



DEPARTMENT OF COMPUTER SYSTEM ENGINEERING

Digital Integrated Circuits - ENCS333

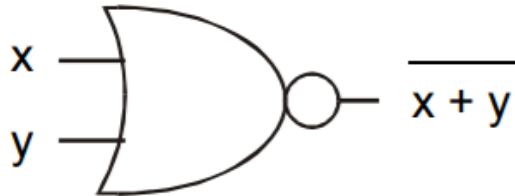
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Lecture #8 Combinational Circuit

Integrated-Circuit Devices and Modeling

CMOS NOR Gate

- NOR Symbol



- NOR Truth Table

x	y	$\overline{x+y}$
0	0	1
0	1	0
1	0	0
1	1	0

- Karnaugh map

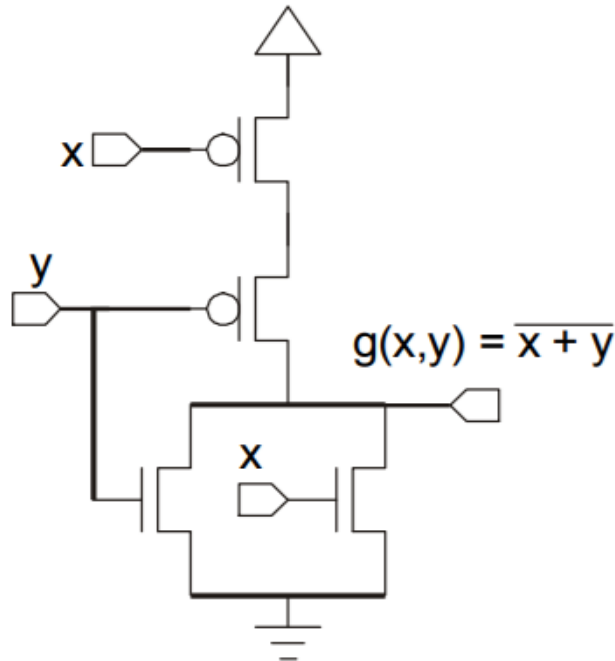
x \ y	0	1
0	1	0
1	0	0

$$g(x,y) = \overline{x} \cdot \overline{y} \cdot 1 + x \cdot 0 + y \cdot 0$$

- construct Sum of Products equation with all terms
- each term represents a MOSFET path to the output
- '1' terms are connected to VDD via pMOS
- '0' terms are connected to ground via nMOS

CMOS NOR Gate

- CMOS NOR Schematic



$$g(x,y) = \overline{x} \cdot \overline{y} \cdot 1 + x \cdot 0 + y \cdot 0$$

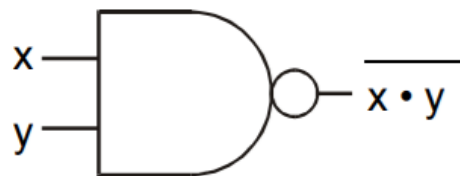
- output is LOW if x OR y is true
 - parallel nMOS
- output is HIGH when x AND y are false
 - series pMOS

- Important Points

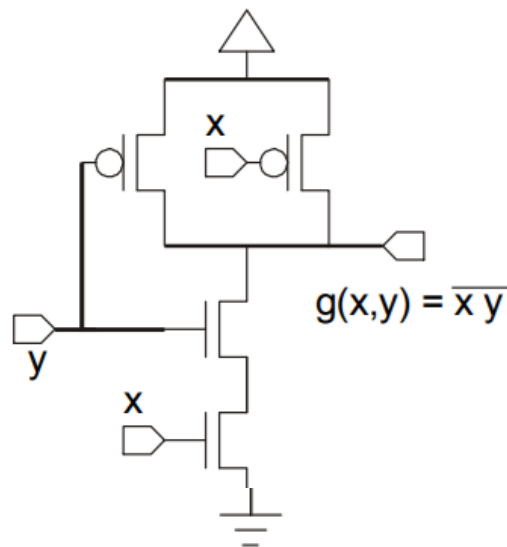
- series-parallel arrangement
 - when nMOS in series, pMOS in parallel, and visa versa
 - true for all CMOS logic gates
 - allows us to construct more complex logic functions

CMOS NAND Gate

- NAND Symbol



- CMOS Schematic



- Truth Table

x	y	$\overline{x \cdot y}$
0	0	1
0	1	1
1	0	1
1	1	0

- K-map

x \ y	0	1
0	1	1
1	1	0

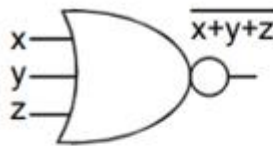
$$g(x,y) = (\overline{x} \cdot \overline{y} \cdot 1) + (\overline{x} \cdot y \cdot 1) + (x \cdot \overline{y} \cdot 1) + (x \cdot y \cdot 0)$$

