Transient Response via Gain Adjustment (Design P-Controller)

Design Procedure

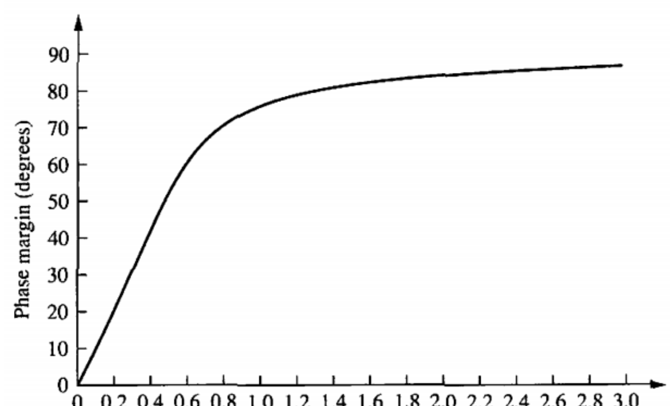
1. Draw the Bode magnitude and phase plots for a convenient value of gain.
2. Using Eqs. (4.39) and (10.73), determine the required phase margin from the percent overshoot.

$$\Phi_M = 90 - \tan^{-1}\frac{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}}{2\zeta} = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}} \quad (10.73)$$

$$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100$$

Equation (10.73), plotted in Figure 10.48, shows the relationship between phase margin and damping ratio.

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2+\ln^2(\%OS/100)}}$$

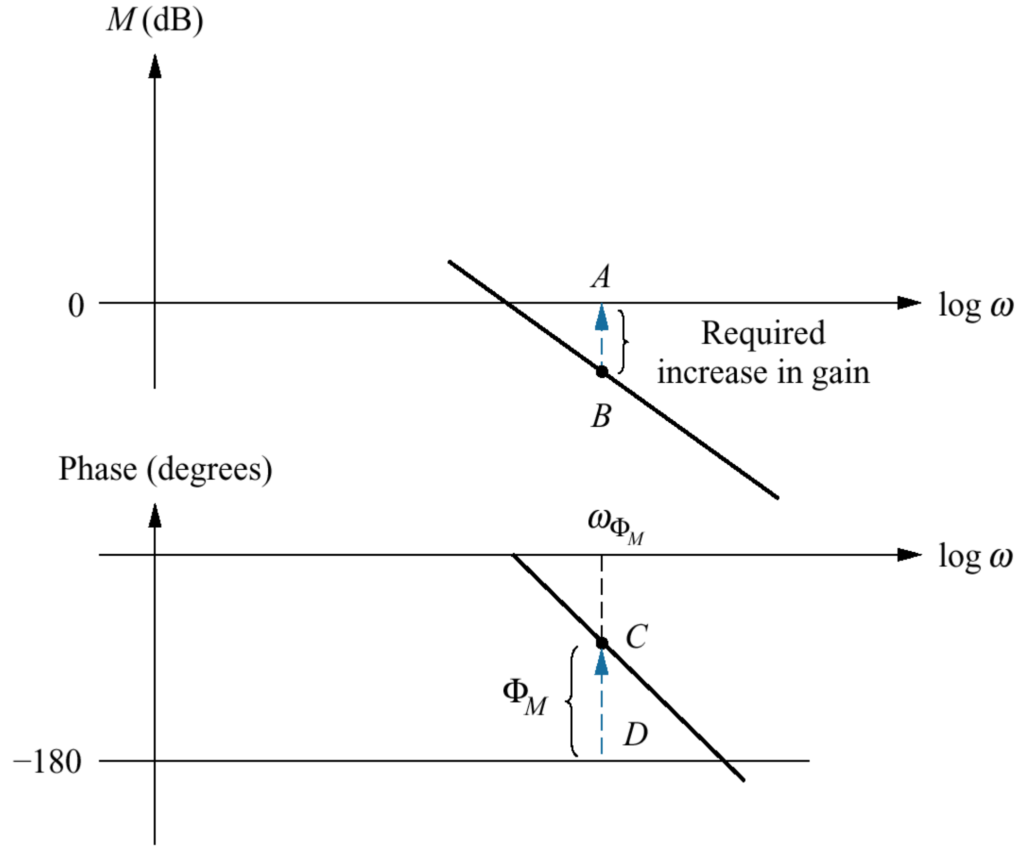


PM=phase angle-(-180)

PM=phase angle+180

Figure 11.1

Bode plots showing gain adjustment for a desired phase margin



3. Find the frequency, ω_{Φ_M}, on the Bode phase diagram that yields the desired phase margin, CD, as shown on Figure 11.1.
4. Change the gain by an amount AB to force the magnitude curve to go through 0 dB at ω_{Φ_M}. The amount of gain adjustment is the additional gain needed to produce the required phase margin.

We now look at an example of designing the gain of a third-order system for percent overshoot.

Transient Response Design via Gain Adjustment

PROBLEM: For the position control system shown in Figure 11.2, find the value of preamplifier gain, K, to yield a 9.5% overshoot in the transient response for a step input. Use only frequency response methods.

