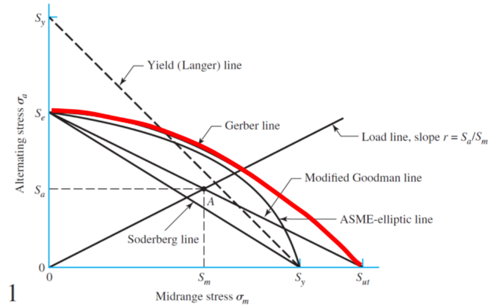
$$\frac{S_a}{S_e} + \frac{S_m}{S_y} = 1 \qquad \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}$$

Modified Goodman line



$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1 \qquad \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

Gerber line

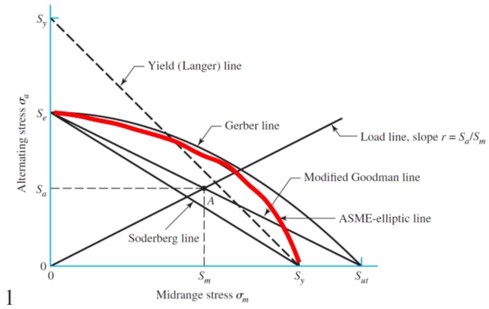


$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\,\right] \qquad \sigma_m > 0$$

ASME-Elliptic line

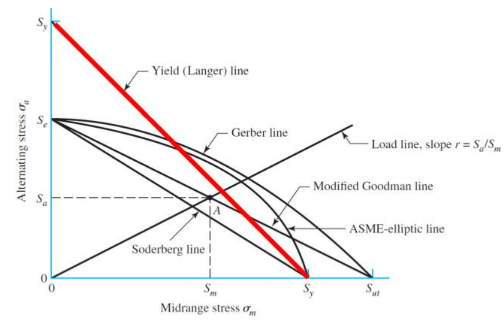


$$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$$

$$\left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1$$

$$n_f = \sqrt{\frac{1}{(\sigma_a/S_e)^2 + (\sigma_m/S_y)^2}}$$

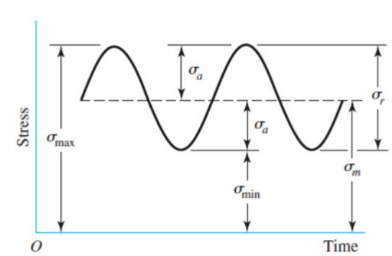
Langer First Cycle Yielding



$$\sigma_a + \sigma_m = \frac{S_y}{n} \qquad S_a + S_m = S_y$$

$$F_m = \frac{F_{max} + F_{min}}{2} \qquad F_a = \left|\frac{F_{max} - F_{min}}{2}\right|$$

σ_{min} = minimum stress
σ_{max} = maximum stress
σ_a = amplitude component



$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_a = \left|\frac{\sigma_{max} - \sigma_{min}}{2}\right|$$

Brittle Materials Smith Dolan

$$\frac{S_a}{S_e} = \frac{1 - S_m/S_{ut}}{1 + S_m/S_{ut}} \qquad (6\text{–}50)$$

$$\frac{n\sigma_a}{S_e} = \frac{1 - n\sigma_m/S_{ut}}{1 + n\sigma_m/S_{ut}} \qquad (6\text{–}51)$$

$$S_a = \frac{rS_{ut} + S_e}{2}\left[-1 + \sqrt{1 + \frac{4rS_{ut}S_e}{(rS_{ut} + S_e)^2}}\right] \qquad (6\text{–}52)$$

6-13 Torsional Fatigue Strength under Fluctuating Stresses

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Joerres uses

$$S_{su} = 0.67S_{ut} \qquad (6\text{–}54)$$

Also, from Chap. 5, $S_{sy} = 0.577S_{yt}$ from distortion-energy theory, and the mean load factor k_c is given by Eq. (6–26), or 0.577.