$$= x \cdot y \cdot 0 + \overline{x} \cdot 1 + \overline{y} \cdot 1$$

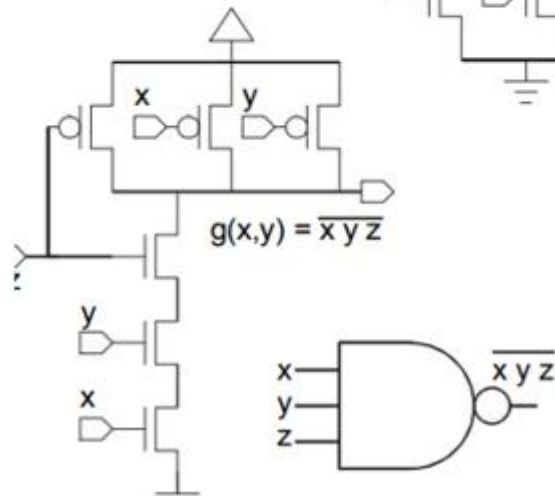
- output is LOW if x AND y are true
 - series nMOS
- output is HIGH when x OR y is false
 - parallel pMOS

3-Input Gates

- NOR3

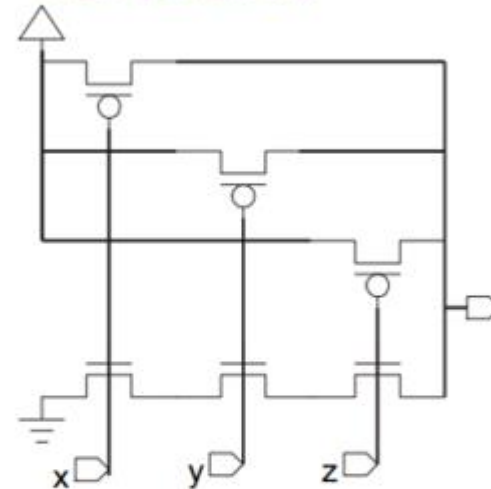


- NAND3



- Alternate Schematic

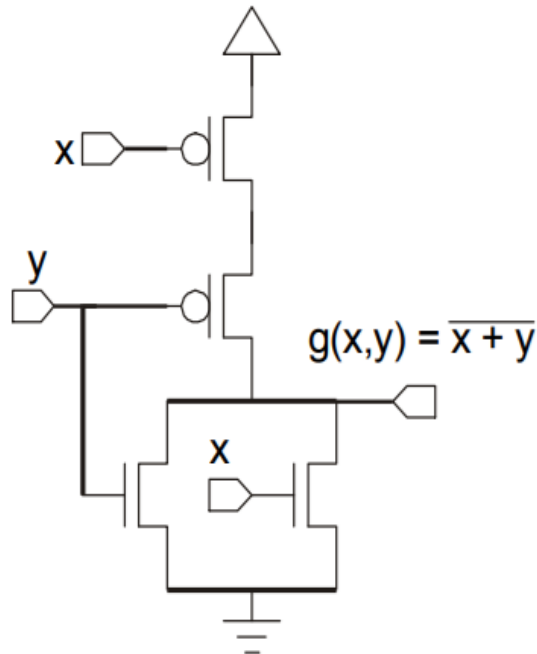
- what function?



- note shared gate inputs
 - is input order important?
 - in series, parallel, both?
- schematic resembles how the circuit will look in *physical layout*

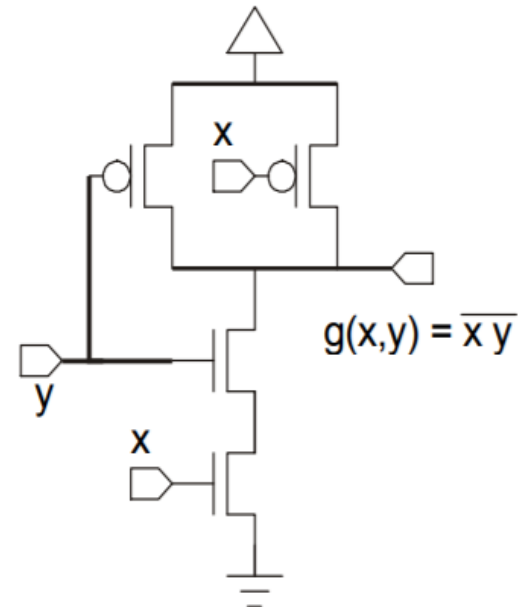
Review: CMOS NAND/NOR Gates

- NOR Schematic



- output is LOW if x OR y is true
 - parallel nMOS
- output is HIGH when x AND y are false
 - series pMOS

- NAND Schematic



- output is LOW if x AND y are true
 - series nMOS
- output is HIGH when x OR y is false
 - parallel pMOS

Complex Combinational Logic

- General logic functions
 - for example

$$f = \overline{a \cdot (b + c)}, \quad f = \overline{(d \cdot e) + a \cdot (\overline{b} + c)}$$