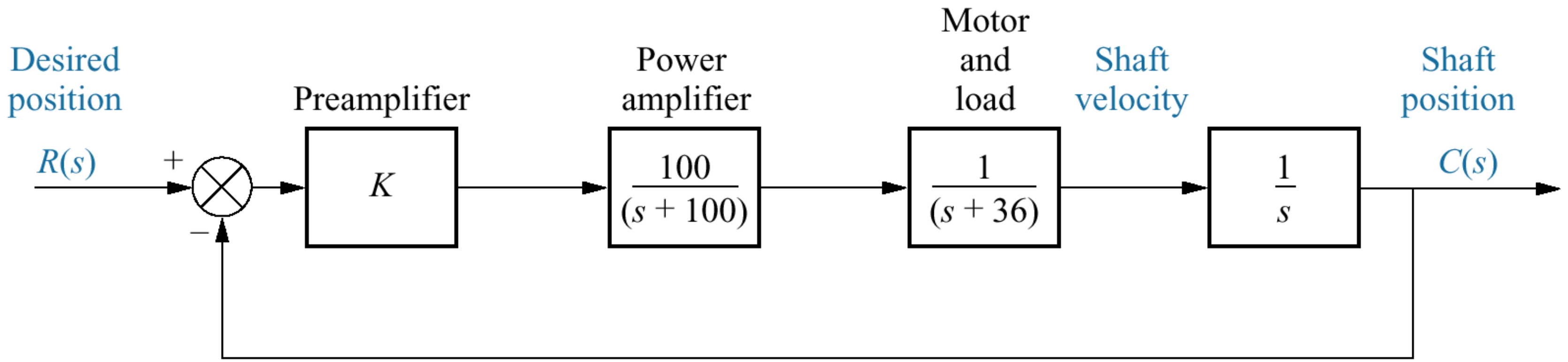


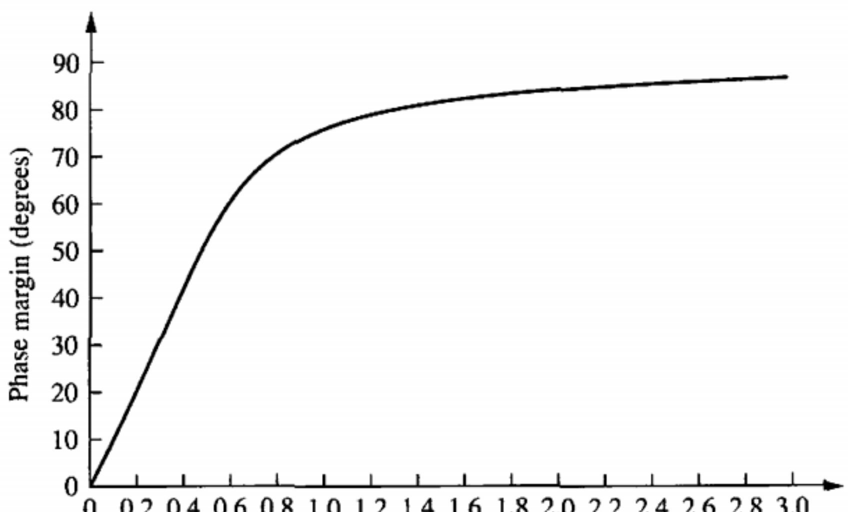
Figure 11.2
System for Example 11.1

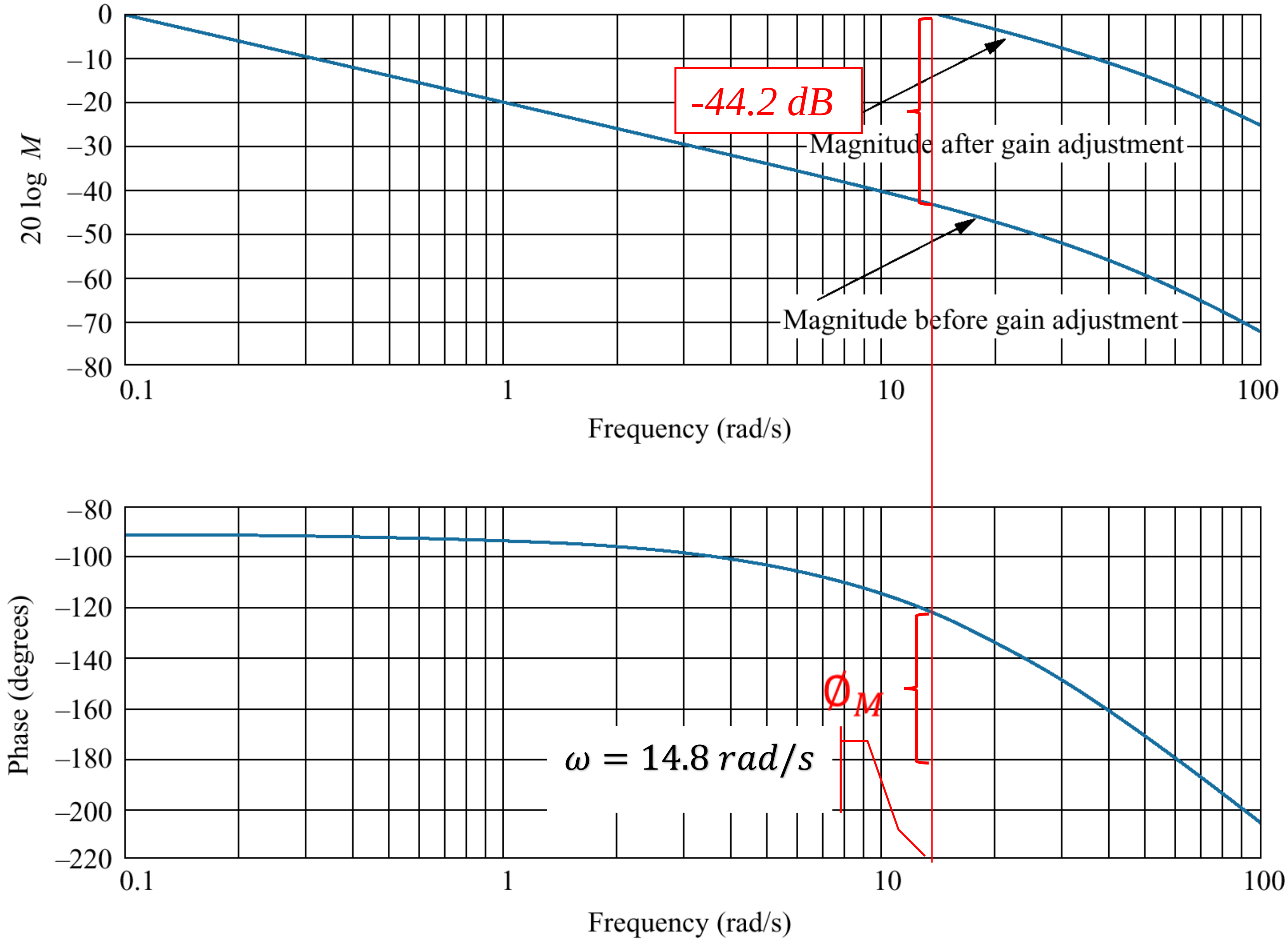
SOLUTION: We will now follow the previously described gain adjustment design procedure.

1. Choose $K = 3.6$ to start the magnitude plot at 0 dB at $\omega = 0.1$ in Figure 11.3.
2. Using Eq. (4.39), a 9.5% overshoot implies $\zeta = 0.6$ for the closed-loop dominant poles. Equation (10.73) yields a 59.2° phase margin for a damping ratio of 0.6.

$$\Phi_M = 90 - \tan^{-1}\frac{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}{2\zeta} = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}} \qquad (10.73)$$

Equation (10.73), plotted in Figure 10.48, shows the relationship between phase margin and damping ratio.





