FSK &QPSK

Binary Frequency Shift Keying (BFSK): Signal Representation

In binary FSK, the frequency of the carrier signal is varied to represent the binary digits and 0 by two distinct frequencies. The amplitude and frequency remain constant during each bit interval.

Signal Representation (coherent FSK)

Send: $s_1(t) = A cos(2\pi(f_c + \Delta f))t)$ if the information bit is "1";

Send: $s_2(t) = A cos(2\pi(f_c - \Delta f))t)$ if the information bit is "0";

Δf is an offset frequency (from the unmodulated carrier f_c) chosen so that $s_1(t)$ and $s_2(t)$ are orthogonal, i.e.,

$$\int_0^{\tau} s_1(t) s_2(t) dt = 0$$

$$\frac{sin(2\pi(2f_c)\tau}{2f_c} + \frac{sin(2\pi(2\Delta f)\tau}{2\Delta f} = 0$$

Binary Frequency Shift Keying (BFSK): Signal Representation

Orthogonality condition

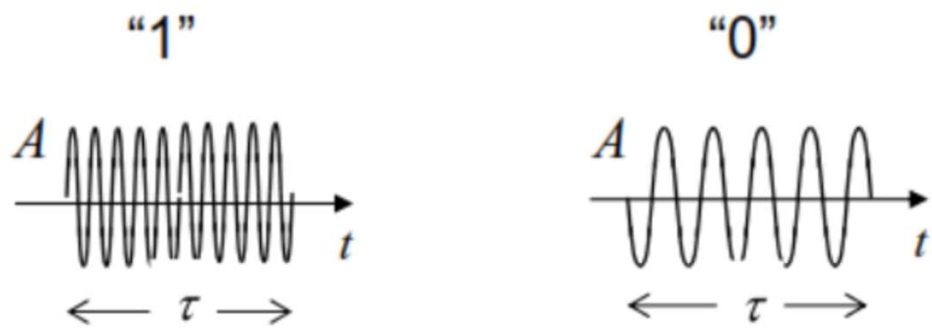
$$\frac{sin(2\pi(2f_c)\tau}{2f_c} + \frac{sin(2\pi(2\Delta f)\tau}{2\Delta f} = 0$$

$$2f_c = \frac{n}{2\tau} = \frac{nR_b}{2}, n = 1, 2, ... \quad \boldsymbol{f_c = \frac{nR_b}{4} = kR_b}$$

$$2\Delta f = \frac{mR_b}{2}, m = 1, 2, ... \quad \boldsymbol{\Delta f = \frac{mR_b}{4}}$$

Note that $sin(x) = 0$ when $x = n\pi$

The minimum frequency separation $\boldsymbol{2\Delta f = R_b/2.}$



$s_1(t) = A\cos(2\pi(f_c + \Delta f)t)$ $\quad$ $s_2(t) = A\cos(2\pi(f_c - \Delta f)t)$

$0 \le t \le \tau$ $\quad$ $0 \le t \le \tau$

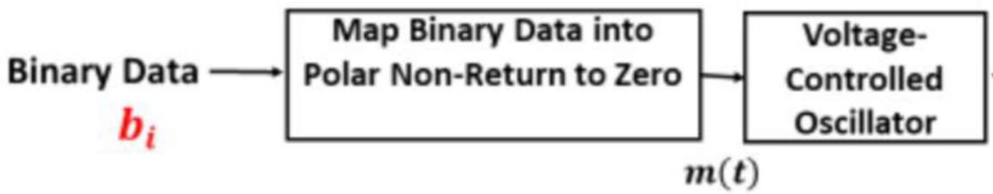
(τ is an integer number of $1/(f_c \pm \Delta f)$)

τ: is the time allocated to transmit the binary digit.