- How do we construct the CMOS gate?
 - use DeMorgan principles to modify expression
 - construct nMOS and pMOS networks

$$\overline{a \cdot b} = \overline{a} + \overline{b}$$

$$\overline{a + b} = \overline{a} \cdot \overline{b}$$

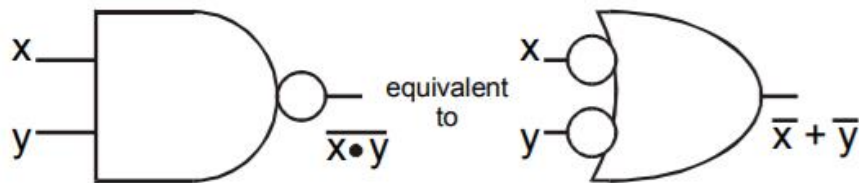
- use Structured Logic
 - AOI (AND OR INV)
 - OAI (OR AND INV)

Using DeMorgan

- DeMorgan Relations

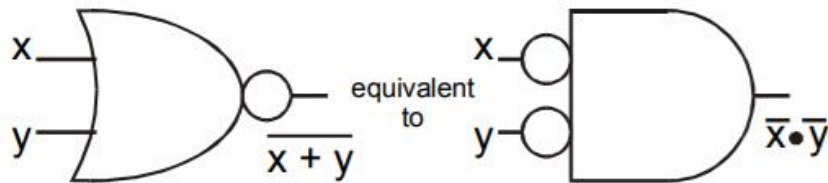
- NAND-OR rule $\boxed{a \cdot b = \overline{a + b}}$

- bubble pushing illustration



- bubbles = inversions

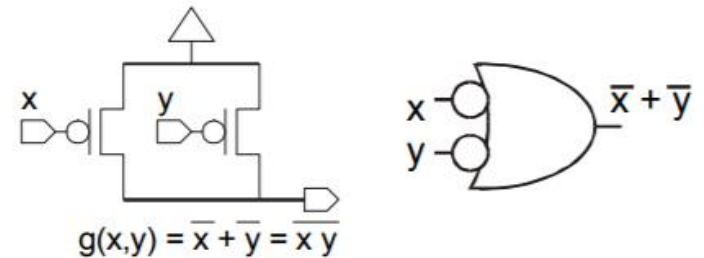
- NOR-AND rule $\boxed{a + b = \overline{a \cdot b}}$



to implement pMOS this way, must push all bubbles to the inputs and remove all NAND/NOR output bubbles

- pMOS and bubble pushing

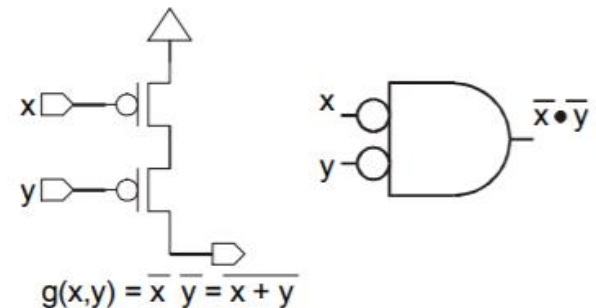
- Parallel-connected pMOS



- assert-low OR

- creates NAND function

- Series-connected pMOS



- assert-low AND

- creates NOR function

Rules for Constructing CMOS Gates

The Mathematical Method

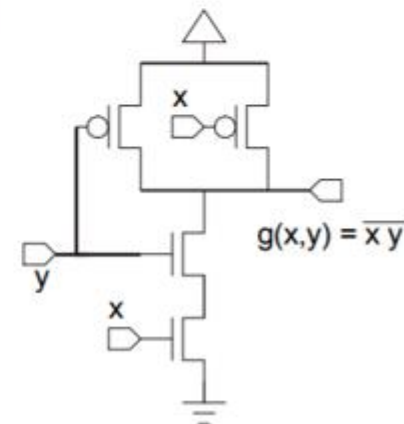
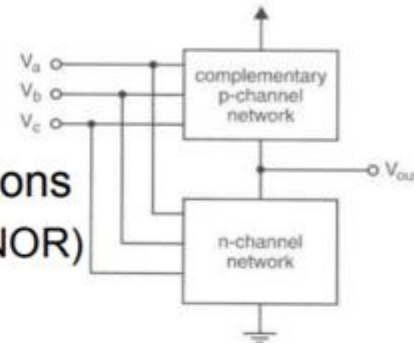
- Given a logic function
 $F = f(a, b, c)$
- Reduce (using DeMorgan) to eliminate inverted operations
 - inverted variables are OK, but not operations (NAND, NOR)
- Form pMOS network by complementing the **inputs**
 $F_p = f(\overline{a}, \overline{b}, \overline{c})$
- Form the nMOS network by complementing the **output**
 $F_n = \overline{f(a, b, c)} = \overline{F}$
- Construct F_n and F_p using AND/OR series/parallel MOSFET structures
 - series = AND, parallel = OR

