③

$y = 1000\, e^{-0.0223t}$

$y(24) = 1000\, e^{-0.0223(24)}$

$= \boxed{585.5 \text{ kg}}$

30 At the end of 3 hours there are 10 000 bacteria

" " 5 hours there are 40 000.

How many bacteria were initially?

$y = y_0 e^{kt}$

$10\,000 = y_0 e^{3k}$ $\longrightarrow$ $4 = e^{2k}$

$40\,000 = y_0 e^{5k}$ $\quad \ln 4 = 2k \longrightarrow k = \frac{\ln 4}{2} = \ln 2$

$10\,000 = y_0 e^{3 \cdot \ln 2}$

$y_0 = \frac{10\,000}{e^{\ln 8}} = \frac{10\,000}{8} = 1250$

$\boxed{y_0 = 1250}$

26 $\frac{dy}{dt} = ky$

1000 kg $\longrightarrow$ 800 kg "1st 10 hours"

How much raw sugar will ~~be~~ remain after another 14 hours

$$\frac{dy}{dt} = ky$$

$$\frac{dy}{y} = k\,dt$$

$$\ln|y| = \cancel{kt}\ kt + c$$

$$y = e^{kt} = \cancel{e^{k} \cdot e^{t}}\ e^{kt+c} = e^{kt} \cdot e^{c}$$

$$\boxed{y = e^{c} \cdot e^{kt}}$$

~~$y = me^{t}$~~ ~~$e^{k} = m$~~

~~$y(0) = 1000 \rightarrow 1000 = me^{0} = m$~~

So ~~$y = 1000\,e^{t}$~~

$$y(0) = 1000$$

$$y(14) = 800$$

$$1000 = e^{c} \cdot 1$$

So $y = 1000\,e^{kt}$

$$y(14) = 1000 \cdot e^{10k} = 800$$

$$e^{10k} = 0.8$$

$$10k = \ln 0.8$$

$$k = \frac{\ln 0.8}{10} = -0.0223$$

7.4 Exponential Change

25 + 26 + 30 + 31 + 36

25 In some Chemecal reaction

$$\frac{dy}{dt} = -0.6y$$

t is measured in hours

there are 100 gm of δ-glucono lacton preset when $t=0$

How many grams will be left after 1st hour?

$$\frac{dy}{y} = -0.6\, dt$$

$$\ln|y| = -0.6t + c$$

$$y = e^{-0.6t+c} = e^{-0.6t} \cdot e^{c}$$

Let $e^{c} = k$

$$y = k\, e^{-0.6t}$$

$y(0) = 100$

$$y = 100\, e^{-0.6t}$$

$$y(1) = 100\, e^{-0.6(1)} = 100\, e^{-0.6} = \boxed{54.88\text{gm}}$$

6

36 The half life of Polonium is 139 days after 95% of your sample will not be useful to your the radio active nuclei preset. on the day the sampk arrives has disintegrated. For about how may days after the sample arrives will you be able to use the polonium.

$$y = y_0 e^{kt}$$

$$\frac{1}{2} y_0 = y_0 e^{k(139)}$$

$$\frac{1}{2} = e^{139k}$$

$$\ln \frac{1}{2} = 139k \rightarrow k = \frac{-\ln 2}{139} = \boxed{-4.986 \times 10^{-3}}$$

$$y = 0.05 y_0 = y_0 e^{-4.986 \times 10^{-3} t}$$

$$t = \frac{\ln 0.05}{-4.986 \times 10^{-3}} \simeq \boxed{600 \text{ days}}$$

5

(b) $\cdot\ 10\,000\ e^{(\ln 0.75 t)} = 1$

$e^{\ln 0.75\ t} = 0.0001$

$\ln 0.75\ t = \ln 0.0001 = -9.2103$

$t = 32.02 \simeq \boxed{32\ \text{years}}$

4

31 25% instead of 20% Example 3

a. How long will it take to reduce the number of cases to 1000?

b. How long will it take to eradicate the disease. That is, reduce the number of cases to less than 1?

there are 10 000 cases today reduced by 25%. so, now we have 7500 cases "after 1 year"

$$y = 10\,000\, e^{kt}$$

$$7500 = 10\,000\, e^{k(1)}$$

$$0.75 = e^{k}$$

$$\boxed{k = \ln 0.75}$$

$$\text{so } \boxed{y = 10000\, e^{\ln 0.75\, t}}$$

(a)

$$1000 = 10\,000\, e^{(\ln 0.75)t}$$

$$0.1 = e^{(\ln 0.75)t}$$

$$\ln 0.1 = (\ln 0.75)t$$

$$t = \frac{\ln 0.1}{\ln 0.75} = 8.004 \simeq \boxed{8 \text{ years}}$$