T_c=1/f_c is the carrier period

$R_b = \frac{1}{\tau}$: $Data\ rate$ bits/sec

Binary FSK : Generation using the Single Oscillator Method

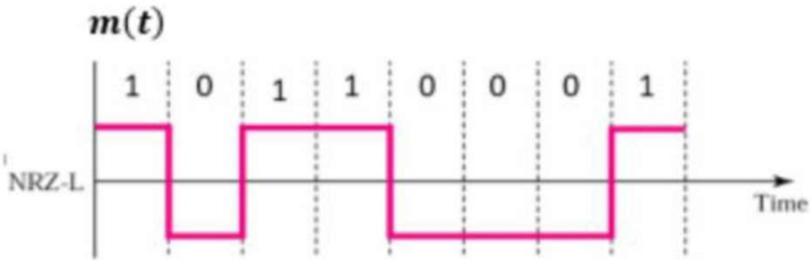


$f_1(t) = f_c + \Delta f; for\ digit\ 1$

$f_2(t) = f_c - \Delta f; for\ digit\ 0$

m(t): Polar non-return to zero

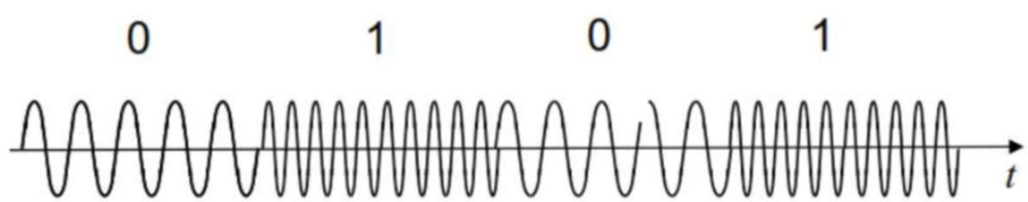
$$f_i(t) = f_c + k_f m(t); for\ VCO$$



Polar Non-return to Zero

$s_1(t) = Acos(2\pi(f_c + \Delta f))t)$

$s_2(t) = Acos(2\pi(f_c - \Delta f))t)$

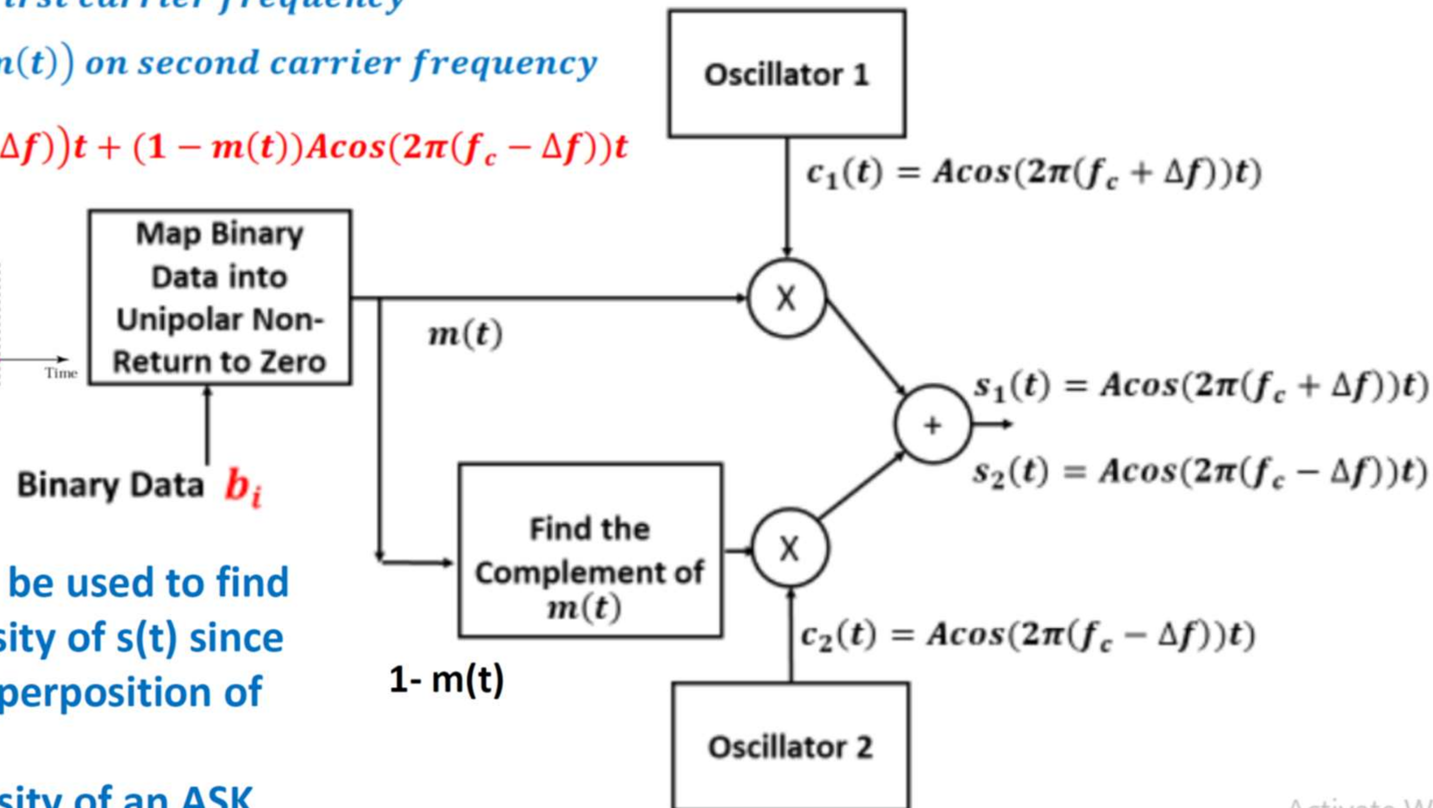


Binary FSK : Generation using the Two-oscillator Method

$$s_{BFSK}(t) = ASK\ of\ m(t)\ on\ first\ carrier\ frequency$$
$$+ ASK\ of\ \left(1 - m(t)\right)\ on\ second\ carrier\ frequency$$

$$s_{BFSK}(t) = m(t)A\cos\left(2\pi(f_c + \Delta f)\right)t + (1 - m(t))A\cos(2\pi(f_c - \Delta f))t$$

Unipolar Non-Return to Zero Signal

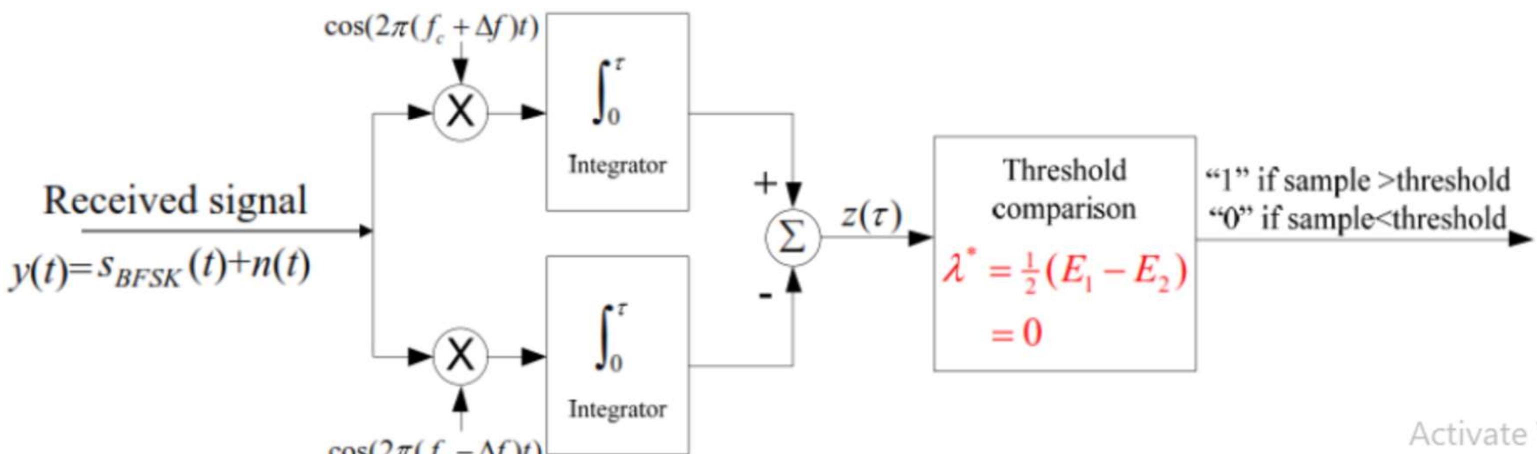


- This representation will be used to find the power spectral density of s(t) since it is envisaged as the superposition of two ASK signals.
- The power spectral density of an ASK signal was derived in a previous video titled: Binary ASK