EXAMPLE:

$$F = \overline{ab} \Rightarrow$$

$$F_p = \overline{\overline{a} \overline{b}} = a + b; \quad \text{OR/parallel}$$

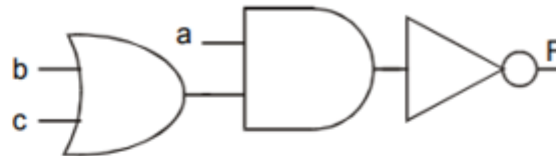
$$F_n = \overline{\overline{ab}} = ab; \quad \text{AND/series}$$



CMOS Combinational Logic Example

- Construct a *CMOS* logic gate to implement the function:

$$F = \overline{a \cdot (b + c)}$$



14 transistors (cascaded gates)

pMOS

- Apply DeMorgan expansions

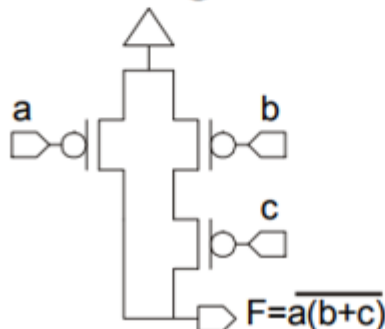
$$F = \overline{a} + \overline{(b + c)}$$

$$F = \overline{a} + (\overline{b} \cdot \overline{c})$$

- Invert inputs for pMOS

$$F_p = a + (b \cdot c)$$

- Resulting Schematic



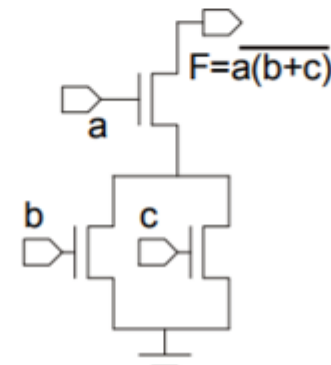
nMOS

- Invert output for nMOS

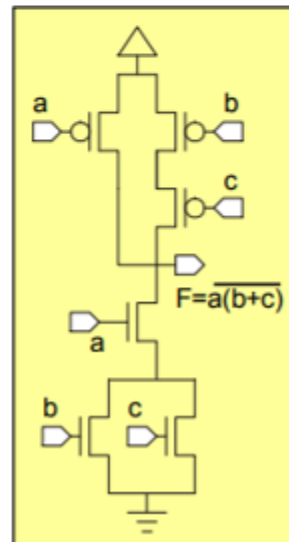
$$F_n = a \cdot (b + c)$$

- Apply DeMorgan
- none needed

- Resulting Schematic



6 transistors
(CMOS)



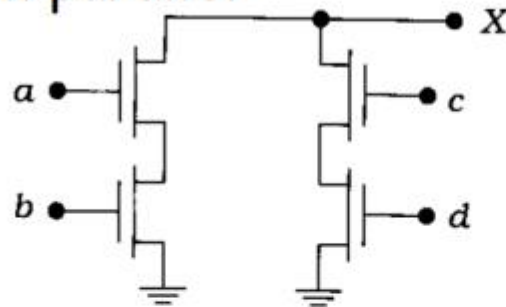
Structured Logic

- Recall CMOS is inherently Inverting logic
 - Can use structured circuits to implement general logic functions
 - **AOI**: implements logic function in the order
AND, OR, NOT (Invert)
 - Example: $F = a \cdot b + c \cdot d$
 - operation order: i) a **AND** b, c **AND** d, ii) (ab) **OR** (cd), iii) **NOT**
 - Inverted Sum-of-Products (SOP) form
 - **OAI**: implements logic function in the order
OR, AND, NOT (Invert)
 - Example: $G = \overline{(x+y)} \cdot (z+w)$
 - operation order: i) x **OR** y, z **OR** w, ii) (x+y) **AND** (z+w), iii) **NOT**
 - Inverted Product-of-Sums (POS) form
- Use a **structured CMOS array** to realize such functions