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases} \qquad (6\text{–}26)$$

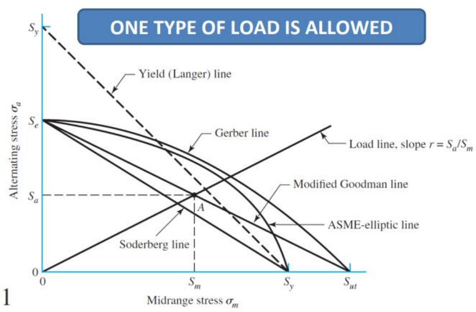
$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right] \qquad \sigma_m > 0$$

Load Factor k_c

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases} \qquad (6\text{–}26)$$

When torsion is combined with other loading, such as bending, set $k_c = 1$, and the combined loading is managed by using the effective von Mises stress as described in Sec. 6-14

Gerber line



$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right] \qquad \sigma_m > 0$$

6-14 Combinations of Loading Modes

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$$\sigma' = (\sigma_x^2 + 3\tau_{xy}^2)^{1/2}.$$

$$\sigma'_a = \left\{\left[(K_f)_{\text{bending}}(\sigma_a)_{\text{bending}} + (K_f)_{\text{axial}}\frac{(\sigma_a)_{\text{axial}}}{0.85}\right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_a)_{\text{torsion}}\right]^2\right\}^{1/2} \qquad (6\text{–}55)$$

$$\sigma'_m = \left\{\left[(K_f)_{\text{bending}}(\sigma_m)_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_m)_{\text{axial}}\right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_m)_{\text{torsion}}\right]^2\right\}^{1/2} \qquad (6\text{–}56)$$

Figure 8–1

Terminology of screw threads. Sharp vee threads shown for clarity; the crests and roots are actually flattened or rounded during the forming operation.

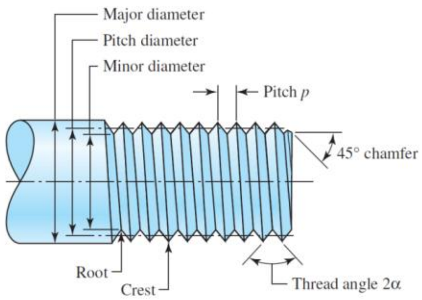


Figure 8–3

(a) Square thread; (b) Acme thread.

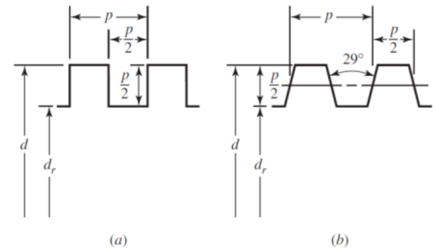


Table 8–3

Preferred Pitches for Acme Threads

d, in	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{2}$	3
p, in	$\frac{1}{16}$	$\frac{1}{14}$	$\frac{1}{12}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$

Table 8–1

Diameters and Areas of Coarse-Pitch and Fine-Pitch Metric Threads.*

Nominal Major Diameter d mm	Coarse-Pitch Series: Pitch p mm	Tensile-Stress Area A_t mm²	Minor-Diameter Area A_r mm²	Fine-Pitch Series: Pitch p mm	Tensile-Stress Area A_t mm²	Minor-Diameter Area A_r mm²
1.6	0.35	1.27	1.07			
2	0.40	2.07	1.79			
2.5	0.45	3.39	2.98			
3	0.5	5.03	4.47			
3.5	0.6	6.78	6.00			
4	0.7	8.78	7.75			
5	0.8	14.2	12.7			
6	1	20.1	17.9			
8	1.25	36.6	32.8	1	39.2	36.0
10	1.5	58.0	52.3	1.25	61.2	56.3
12	1.75	84.3	76.3	1.25	92.1	86.0
14	2	115	104	1.5	125	116
16	2	157	144	1.5	167	157
20	2.5	245	225	1.5	272	259
24	3	353	324	2	384	365
30	3.5	561	519	2	621	596
36	4	817	759	2	915	884
42	4.5	1120	1050	2	1260	1230
48	5	1470	1380	2	1670	1630
56	5.5	2030	1910	2	2300	2250
64	6	2680	2520	2	3030	2980
72	6	3460	3280	2	3860	3800
80	6	4340	4140	1.5	4850	4800
90	6	5590	5360	2	6100	6020
100	6	6990	6740	2	7560	7470
110				2	9180	9080