20 log M
0
–10
–20
–30
–40
–50
–60
–70
–80
-44.2 dB
Magnitude after gain adjustment
Magnitude before gain adjustment
0.1
1
10
100
Frequency (rad/s)
Phase (degrees)
–80
–100
–120
–140
–160
–180
–200
–220
ϕ_M
$\omega = 14.8\ rad/s$
0.1
1
10
100
Frequency (rad/s)

3. Locate on the phase plot the frequency that yields a 59.2° phase margin. This frequency is found where the phase angle is the difference between −180° and 59.2°, or −120.8°. The value of the phase-margin frequency is 14.8 rad/s.
4. At a frequency of 14.8 rad/s on the magnitude plot, the gain is found to be −44.2 dB. This magnitude has to be raised to 0 dB to yield the required phase margin. Since the log-magnitude plot was drawn for $K = 3.6$, a 44.2 dB increase, or $K = 3.6 \times 162.2 = 583.9$, would yield the required phase margin for 9.48% overshoot.

The gain-adjusted open-loop transfer function is

$$G(s) = \frac{58{,}390}{s(s+36)(s+100)} \tag{11.1}$$

Table 11.1 summarizes a computer simulation of the gain-compensated system.

TABLE 11.1 Characteristic of gain-compensated system of Example 11.1

Parameter	Proposed specification	Actual value
K_v	—	16.22
Phase margin	59.2°	59.2°
Phase-margin frequency	—	14.8 rad/s
Percent overshoot	9.5	10
Peak time	—	0.18 second

Now at ($\omega_n = 14.8$) *the Magnitude (M=0 dB)*

$$M = 0\ dB = 20\log\left(\frac{100*3.6\ K_n}{36*100*(s)(\frac{s}{36}+1)(\frac{s}{100}+1)}\right)\ dB$$

$$G(jw) = 1 = \left(\frac{100*3.6\ K_n}{36*100*(jw)(\frac{jw}{36}+1)(\frac{jw}{100}+1)}\right) = \frac{0.1K_n}{\sqrt{0+\omega^2}\sqrt{1+\left(\frac{\omega}{100}\right)^2}\sqrt{1+\left(\frac{\omega}{36}\right)^2}}$$

Now at ($\omega_n = 14.8$)

$$\frac{0.1K_n}{(14.8)\sqrt{1+\left(\frac{14.8}{36}\right)^2}\sqrt{1+\left(\frac{14.8}{100}\right)^2}} = \frac{0.1K_n}{16.14} = 1$$

→ $K_n = 162.2$

$$G(s) = \left(\frac{100*3.6\ K_n}{s(s+36)(s+100)}\right) = \left(\frac{100*3.6*162.2}{s(s+36)(s+100)}\right) = \left(\frac{58390}{s(s+36)(s+100)}\right)$$

Table 11.1
Characteristics of gain-compensated system of Example 11.1

Parameter	Proposed Specification	Actual Value
K_v	—	16.22
Phase margin	59.2°	59.2°
Phase-margin frequency	—	14.8 rad/s
Percent overshoot	9.5	10
Peak time	—	0.18 second

PROBLEM: For a unity feedback system with a forward transfer function

$$G(s) = \frac{K}{s(s+50)(s+120)}$$

use frequency response techniques to find the value of gain, K, to yield a closed-loop step response with 20% overshoot.

