⑫ $y = f(x) = 1 - \dfrac{1}{x}$, $x > 0$

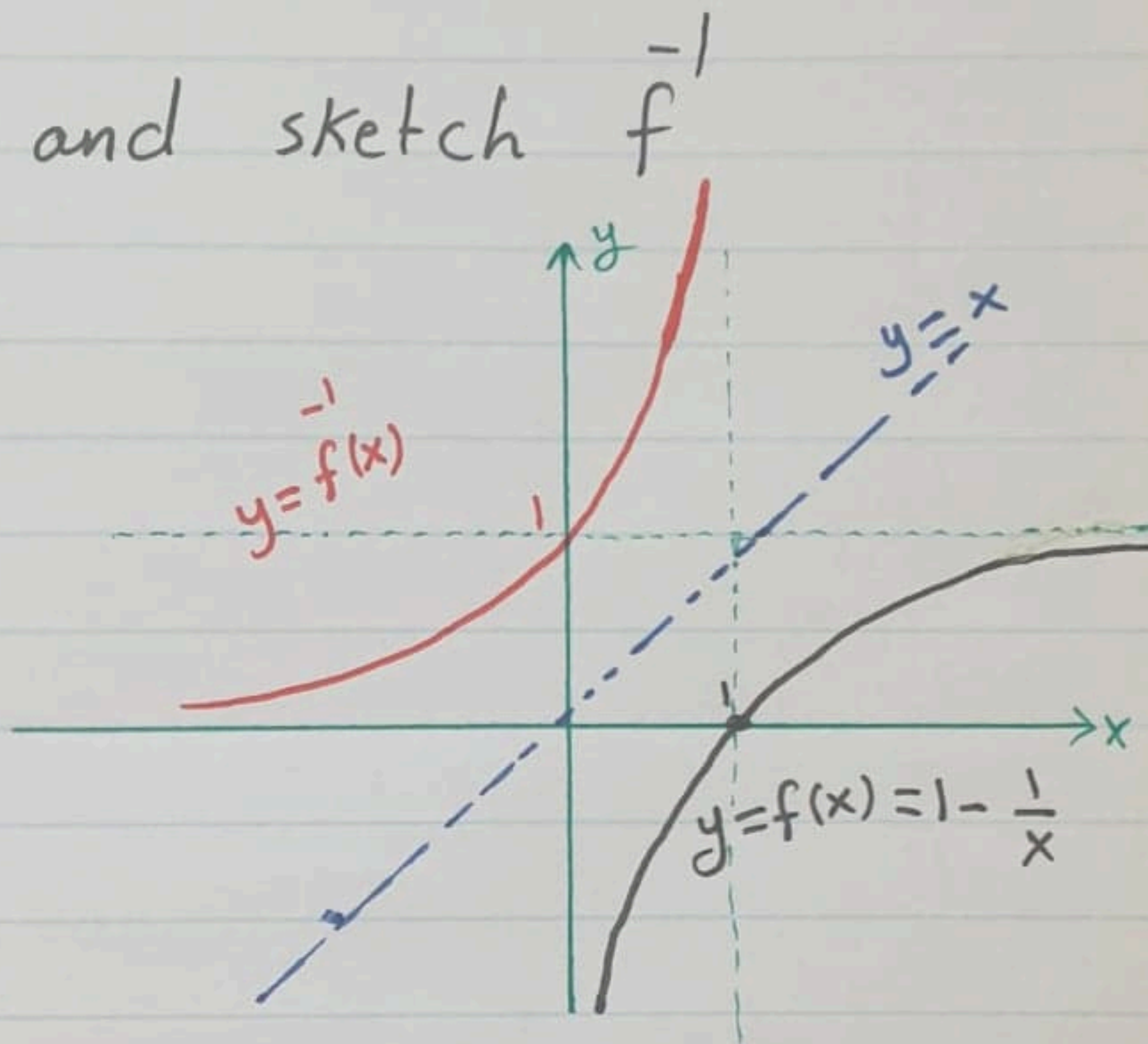Find $D(\bar{f}^{-1})$, $R(\bar{f}^{-1})$ and sketch $\bar{f}^{-1}$