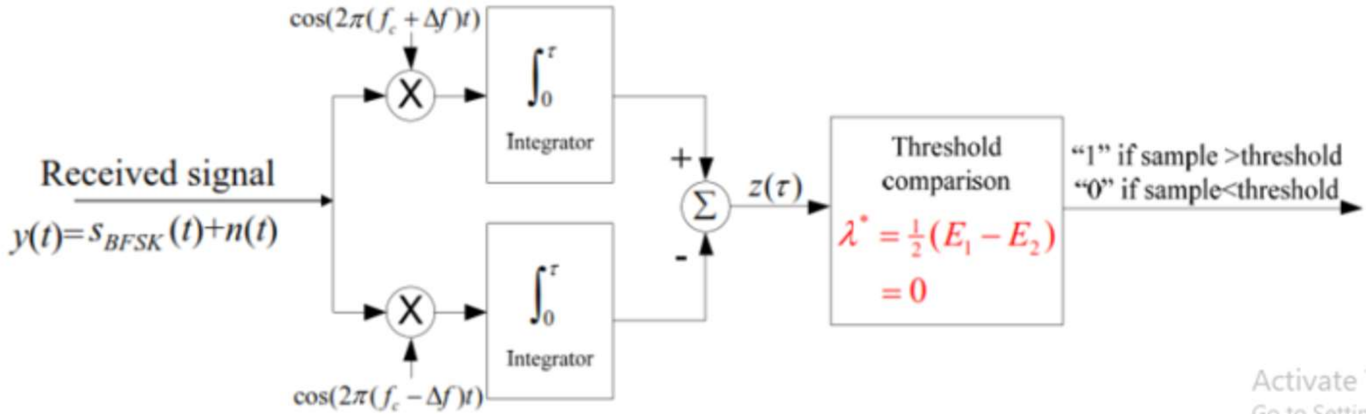
FSK: modeled as a sum of two ASK signals

Binary FSK : Coherent Demodulation

The optimum coherent receiver consists of two correlators. The operation of the receiver makes use of the orthogonality condition imposed on the signals $s_1(t)$ and $s_2(t)$. In the absence of noise, if $s_1(t)$ is received, then the ouput of the upper correlator will have a value greater than zero, while the output of the lower correlator is zero. The converse is true when $s_2(t)$ is received. In the presence of noise, the system decides 1 when $z(\tau) > 0$. That is, when the output of the upper correlator is greater than the output of the lower one. Otherwise, it decides 0.



Binary FSK : Probability of Error



Probability of Error

Energy of $s_i(t)$: $E_1 = E_2 = \frac{1}{2}A^2\tau$

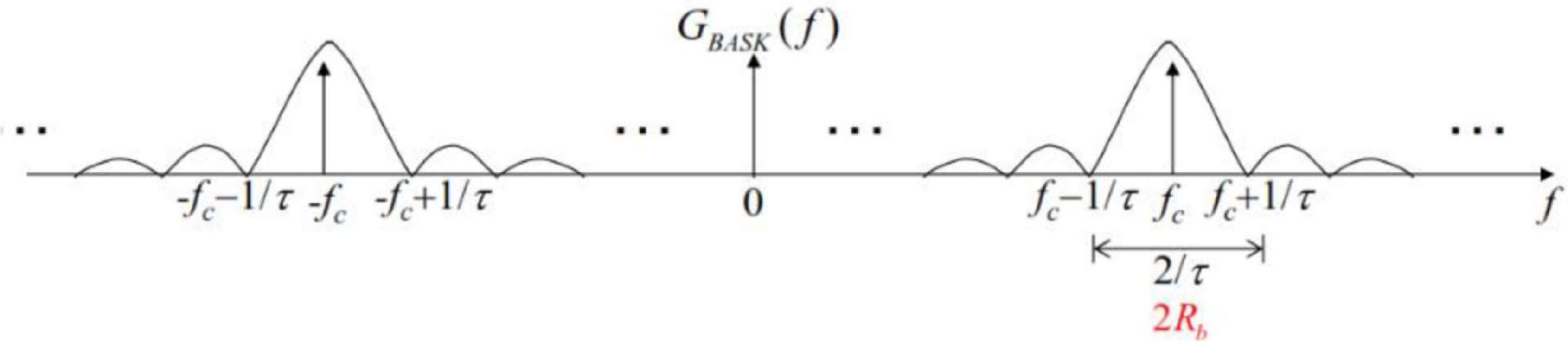
Average Energy per bit: $E_b = \frac{1}{2}(E_1 + E_2) = \frac{1}{2}A^2\tau$

When the signals are orthogonal, i.e., when $\int_0^\tau s_1(t)s_2(t)dt = 0$, the probability of error is given by

$$P_b^* = Q\left(\sqrt{\frac{A^2\tau}{2N_0}}\right) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Binary FSK : Power Spectral Density