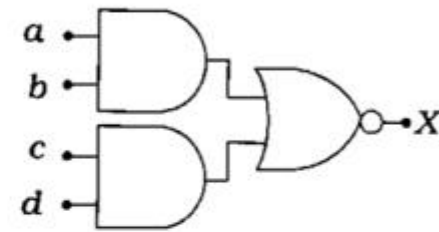
AOI/OAI nMOS Circuits

- nMOS AOI structure

- series txs in parallel

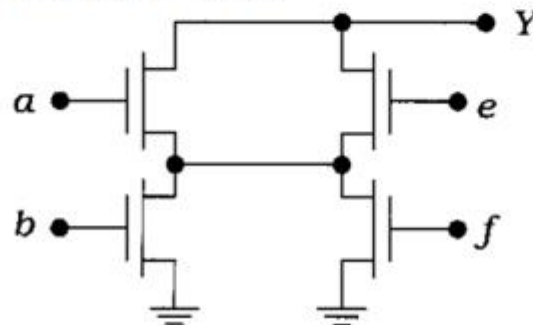


$$F = \overline{a \cdot b + c \cdot d}$$

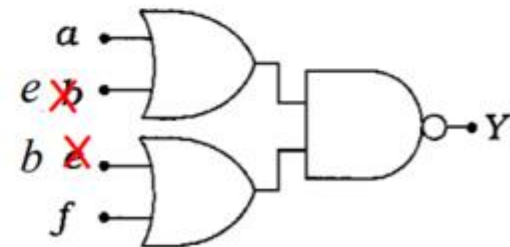


- nMOS OAI structure

- series of parallel txs



$$F = \overline{(a + e) \cdot (b + f)}$$

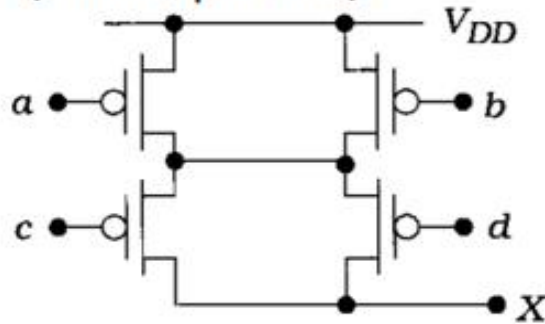


error in textbook Figure 2.45

AOI/OAI pMOS Circuits

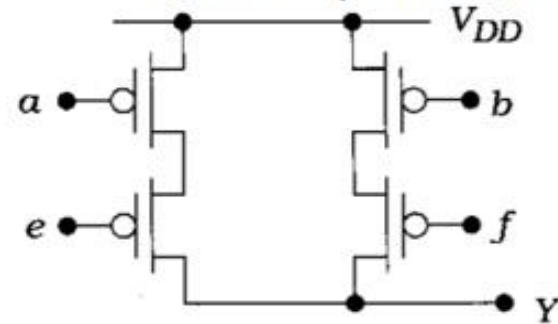
- pMOS AOI structure

- series of parallel txs
- opposite of nMOS (series/parallel)

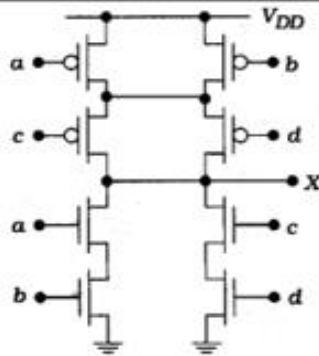


- pMOS OAI structure

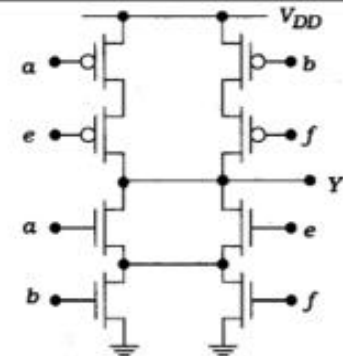
- series txs in parallel
- opposite of nMOS (series/parallel)



Complete CMOS
AOI/OAI circuits



1a) AOI circuit



1b) OAI circuit

Implementing Logic in CMOS

- Reducing Logic Functions
 - fewest operations $\Rightarrow$ fewest txs
 - minimized function to eliminate txs
 - Example: $x y + x z + x v = x (y + z + v)$

5 operations:	3 operations:
3 AND, 2 OR	1 AND, 2 OR
# txs =	# txs =
- Suggested approach to implement a CMOS logic function
 - create nMOS network
 - invert output
 - reduce function, use DeMorgan to eliminate NANDs/NORs
 - implement using **series for AND** and **parallel for OR**
 - create pMOS network
 - complement each operation in nMOS network
 - i.e. make parallel into series and visa versa