Table 8–2

Diameters and Area of Unified Screw Threads UNC and UNF*

Size Designation	Nominal Major Diameter in	Coarse Series—UNC: Threads per Inch N	Tensile-Stress Area A_t in²	Minor-Diameter Area A_r in²	Fine Series—UNF: Threads per Inch N	Tensile-Stress Area A_t in²	Minor-Diameter Area A_r in²
0	0.0600				80	0.001 80	0.001 51
1	0.0730	64	0.002 63	0.002 18	72	0.002 78	0.002 37
2	0.0860	56	0.003 70	0.003 10	64	0.003 94	0.003 39
3	0.0990	48	0.004 87	0.004 06	56	0.005 23	0.004 51
4	0.1120	40	0.006 04	0.004 96	48	0.006 61	0.005 66
5	0.1250	40	0.007 96	0.006 72	44	0.008 80	0.007 16
6	0.1380	32	0.009 09	0.007 45	40	0.010 15	0.008 74
8	0.1640	32	0.014 0	0.011 96	36	0.014 74	0.012 85
10	0.1900	24	0.017 5	0.014 50	32	0.020 0	0.017 5
12	0.2160	24	0.024 2	0.020 6	28	0.025 8	0.022 6
$\frac{1}{4}$	0.2500	20	0.031 8	0.026 9	28	0.036 4	0.032 6
$\frac{5}{16}$	0.3125	18	0.052 4	0.045 4	24	0.058 0	0.052 4
$\frac{3}{8}$	0.3750	16	0.077 5	0.067 8	24	0.087 8	0.080 9
$\frac{7}{16}$	0.4375	14	0.106 3	0.093 3	20	0.118 7	0.109 0
$\frac{1}{2}$	0.5000	13	0.141 9	0.125 7	20	0.159 9	0.148 6
$\frac{9}{16}$	0.5625	12	0.182	0.162	18	0.203	0.189
$\frac{5}{8}$	0.6250	11	0.226	0.202	18	0.256	0.240
$\frac{3}{4}$	0.7500	10	0.334	0.302	16	0.373	0.351
$\frac{7}{8}$	0.8750	9	0.462	0.419	14	0.509	0.480
1	1.0000	8	0.606	0.551	12	0.663	0.625
$1\frac{1}{4}$	1.2500	7	0.969	0.890	12	1.073	1.024
$1\frac{1}{2}$	1.5000	6	1.405	1.294	12	1.581	1.521

Figure 8–5

Portion of a power screw.

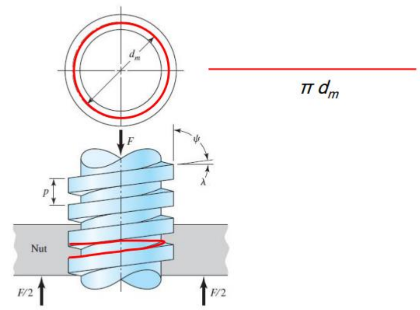


Figure 8–6

Force diagrams: (a) lifting the load; (b) lowering the load.