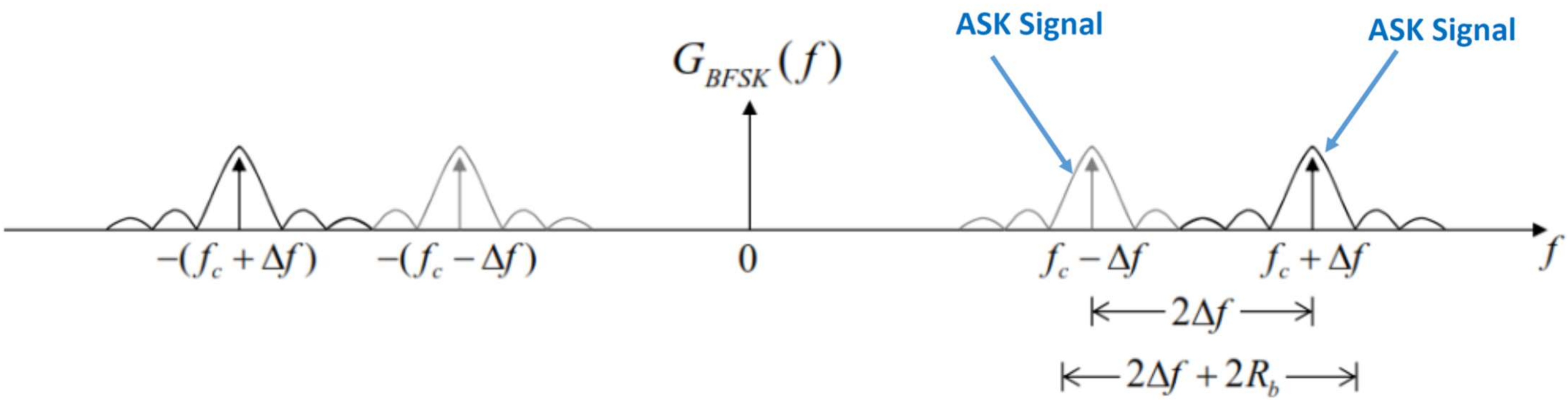
Since the FSK signal is the superposition of two ASK signals on two orthogonal frequencies, the spectrum is also the superposition of that of the ASK signals. We recall that the spectrum of the ASK signal is as shown below



$$s_{BFSK}(t) = ASK\ of\ m(t)\ on\ first\ carrier\ frequency$$
$$+ ASK\ of\ \big(1 - m(t)\big)\ on\ second\ carrier\ frequency$$

$$s_{BFSK}(t) = m(t)A\cos\big(2\pi(f_c + \Delta f)\big)t + (1 - m(t))A\cos(2\pi(f_c - \Delta f))t$$

Binary FSK : Power Spectral Density and Bandwidth

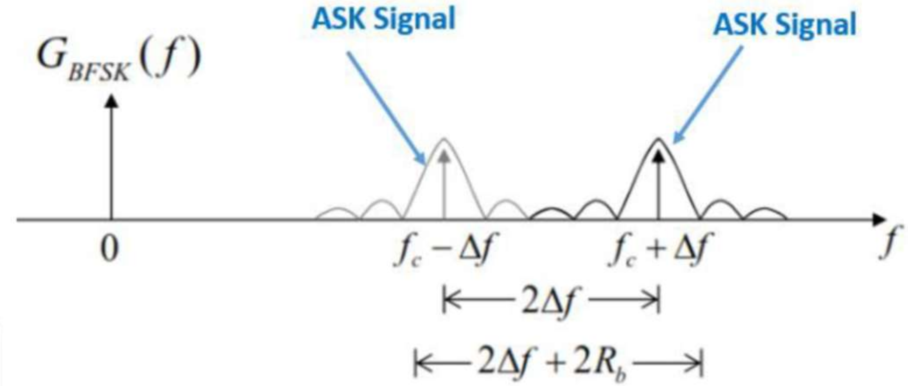


The required channel bandwidth for 90% in-band power

$$B_{h\_90\%}=2\Delta f+2R_b$$

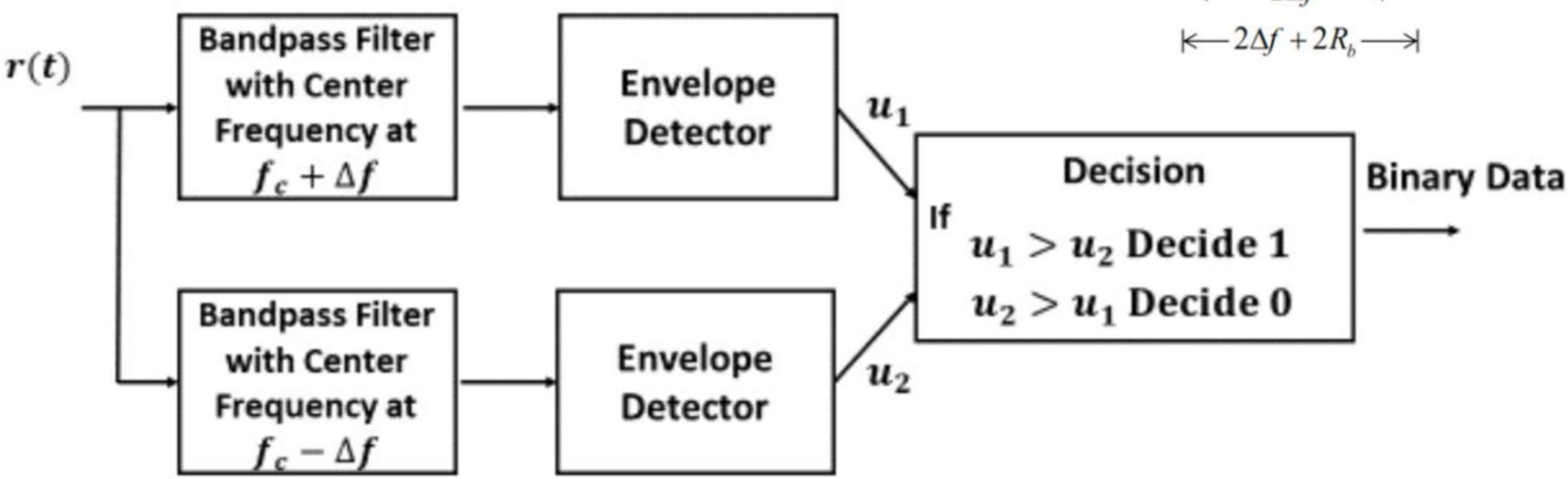
$$\boldsymbol{B.W=(f_1-f_2)+2R_b=\frac{R_b}{2}+2R_b}$$