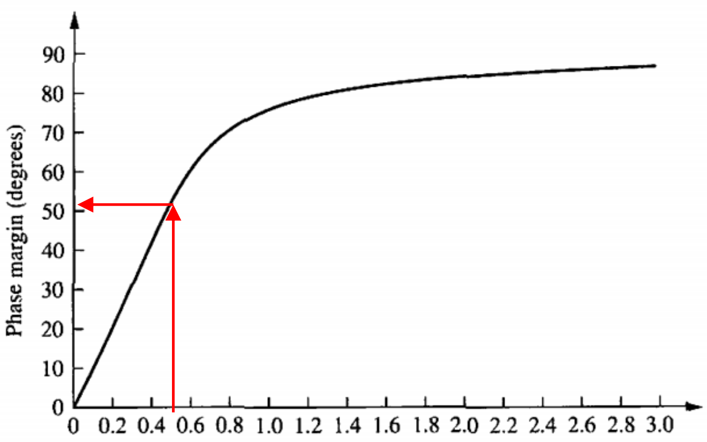
Let K=1

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}$$

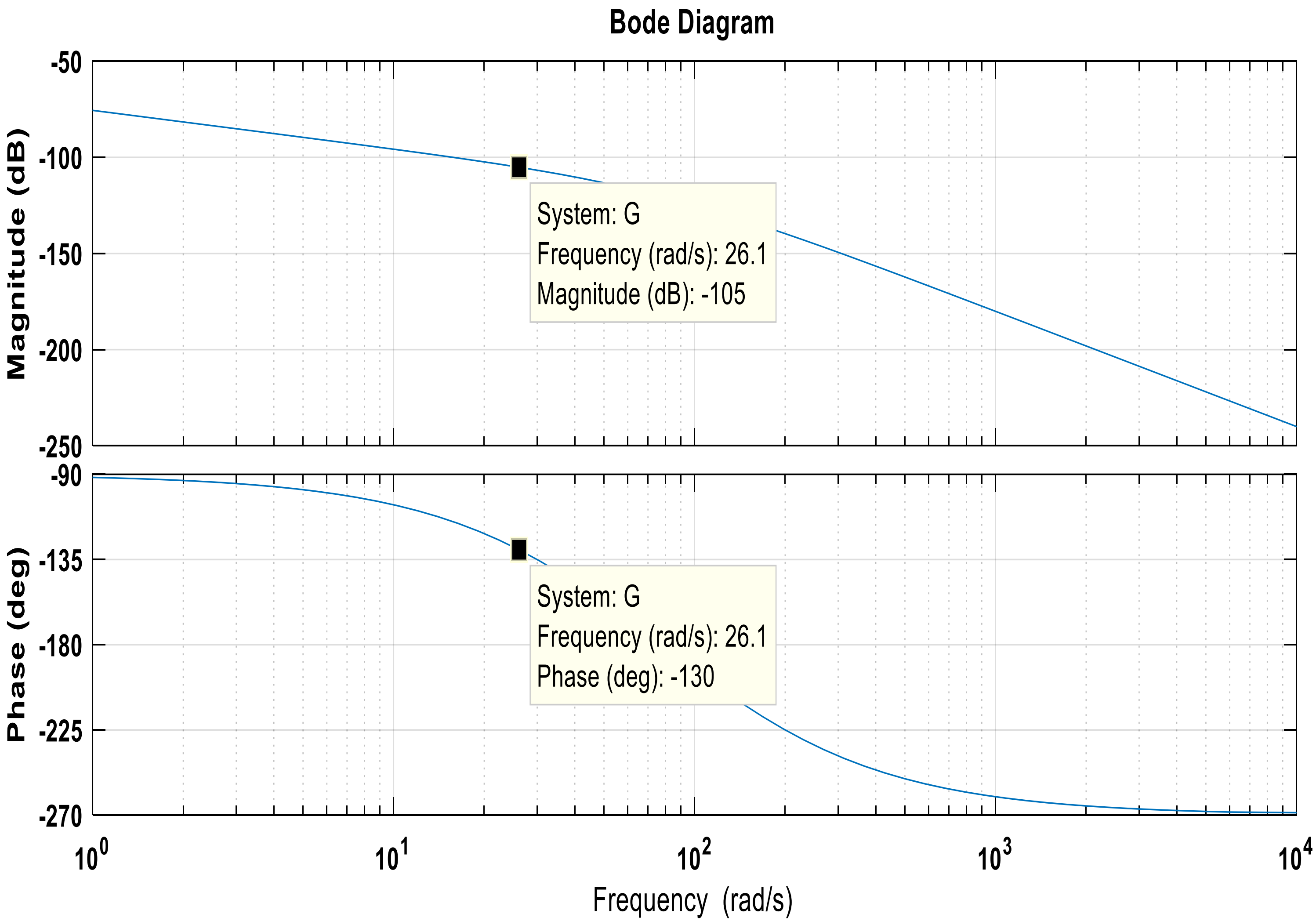
$$\zeta = 0.456$$

$$\Phi_M = 90 - \tan^{-1}\frac{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}{2\zeta} = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}} \quad (10.73)$$

Equation (10.73), plotted in Figure 10.48, shows the relationship between phase margin and damping ratio.



PM=50
phase angle=-180+50
=-130


Bode Diagram
Magnitude (dB)
-50
-100
-150
-200
-250
System: G
Frequency (rad/s): 26.1
Magnitude (dB): -105
Phase (deg)
-90
-135
-180
-225
-270
System: G
Frequency (rad/s): 26.1
Phase (deg): -130
10^0
10^1
10^2
10^3
10^4
Frequency (rad/s)

Now at ($\omega_{\phi m} = 26.1$) *the Magnitude (M=0 dB)*

$$M = 0\ dB = 20\log\left(\frac{K_n}{6000*(s)(\frac{s}{50}+1)(\frac{s}{120}+1)}\right)\ dB$$

$$G(jw) = 1 = \left(\frac{K_n}{6000*(jw)(\frac{jw}{50}+1)(\frac{jw}{120}+1)}\right) = \frac{0.00016667K_n}{\sqrt{0+\omega^2}\sqrt{1+\left(\frac{\omega}{50}\right)^2}\sqrt{1+\left(\frac{\omega}{120}\right)^2}}$$

Now at ($\omega_{\phi m} = 26.1$)

$$\frac{0.00016667K_n}{(26.1)\sqrt{1+\left(\frac{26.1}{50}\right)^2}\sqrt{1+\left(\frac{26.1}{120}\right)^2}} = \frac{0.00016667K_n}{30.13} = 1$$

$$\rightarrow K_n = 180776$$

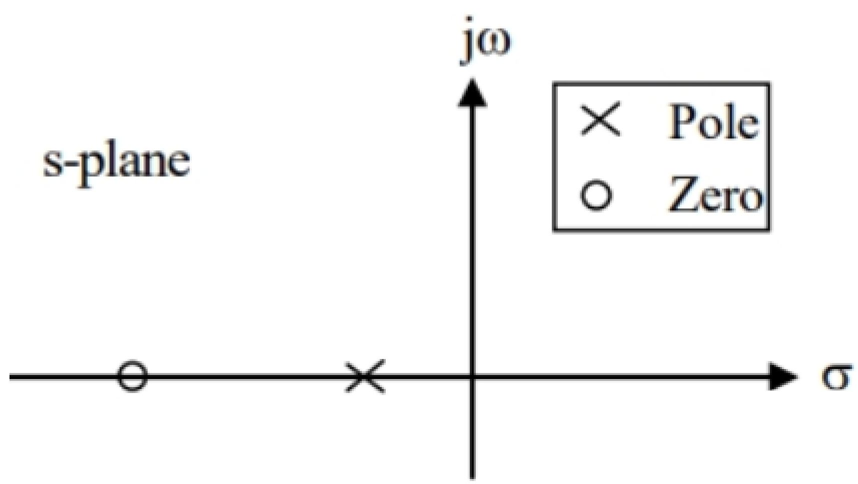
$$G(s) = \left(\frac{180776}{s(s+50)(s+120)}\right)$$

Lag Compensation

The transfer function of the lag compensator is

$$G_c(s) = \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} \qquad \text{where } \alpha > 1.$$