CMOS Logic Example

- Construct the function below in CMOS

$F = a + b \cdot (c + d)$; remember AND operations occur before OR

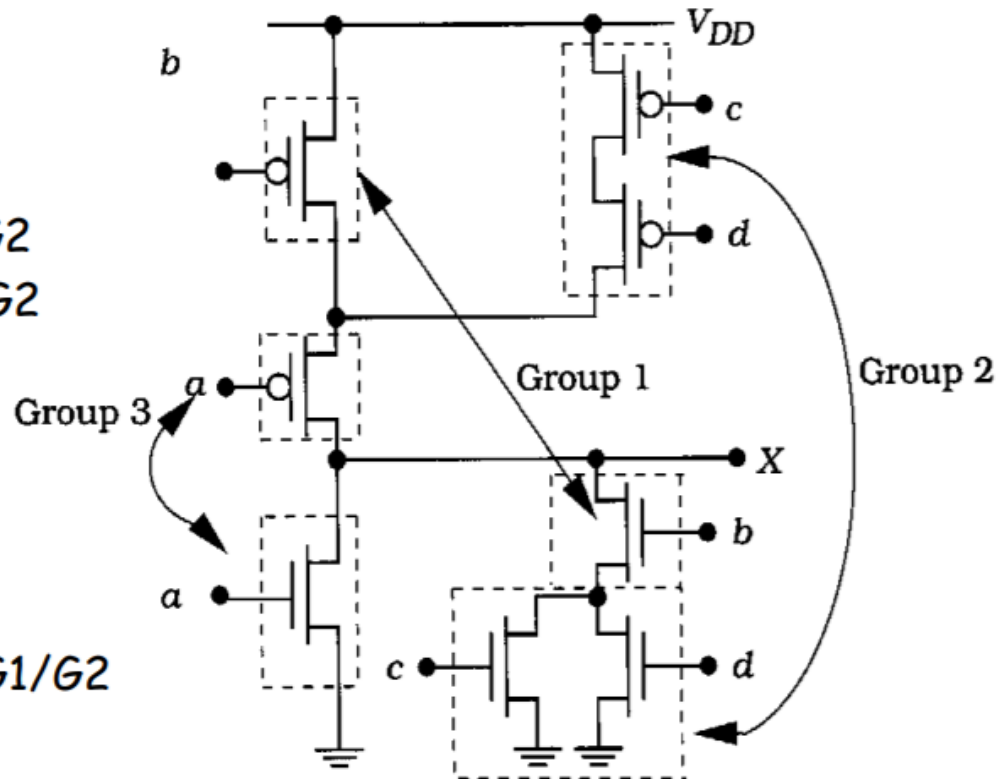
$F_n = a + b \cdot (c + d)$

- nMOS

- Group 2: c & d in parallel
- Group 1: b in series with G2
- Group 3: a parallel to G1/G2

- pMOS

- Group 2: c & d in series
- Group 1: b parallel to G2
- Group 3: a in series with G1/G2



- Circuit has an OAOI organization (AOI with extra OR)

Another Combinational Logic Example

- Construct a CMOS logic gate which implements the function:

$$F = \overline{a} \cdot (b + \overline{c})$$

- pMOS

- Apply DeMorgan expansions
none needed
- Invert inputs for pMOS
 $F_p = a \cdot (\overline{b} + c)$
- Resulting Schematic ?

- nMOS

- Invert output for nMOS
 $F_n = \overline{a} \cdot (b + \overline{c})$
- Apply DeMorgan
 $F_n = a + \overline{(b + \overline{c})}$
 $F_n = a + (\overline{b} \cdot c)$
- Resulting Schematic ?

Another Combinational Logic Example

- Implement the function below by constructing the nMOS network and complementing operations for the pMOS:

$$F = \overline{\overline{a}} \cdot b \cdot (a + c)$$

- nMOS**

- Invert Output

- $F_n = \overline{\overline{a} \cdot b \cdot (a + c)} = \overline{\overline{a} \cdot b} + \overline{(a + c)}$

- Eliminate NANDs and NORs

- $F_n = \overline{a} \cdot b + (\overline{a} \cdot \overline{c})$

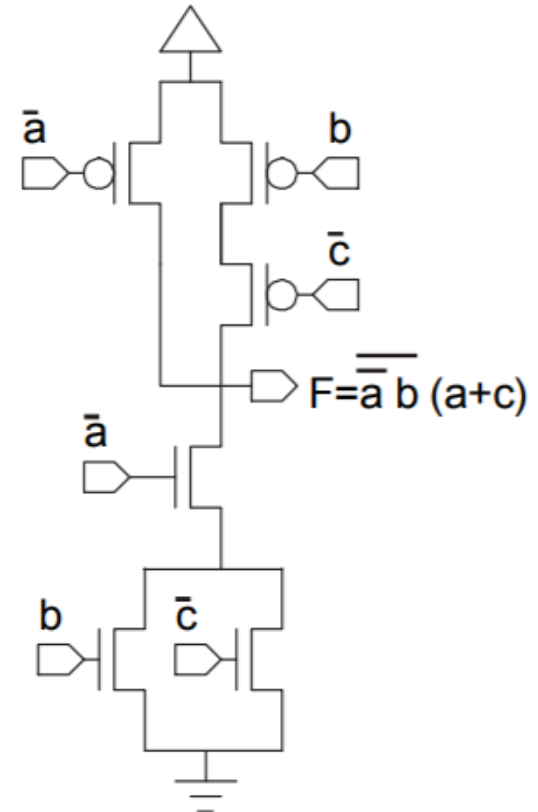
- Reduce Function

- $F_n = \overline{a} \cdot (b + \overline{c})$

- Resulting Schematic ?

- Complement operations for pMOS

- $F_p = \overline{a} + (b \cdot \overline{c})$

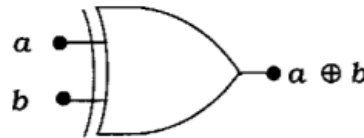


XOR and XNOR

- Exclusive-OR (XOR)