Binary FSK : Non-coherent Demodulation



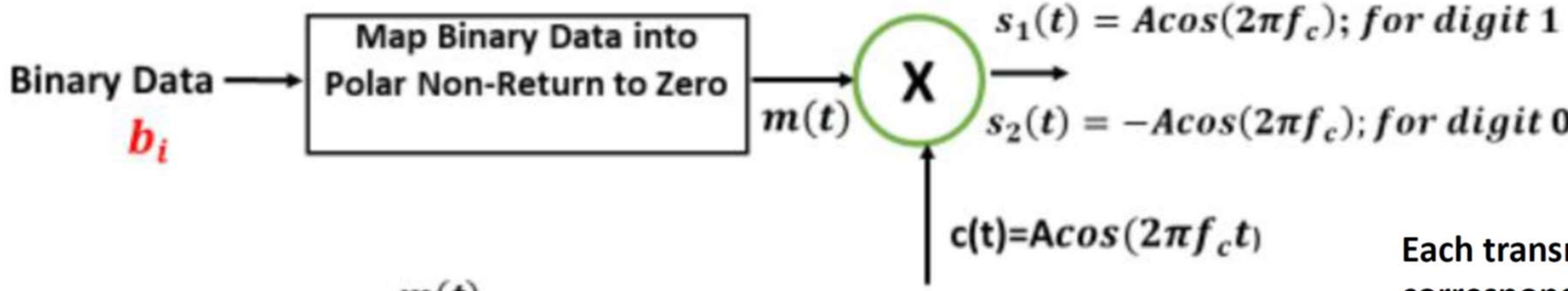
$$\mathbf{r(t)} = A\cos(2\pi(f_c + \Delta f)t) + n(t)$$

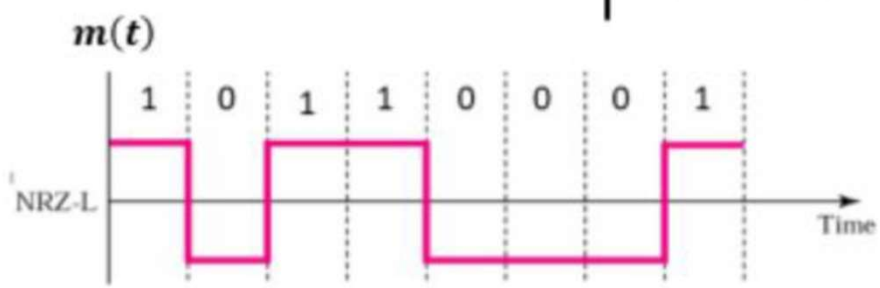
$$\mathbf{r(t)} = A\cos(2\pi(f_c - \Delta f)t) + n(t)$$



Quadri-phase Shift Keying (QPSK)

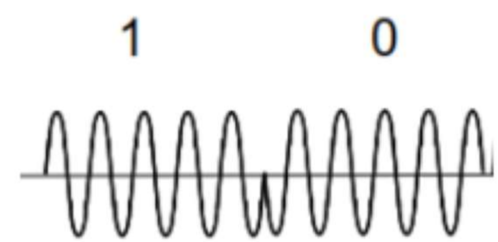
Review: Binary Phase Shift Keying Modulation and Demodulation



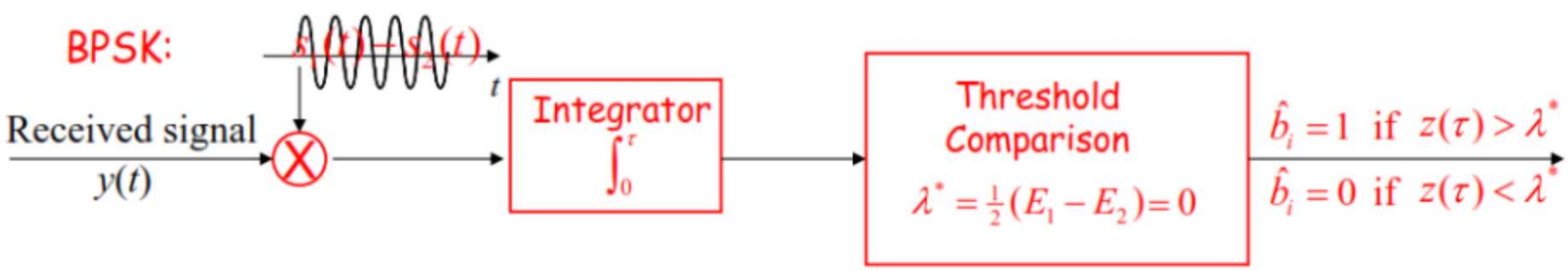


Polar Non-return to Zero

Each transmitted signal corresponds to one binary digit



$$s_1(t) - s_2(t) = 2Acos(2\pi f_c)t$$



Quadri-phase Shift Keying (QPSK): Signal Representation

"1 1" $s_1(t) = A\cos(2\pi f_c t - \pi/4) = +\frac{A}{\sqrt{2}}\cos(2\pi f_c t) + \frac{A}{\sqrt{2}}\sin(2\pi f_c t)$

"1 0" $s_2(t) = A\cos(2\pi f_c t + \pi/4) = +\frac{A}{\sqrt{2}}\cos(2\pi f_c t) - \frac{A}{\sqrt{2}}\sin(2\pi f_c t)$

"0 0" $s_3(t) = A\cos(2\pi f_c t + 3\pi/4) = -\frac{A}{\sqrt{2}}\cos(2\pi f_c t) - \frac{A}{\sqrt{2}}\sin(2\pi f_c t)$