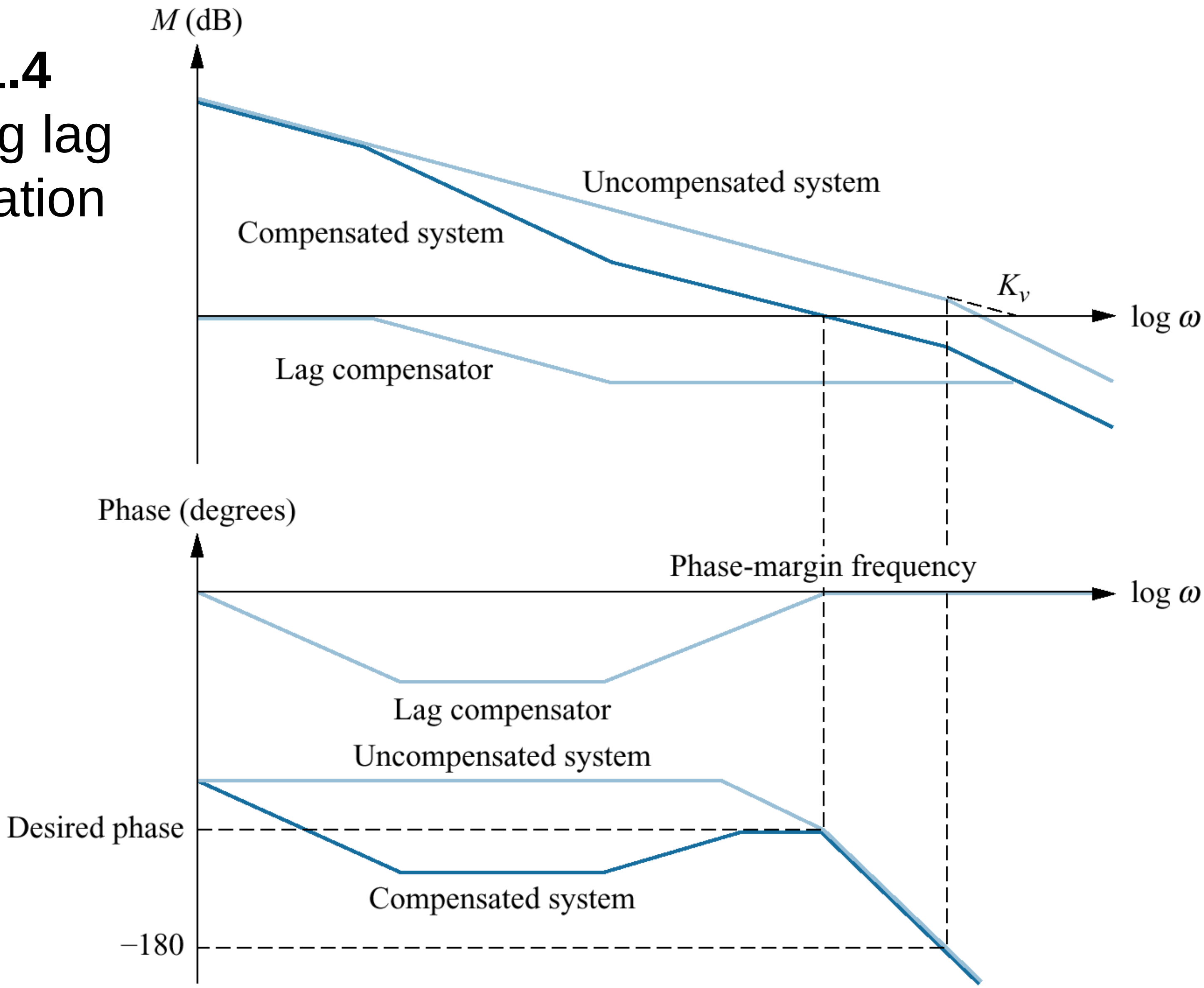
The function of the lag compensator as seen on Bode diagrams is to (1) improve the static error constant by increasing only the low-frequency gain without any resulting instability, and (2) increase the phase margin of the system to yield the desired transient response. These concepts are illustrated in Figure 11.4.



Design Procedure

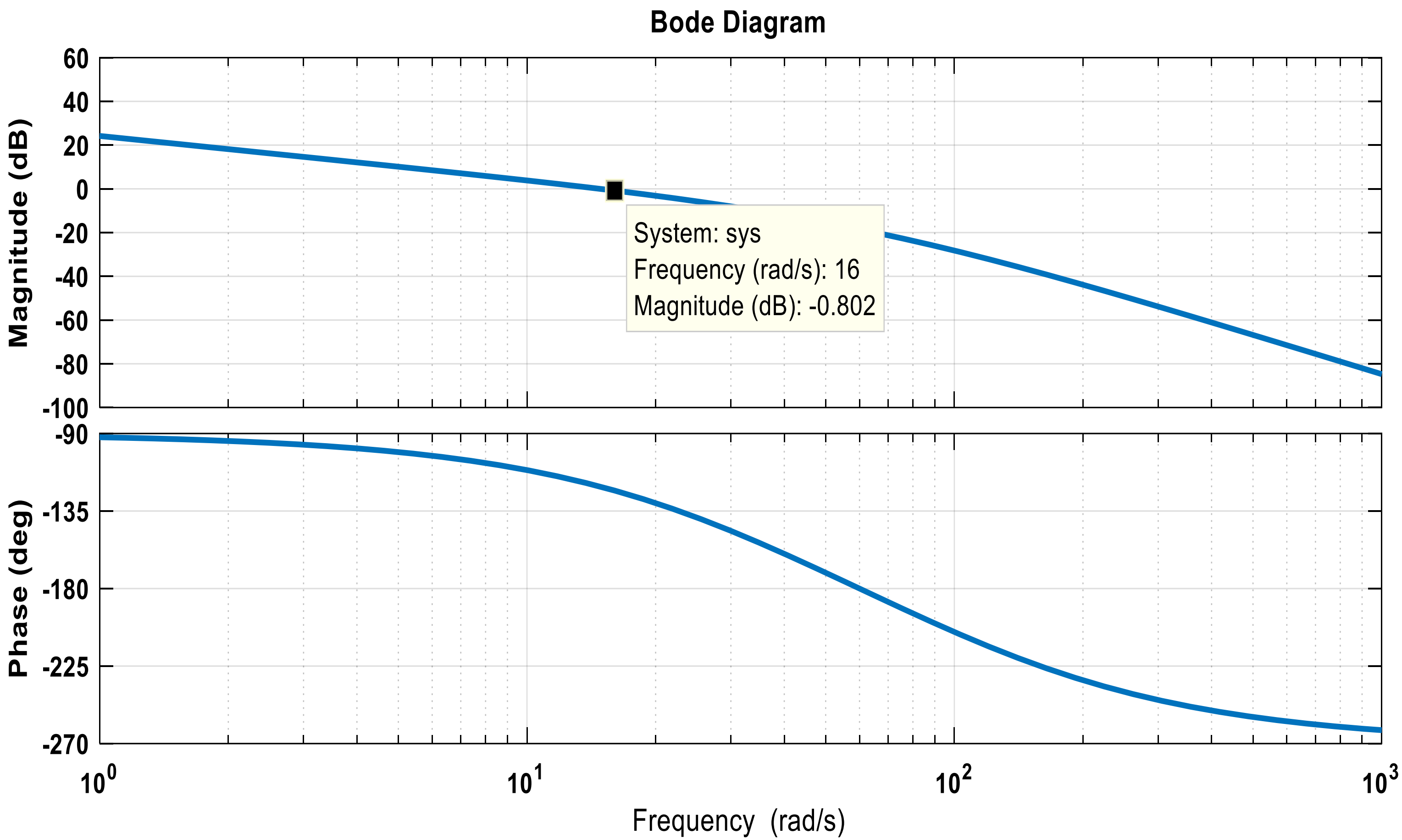
1. Set the gain, K, to the value that satisfies the steady-state error specification and plot the Bode magnitude and phase diagrams for this value of gain.
2. Find the frequency where the phase margin is 5° to 12° greater than the phase margin that yields the desired transient response (*Ogata, 1990*). This step compensates for the fact that the phase of the lag compensator may still contribute anywhere from −5° to − 12° of phase at the phase-margin frequency.
3. Select a lag compensator whose magnitude response yields a composite Bode magnitude diagram that goes through 0 dB at the frequency found in Step 2 as follows: Draw the compensator's high-frequency asymptote to yield 0 dB for the compensated system at the frequency found in Step 2. Thus, if the gain at the frequency found in Step 2 is $20 \log K_{PM}$, then the compensator's high-frequency asymptote will be set at $-20 \log K_{PM}$; select the upper break frequency to be 1 decade below the frequency found in Step 2;[2] select the low-frequency asymptote to be at 0 dB; connect the compensator's high- and low-frequency asymptotes with a −20 dB/decade line to locate the lower break frequency.
4. Reset the system gain, K, to compensate for any attenuation in the lag network in order to keep the static error constant the same as that found in Step 1.