- $a \oplus b = \bar{a} \cdot b + a \cdot \bar{b}$

- not AOI form



a	b	$a \oplus b$
0	0	0
0	1	1
1	0	1
1	1	0

- Exclusive-NOR

- $\overline{a \oplus b} = a \cdot b + \bar{a} \cdot \bar{b}$

- inverse of XOR

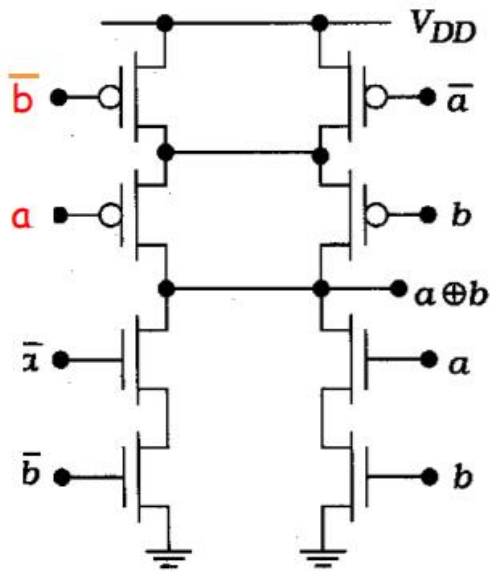
- XOR/XNOR in AOI form

- XOR: $\overline{\overline{a \oplus b}} = \overline{a \cdot b + \bar{a} \cdot \bar{b}}$, formed by complementing XNOR above

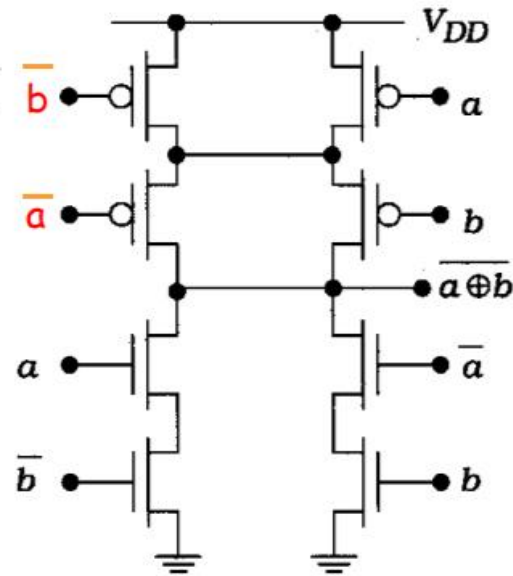
- XNOR: $\overline{a \oplus b} = \overline{a \cdot b + \bar{a} \cdot \bar{b}}$, formed by complementing XOR

thus, interchanging a and $\bar{a}$ (or b and $\bar{b}$) converts from XOR to XNOR

XOR and XNOR AOI Schematic



(a) Exclusive-OR



(b) Exclusive-NOR

note: see textbook, figure 2.57

$$\text{-XOR: } a \oplus b = \overline{a \cdot b + \overline{a} \cdot \overline{b}}$$

$$\text{-XNOR: } \overline{a \oplus b} = \overline{\overline{a} \cdot b + a \cdot \overline{b}}$$

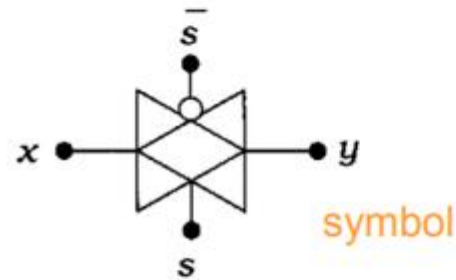
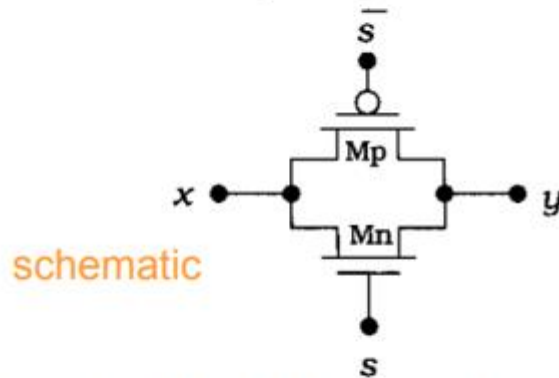
CMOS Transmission Gates

- Function

- gated switch, capable of passing both '1' and '0'

recall: pMOS passes a good '1'
and nMOS passes a good '0'

- Formed by a parallel nMOS and pMOS tx



- Controlled by gate select signals, s and $\bar{s}$

- if $s = 1$, $y = x$, switch is **closed**, txs are **on**

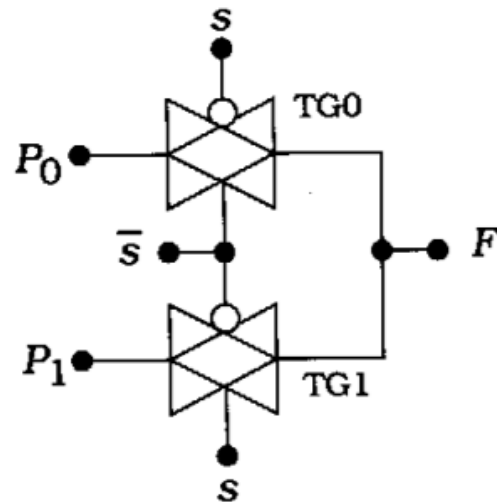
- if $s = 0$, $y = \text{unknown}$ (high impedance),
switch **open**, txs **off**

$$y = x s, \text{ for } s=1$$

Transmission Gate Logic Functions

- TG circuits used extensively in CMOS
 - good switch, can pass full range of voltage (VDD-ground)
- **2-to-1 MUX** using TGs