"0 1" $s_4(t) = A\cos(2\pi f_c t + 5\pi/4) = -\frac{A}{\sqrt{2}}\cos(2\pi f_c t) + \frac{A}{\sqrt{2}}\sin(2\pi f_c t)$

$$s_{QPSK}(t) = d_I \frac{A}{\sqrt{2}}\cos(2\pi f_c t) + d_Q \frac{A}{\sqrt{2}}\sin(2\pi f_c t)$$

Odd bits: $d_I = \begin{cases} 1 & \text{if } b_{2i-1} = 1 \\ -1 & \text{if } b_{2i-1} = 0 \end{cases}$

Even bits: $d_Q = \begin{cases} 1 & \text{if } b_{2i} = 1 \\ -1 & \text{if } b_{2i} = 0 \end{cases}$

$$s_{QPSK}(t) = A_k cos(2\pi f_c t) + B_k sin(2\pi f_c t)$$

$$s_{QPSK}(t) = Binary\ PSK\ on\ cos(2\pi f_c t) + Binary\ PSK\ on\ sin(2\pi f_c t)$$

- **In this type of modulation two binary digits are grouped together to form one message that phase modulates the carrier $Acos(2\pi f_c t)$.**
- **The transmitted signal assumes one of four possible phases (+ 45, -45, +135, -135) $Acos(2\pi f_c t + \theta_i)$**
- **A QPSK signal can be decomposed into a sum of two PSK signals; an in-phase component and a quadrature component. The serial to parallel converter splits the incoming data sequence into two sequences that consist of the odd and even bits of the main sequence. The odd bit stream sequence modulates the in-phase carrier, while the even bit stream sequence modulates the quadrature carrier.**

Quadri-phase Shift Keying (QPSK): Signal Representation

$$s_{QPSK}(t) = d_I \frac{A}{\sqrt{2}} \cos(2\pi f_c t) + d_Q \frac{A}{\sqrt{2}} \sin(2\pi f_c t)$$

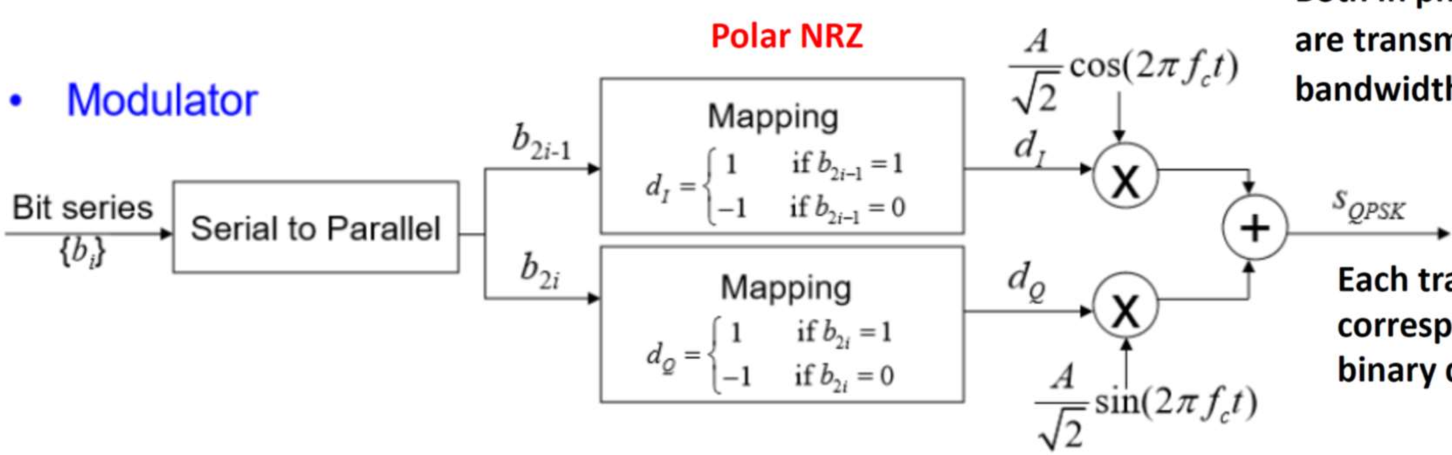
$$d_I = \begin{cases} 1 & \text{if } b_{2i-1} = 1 \\ -1 & \text{if } b_{2i-1} = 0 \end{cases} \qquad d_Q = \begin{cases} 1 & \text{if } b_{2i} = 1 \\ -1 & \text{if } b_{2i} = 0 \end{cases}$$

$$s_{QPSK}(t) = A_k cos(2\pi f_c t) + B_k sin(2\pi f_c t)$$

$$s_{QPSK}(t) = Binary\ PSK\ on\ cos(2\pi f_c t) + Binary\ PSK\ on\ sin(2\pi f_c t)$$

- **The serial to parallel converter splits the incoming data sequence into two sequences that consist of the odd (A_k) and even bits (B_k) of the main sequence. The odd bit stream sequence modulates the in-phase carrier, while the even bit stream sequence modulates the quadrature carrier.**
- **Both in phase and quadrature BPSK are transmitted over the same bandwidth.**