Figure 11.4
Visualizing lag compensation



PROBLEM: Given the system of Figure 11.2, use Bode diagrams to design a lag compensator to yield a tenfold improvement in steady-state error over the gain-compensated system while keeping the percent overshoot at 9.5%.

$$G(s) = \frac{58{,}390}{s(s+36)(s+100)}$$



For uncompensated system

$$K_v = \lim_{s\to 0} sG(s) = K\frac{\prod_{i=1}^{n} z_i}{\prod_{i=1}^{m} p_i} = \lim_{s\to 0}\frac{\cancel{s}\,K}{\cancel{s}(s+36)(s+100)} = \quad 16.2$$

1. From Example 11.1 a gain, K, of 583.9 yields a 9.5% overshoot. Thus, for this system, $K_v = 16.22$. For a tenfold improvement in steady-state error, K_v must increase by a factor of 10, or $K_v = 162.2$.

For compensated system

$$K_v = \lim_{s\to 0} sG(s) = K\frac{\prod_{i=1}^{n} z_i}{\prod_{i=1}^{m} p_i} = \lim_{s\to 0}\frac{\not{s}K}{\not{s}(s+36)(s+100)} = 162.2$$

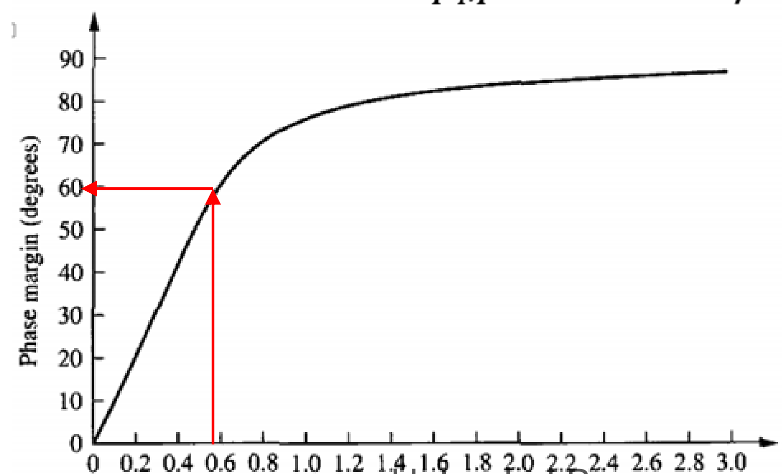
$$K = 162.2*36*100 = 583{,}900$$

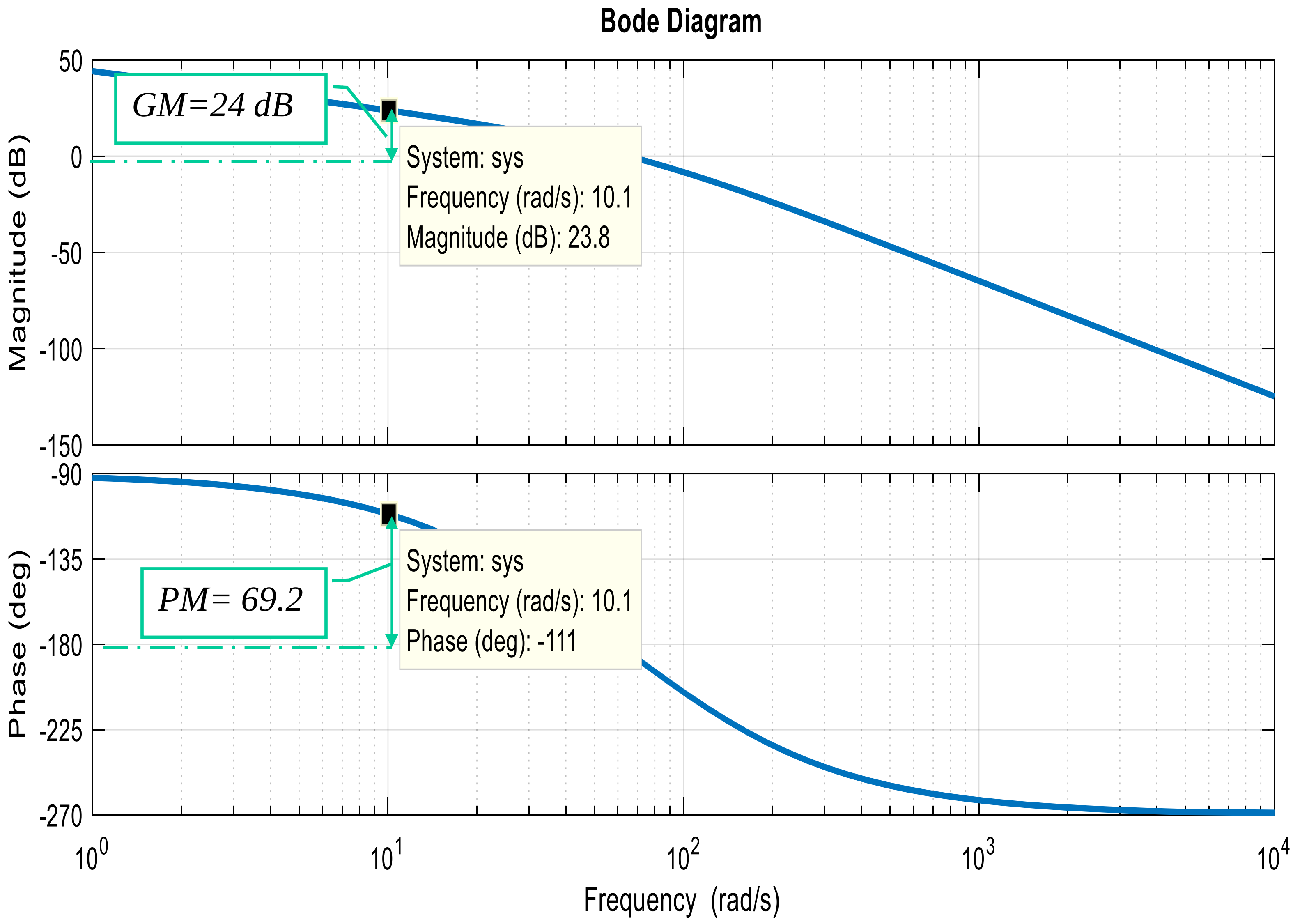
$$G(s) = \frac{583{,}900}{s(s+36)(s+100)}$$