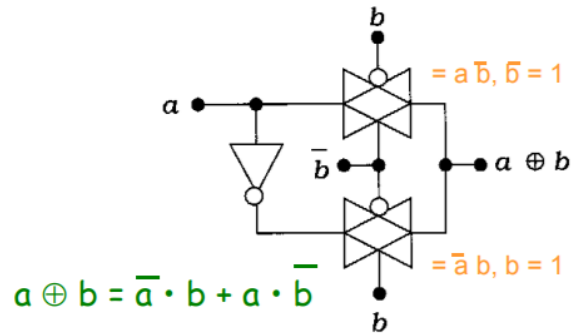
$$F = P_0 \cdot \bar{s} + P_1 \cdot s$$



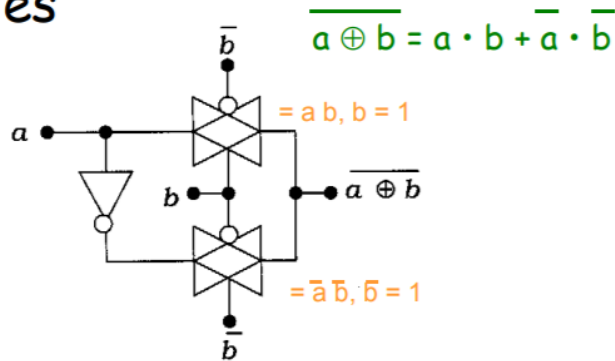
s	TG0	TG1	F
0	Closed	Open	P_0
1	Open	Closed	P_1

More TG Functions

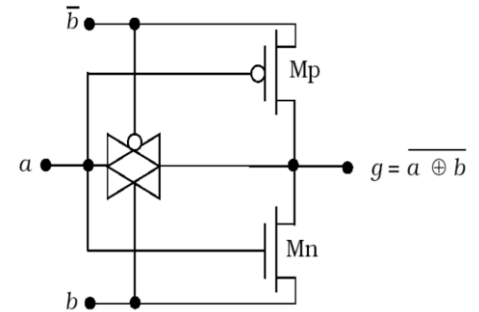
- TG XOR and XNOR Gates



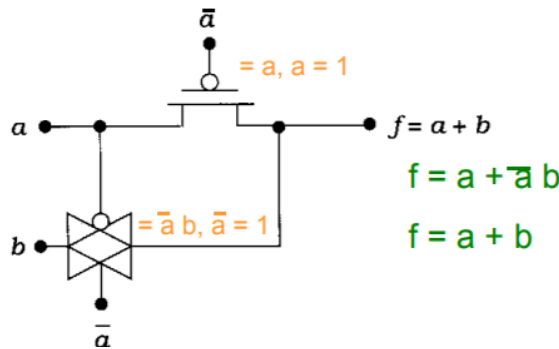
(a) XOR circuit



(b) XNOR circuit



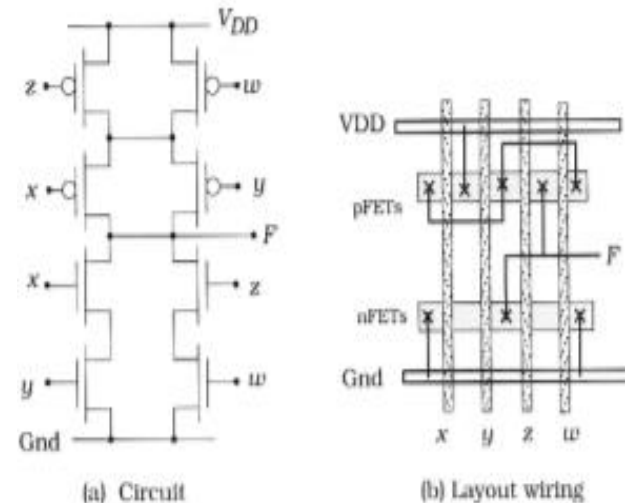
- Using TGs instead of "static CMOS"
- TG OR gate



Structured Layout

General Approach

- power rails
- horizontal Active
- vertical Poly (inputs from top/bottom)
 - Metal1 connects nodes as needed in schematic
- Structured Layout
 - AOI circuit figure
 - useful for many logic functions
 - see examples in textbook
- Disadvantages
 - not optimized for speed
 - large S/D regions = higher capacitance
 - interconnect paths could be shorter
 - not optimized for area/size



Euler Graphs

- Euler Graph

- method for determining what order to layout txs
- assign each circuit node as a point
- assign each tx as a line between points

- Method

- locate starting point
- trace a loop from starting point through each transistor
 - can only re-cross a point once, separate nMOS loop must cross each pMOS
- if above is possible, all tx can be in same Active
- if not, will require multiple Active regions