- Modulator



Each transmitted signal corresponds to two binary digits

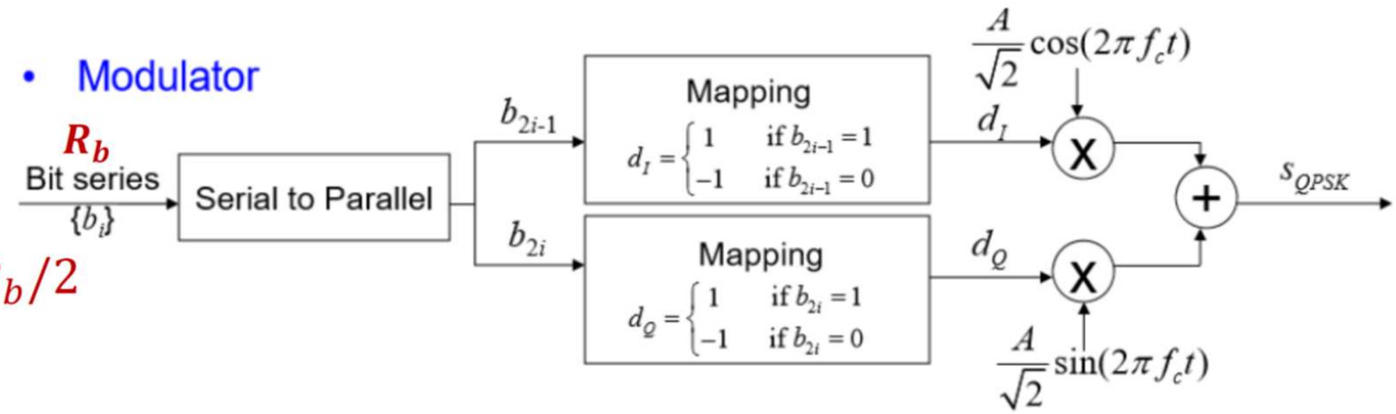
Quadri-phase Shift Keying (QPSK): Modulation

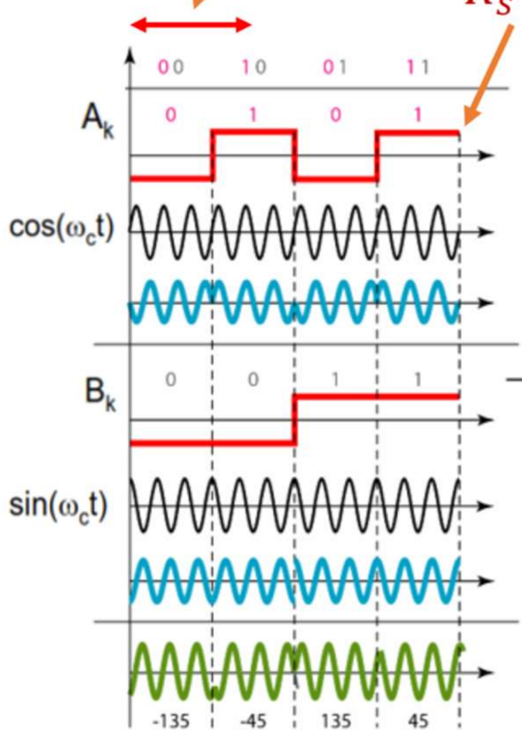
One message (or symbol) consists of two bits

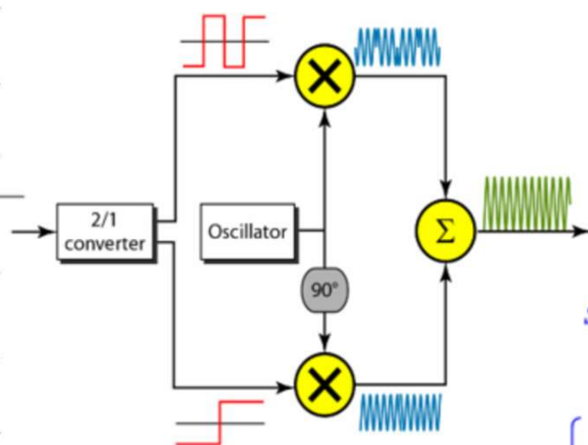
$T_s = 2T_b$

$R_s = R_b/2$

- Modulator







Message duration $T_s = 2T_b$

Message Rate $R_s = R_b/2$

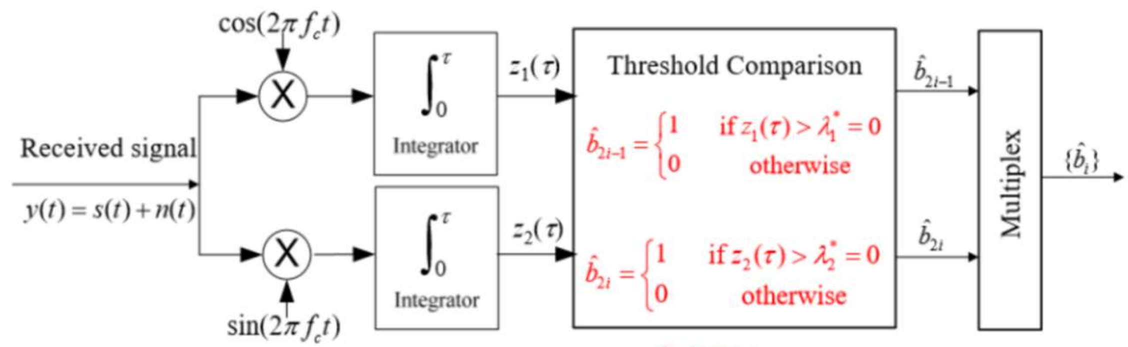
$$s_{QPSK}(t) = d_I \frac{A}{\sqrt{2}}\cos(2\pi f_c t) + d_Q \frac{A}{\sqrt{2}}\sin(2\pi f_c t)$$

$$d_I = \begin{cases} 1 & \text{if } b_{2i-1} = 1 \\ -1 & \text{if } b_{2i-1} = 0 \end{cases} \qquad d_Q = \begin{cases} 1 & \text{if } b_{2i} = 1 \\ -1 & \text{if } b_{2i} = 0 \end{cases}$$