- $OS\% = 9.5\%$

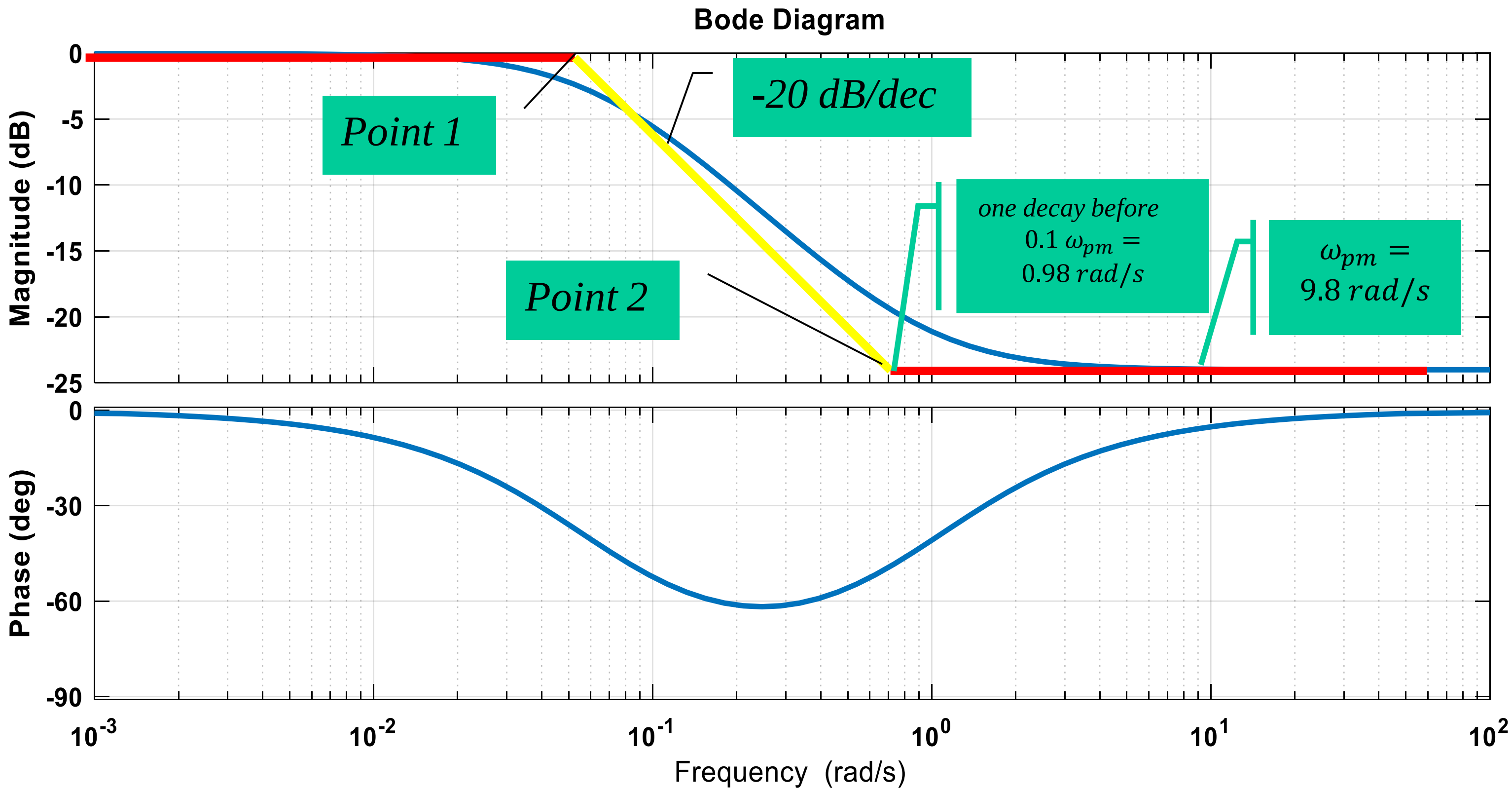
$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} = 0.6$$

- $PM = 59.2$ *from the following figure.*
- *we increase 5-12 to the phase margin in order to compensate for the phase angle contribution of the lag compensator.*
- *So PM=59.2+10=69.2.*
- *Now find where the phase margin is 69.2. You can find it from the bode diagrams*
- *This frequency occurs at phase angle of -180+69.2=-110.8. Therefore* $\omega_{PM} = 9.8\, rad/s$.
- $M(\omega_{PM})$=+24 dB.
- *Thus the lag compensator must provide -24 dB attenuation at* $\omega_{PM} = 9.8\, rad/s$