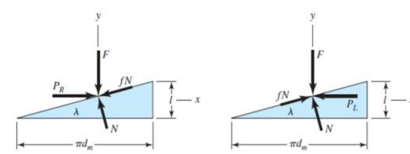
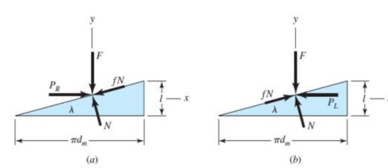


Figure 8–6

Force diagrams: (a) lifting the load; (b) lowering the load.



for raising the load, we have

$$\sum F_x = P_R - N\sin\lambda - fN\cos\lambda = 0$$

$$\sum F_y = -F - fN\sin\lambda + N\cos\lambda = 0$$

In a similar manner, for lowering the load, we have

$$\sum F_x = -P_L - N\sin\lambda + fN\cos\lambda = 0$$

$$\sum F_y = -F + fN\sin\lambda + N\cos\lambda = 0$$

Next, divide the numerator and the denominator of these equations by cos λ and use the relation $\tan\lambda = l/\pi d_m$ (Fig. 8–6). We then have, respectively,

$$P_R = \frac{F(\sin\lambda + f\cos\lambda)}{\cos\lambda - f\sin\lambda} \quad\Rightarrow\quad P_R = \frac{F[(l/\pi d_m) + f]}{1 - (fl/\pi d_m)}$$

$$P_L = \frac{F(f\cos\lambda - \sin\lambda)}{\cos\lambda + f\sin\lambda} \quad\Rightarrow\quad P_L = \frac{F[f - (l/\pi d_m)]}{1 + (fl/\pi d_m)}$$

- mean radius $d_m/2$,

$$P_R = \frac{F[(l/\pi d_m) + f]}{1 - (fl/\pi d_m)}$$

$$P_L = \frac{F[f - (l/\pi d_m)]}{1 + (fl/\pi d_m)}$$

$$T_R = \frac{Fd_m}{2}\left(\frac{l + \pi f d_m}{\pi d_m - fl}\right)$$

$$T_L = \frac{Fd_m}{2}\left(\frac{\pi f d_m - l}{\pi d_m + fl}\right)$$

$$T_L = \frac{Fd_m}{2}\left(\frac{\pi f d_m - l}{\pi d_m + fl}\right)$$

This is the torque required to overcome a part of the friction in lowering the load. It may turn out, in specific instances where the lead is large or the friction is low, that the load will lower itself by causing the screw to spin without any external effort. In such cases, the torque T_L from Eq. (8–2) will be negative or zero. When a positive torque is obtained from this equation, the screw is said to be *self-locking*. Thus the condition for self-locking is

$$\pi f d_m > l$$

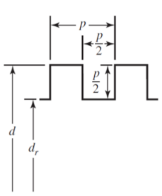
Overhauling <> Self Locking

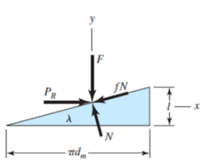
Power Screw Efficiency – Raising the load

$$T_R = \frac{Fd_m}{2}\left(\frac{l + \pi f d_m}{\pi d_m - fl}\right)$$ With Friction

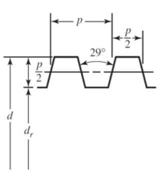
$$T_0 = \frac{Fl}{2\pi}$$ Without Friction $f = 0$

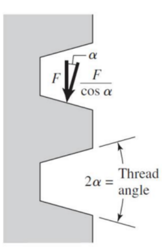
$$e = \frac{T_0}{T_R} = \frac{Fl}{2\pi T_R}$$





$$T_R = \frac{Fd_m}{2}\left(\frac{l + \pi f d_m}{\pi d_m - fl}\right)$$





$$T_R = \frac{Fd_m}{2}\left(\frac{l + \pi f d_m \sec\alpha}{\pi d_m - fl\sec\alpha}\right)$$