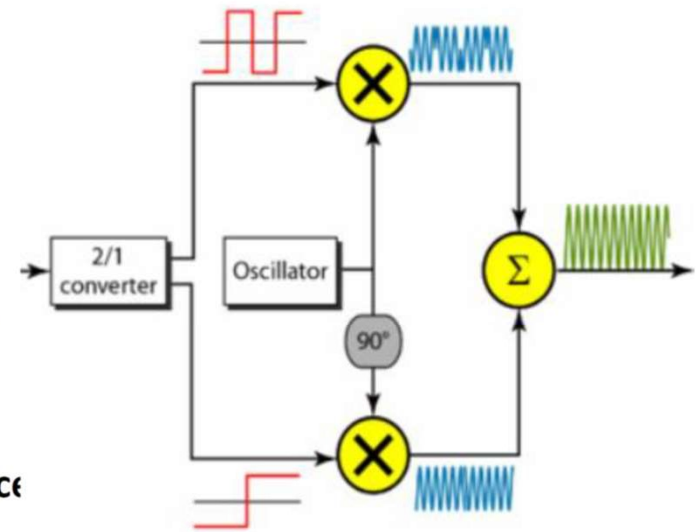
Quadri-phase Shift Keying (QPSK): Demodulation

$$z_1(\tau) = \int_0^{\tau} [s_{QPSK}(t) + n(t)]cos(2\pi f_c t)dt$$

- $z_2(\tau) = \int_0^{\tau} [s_{QPSK}(t) + n(t)]sin(2\pi f_c t)dt$



- $z_1(\tau) = \int_0^{\tau} [A_k cos(2\pi f_c t) + B_k sin(2\pi f_c t) + n(t)]cos(2\pi f_c t)dt$
- $z_2(\tau) = \int_0^{\tau} [A_k cos(2\pi f_c t) + B_k sin(2\pi f_c t) + n(t)]sin(2\pi f_c t)dt$
- $z_1(\tau) = \frac{A_k}{2}\tau + N_1 = \frac{Ad_I}{2\sqrt{2}}\tau + N_1$ signal component proportional to d_i
- $z_2(\tau) = \frac{B_k}{2}\tau + N_2; = \frac{Ad_Q}{2\sqrt{2}}\tau + N_2$ signal component proportional to d_q
- The odd and even bits can be recovered using the decision rules
- $d_I = \begin{cases} 1, & z_1(\tau) \geq 0 \\ -1, & z_1(\tau) < 0 \end{cases} \Rightarrow b_{2i-1} = \begin{cases} 1, & d_I = 1 \\ 0, & d_I = -1 \end{cases};$ Odd sequence
- $d_Q = \begin{cases} 1, & z_2(\tau) \geq 0 \\ -1, & z_2(\tau) < 0 \end{cases} \Rightarrow b_{2i} = \begin{cases} 1, & d_Q = 1 \\ 0, & d_Q = -1 \end{cases};$ Even sequence



QPSK: Probability of Error

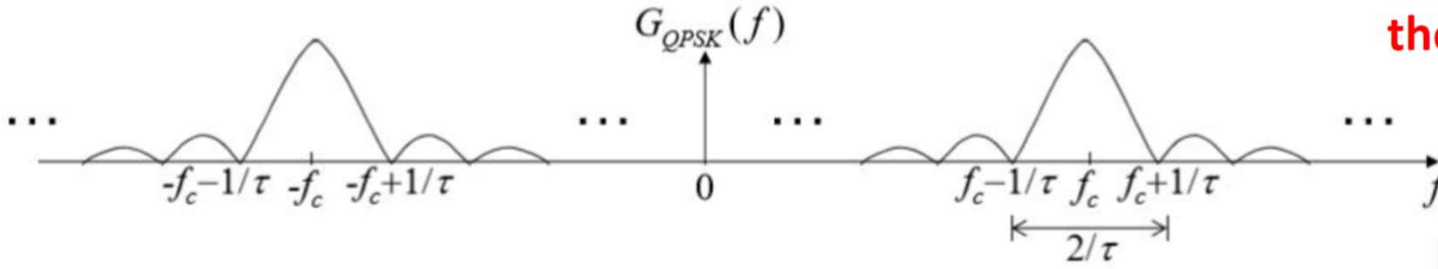
- The symbol error probability is twice the bit error probability, and is given as (will also be derived in the next chapter when we consider M-ary PSK).

$$P_b^* = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

- This is the same as that for binary PSK, provided that both message bits have the same energy. The advantage of QPSK is that it is more bandwidth efficient than BPSK (**can transmit twice the data rate within the same bandwidth)**

QPSK: Power Spectral Density and Bandwidth

The power spectral density has the same shape as that for BPSK (the QPSK is the sum of two BPSK signals one modulated on $cos(2\pi f_c t)$ and the other on $sin(2\pi f_c t)$.



Remember that the symbol duration τ is twice the bit duration.

$$R_s = \frac{1}{\tau} = \frac{R_b}{2}.$$

Here, $\tau = T_s = 2T_b$

B.W $= \frac{2}{\tau} = \frac{2}{2T_b} = R_b$

$$s_{QPSK}(t) = A_k cos(2\pi f_c t) + B_k sin(2\pi f_c t)$$

$$s_{QPSK}(t) = Binary\ PSK\ on\ cos(2\pi f_c t) + Binary\ PSK\ on\ sin(2\pi f_c t)$$

Note that for QPSK, the data rate is R_b bits/sec and the B.W $= R_b$ Hz
Which means we can transmit $2R_b$ bits/sec and the B.W $= 2R_b$ Hz
While for regular BPSK, when the data rate is R_b bits/sec the B.W $= 2R_b$ Hz
Hence, QPSK is more bandwidth efficient than BPSK since in W Hz, we can transmit W bits/sec while in BPSK we can transmit half that value W/2 bits/sec for the same probability of error.

	Bandwidth Efficiency (90% in-band power)
Binary ASK	0.5
Binary FSK	$0.5 \cdot \frac{1}{1+\Delta f / R_b}$
Binary PSK	0.5
QPSK	1

Final Remarks

- **Advantage: higher data rate than in PSK (2 bits per bit interval), while bandwidth occupancy remains the same**
- **4-PSK can easily be extended to 8-PSK, i.e. n-PSK**
- **However, higher rate PSK schemes are limited by the ability of equipment to distinguish small differences in phase**