$D(f) = (0, \infty) = R(\bar{f}^{-1})$

$D(\bar{f}^{-1}) = (-\infty, 1) = R(f)$

$y = 1 - \dfrac{1}{x}$ $\Rightarrow$ $1 - y = \dfrac{1}{x}$

$\dfrac{1}{1-y} = x$ $\Rightarrow$ $y = \dfrac{1}{1-x}$

$\bar{f}^{-1}(x) = \dfrac{1}{1-x}$

$y = \bar{f}^{-1}(x)$

$y = x$

$y = f(x) = 1 - \dfrac{1}{x}$

㉛ $f(x) = \dfrac{x+3}{x-2}$ Find $\bar{f}^{-1}(x)$, $D(\bar{f}^{-1})$, $R(\bar{f}^{-1})$ and show that
$f(\bar{f}^{-1}(x)) = f^{-1}(f(x)) = x$

• $y = \dfrac{x+3}{x-2}$ $\Rightarrow$ $yx - 2y = x + 3$ $\Rightarrow$ $x(y-1) = 3 + 2y$

$x = \dfrac{3+2y}{y-1}$ $\Rightarrow$ $y = \dfrac{3+2x}{x-1}$ $\Rightarrow$ $\bar{f}^{-1}(x) = \dfrac{2x+3}{x-1}$

• $D(f) = \mathbb{R} \setminus \{2\} = R(\bar{f}^{-1})$
• $D(\bar{f}^{-1}) = \mathbb{R} \setminus \{1\} = R(f)$

• $f(\bar{f}^{-1}(x)) = f\left(\dfrac{2x+3}{x-1}\right) = \dfrac{\dfrac{2x+3}{x-1} + 3}{\dfrac{2x+3}{x-1} - 2} = \dfrac{\dfrac{2x+3+3x-3}{x-1}}{\dfrac{2x+3-2x+2}{x-1}} = \dfrac{5x}{5} = x$

• $\bar{f}^{-1}(f(x)) = \bar{f}^{-1}\left(\dfrac{x+3}{x-2}\right) = \dfrac{2\left(\dfrac{x+3}{x-2}\right) + 3}{\dfrac{x+3}{x-2} - 1} = \dfrac{\dfrac{2x+6+3x-6}{x-2}}{\dfrac{x+3-x+2}{x-2}} = \dfrac{5x}{5} = x$

[41] $f(x) = x^3 - 3x^2 - 1$, $x \geq 2$ Find $\dfrac{df^{-1}}{dx}\Big|$

$f'(x) = 3x^2 - 6x$

$ x = -1 = f(3)$

$\dfrac{df^{-1}}{dx}\Big|_{x=\boxed{-1}=f(3)} = \dfrac{1}{\dfrac{df}{dx}\Big|_{x=f^{-1}(-1)}} = \dfrac{1}{\dfrac{df}{dx}\Big|_{x=f^{-1}(f(3))}} = \dfrac{1}{\dfrac{df}{dx}\Big|_{x=3}} = \dfrac{1}{3(9)-6(3)}$

$\underset{b}{\downarrow}\quad\underset{a}{\uparrow} = \dfrac{1}{9}$

Exp Let $f(x) = 2x + e^x$ Find $(f^{-1})'(1)$

$ \underset{\hookrightarrow}{} b = 1$

$f'(x) = 2 + e^x f(a) = 1$

$ 2a + e^a = 1$

$(f^{-1})'(1) = \dfrac{1}{f'(f^{-1}(1))} = \dfrac{1}{f'(0)} = \dfrac{1}{2+e^0} = \dfrac{1}{3} \Bigg| a = 0$

$ f(a) = b$

Exp Find f^{-1} if $f(x) = x^2 - 2x$, $x \leq 1$ $\Bigg| a = f^{-1}(b)$

$ 0 = f^{-1}(1)$

$y = x^2 - 2x = (x-1)^2 - 1$

$y + 1 = (x-1)^2 \Longrightarrow |x-1| = \sqrt{y+1}$

$|x-1| = \begin{cases} x-1 & , \ x > 1 \\ 1-x & , \ x \leq 1 \end{cases}$

$\overset{\Longrightarrow}{} 1 - x = \sqrt{y+1}$

$ x = 1 - \sqrt{y+1}$

$ y = 1 - \sqrt{x+1}$

$ f^{-1}(x) = 1 - \sqrt{x+1}$