When the screw is loaded axially, a thrust or collar bearing must be employed between the rotating and stationary members in order to carry the axial component. The right figure shows a typical thrust collar in which the load is assumed to be concentrated at the mean collar diameter *dc*. If *fc* is the coefficient of collar friction, the torque required is

$$T_c = \frac{F f_c d_c}{2}$$

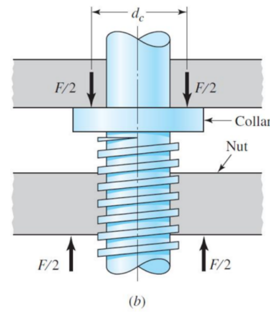


Table 8–5

Coefficients of Friction f for Threaded Pairs

Source: H. A. Rothbart and T. H. Brown, Jr., *Mechanical Design Handbook*, 2nd ed., McGraw-Hill, New York, 2006.

Screw Material	Nut Material: Steel	Bronze	Brass	Cast Iron
Steel, dry	0.15–0.25	0.15–0.23	0.15–0.19	0.15–0.25
Steel, machine oil	0.11–0.17	0.10–0.16	0.10–0.15	0.11–0.17
Bronze	0.08–0.12	0.04–0.06	—	0.06–0.09

Table 8–6

Thrust-Collar Friction Coefficients

Source: H. A. Rothbart and T. H. Brown, Jr., *Mechanical Design Handbook*, 2nd ed., McGraw-Hill, New York, 2006.

Combination	Running	Starting
Soft steel on cast iron	0.12	0.17
Hard steel on cast iron	0.09	0.15
Soft steel on bronze	0.08	0.10
Hard steel on bronze	0.06	0.08

The maximum nominal shear stress τ in torsion of the screw body can be expressed as

$$\tau = \frac{16T}{\pi d_r^3} \tag{8–7}$$

The axial stress σ in the body of the screw due to load F is

$$\sigma = \frac{F}{A} = \frac{4F}{\pi d_r^2} \tag{8–8}$$

in the absence of column action. For a short column the J. B. Johnson buckling formula is given by Eq. (4–43), which is

$$\left(\frac{F}{A}\right)_{crit} = S_y - \left(\frac{S_y}{2\pi}\frac{l}{k}\right)^2 \frac{1}{CE} \tag{8–9}$$

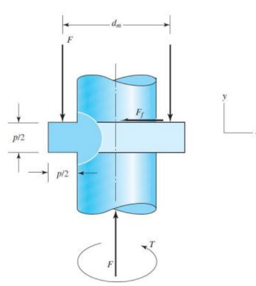
Nominal thread stresses in power screws can be related to thread parameters as follows. The bearing stress in Fig. 8–8, σ_B, is

$$\sigma_B = -\frac{F}{\pi d_m n_t p/2} = -\frac{2F}{\pi d_m n_t p} \tag{8–10}$$

where n_t is the number of engaged threads.

Figure 8–8

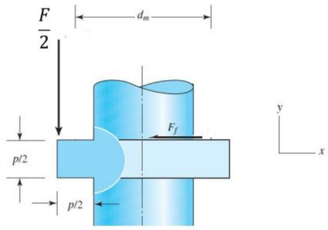
Geometry of square thread useful in finding bending and transverse shear stresses at the thread root.



The bending stress at the root of the thread σ_b is found from

$$Z = \frac{I}{c} = \frac{(\pi d_r n_t)(p/2)^2}{6} = \frac{\pi}{24} d_r n_t p^2 \qquad M = \frac{Fp}{4} \qquad M = \frac{F}{2}\frac{p}{2}$$

$$\sigma_b = \frac{M}{Z} = \frac{Fp}{4}\frac{24}{\pi d_r n_t p^2} = \frac{6F}{\pi d_r n_t p} \tag{8–11}$$



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The transverse shear stress τ at the center of the root of the thread due to load F is

$$\tau = \frac{3V}{2A} = \frac{3}{2}\frac{F}{\pi d_r n_t p/2} = \frac{3F}{\pi d_r n_t p} \tag{8–12}$$

and at the top of the root it is zero.