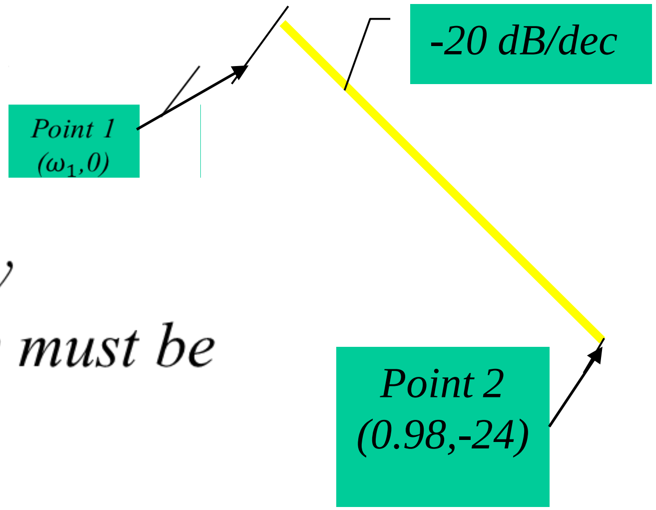




Bode Diagram
GM=24 dB
System: sys
Frequency (rad/s): 10.1
Magnitude (dB): 23.8
PM= 69.2
System: sys
Frequency (rad/s): 10.1
Phase (deg): -111
Magnitude (dB)
Phase (deg)
Frequency (rad/s)



- First draw the high frequency asymptote at -24 dB.
- Arbitrarily select the higher break frequency to be about one decade below the phase-margin frequency, or 0.98 rad/s.
- Starting at the intersection of this frequency with the lag compensator's high-frequency asymptote, draw a -20 dB/decade line until 0 dB is reached.



- *at point 1: (ω_1,0 dB)*
- *at point 2: (0.98,-24 dB)*
- *the slope of the yellow line is -20 dB/decay*
- *To compute ω_1 by using the slope the draw must be 20log (G(jw)) vs log(ω). So:*

$$-20 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-20 = \frac{-24 - 0}{\log(0.98) - \log(\omega_1)}$$

$$0.1760 + 20\log(\omega_1) = -24 \longrightarrow \frac{-24.176}{20} = \log(\omega_1)$$

$$\omega_1 = 10^{-1.2088} = 0.062$$

$$C_c = \frac{K_c(s + 0.98)}{(s + 0.062)}$$

$$C_c(s) = \frac{K_c(s + 0.98)}{(s + 0.062)}$$

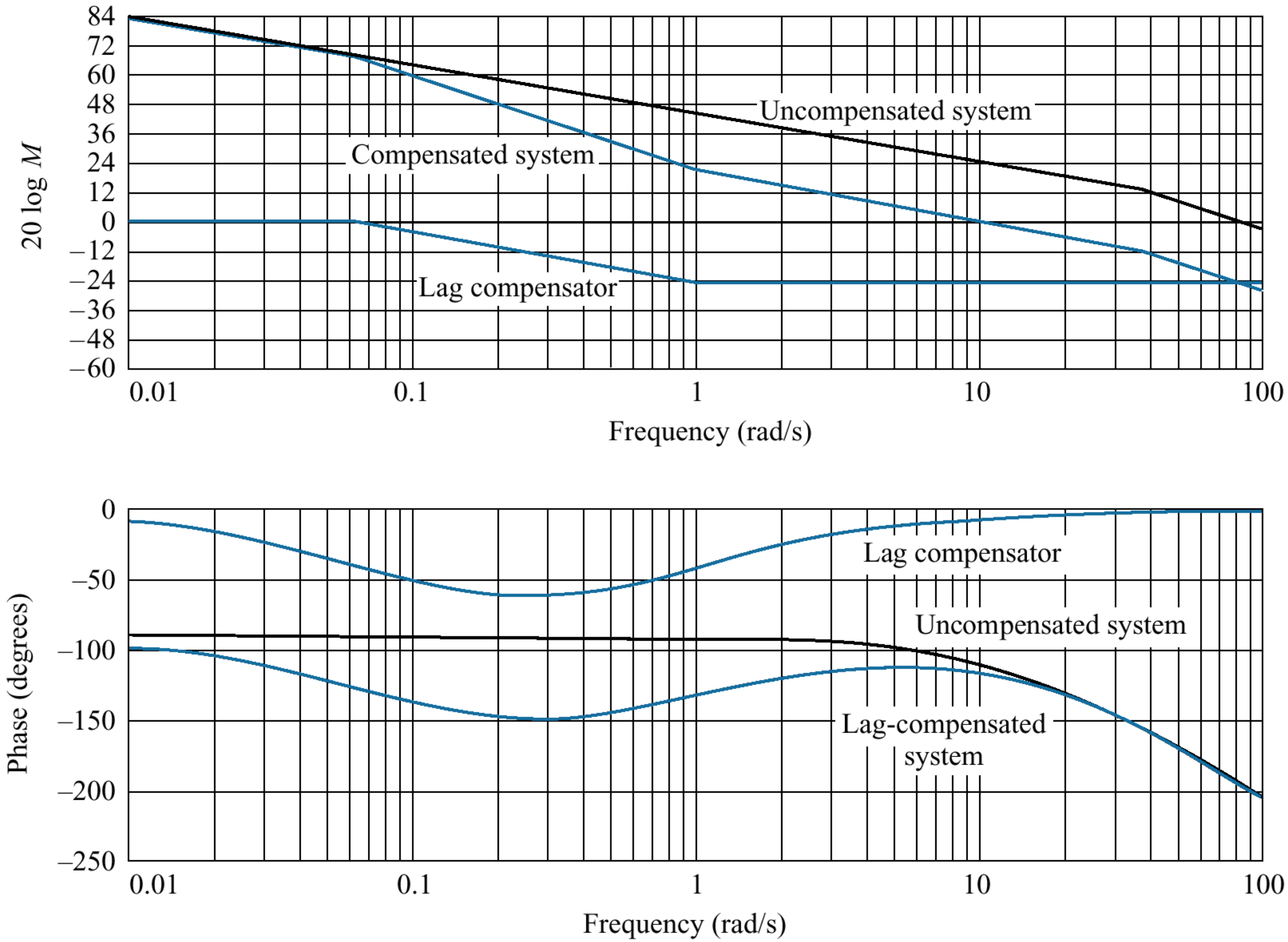
The dc gain for $C_c(s)$ must be unity

$$C_c(s) = \frac{0.98\, K_c(\frac{s}{0.98} s + 1)}{0.062(\frac{s}{0.062} + 1)}$$

$$Dc\ gain = \lim_{s\to 0}(C_c(s)) = 1 = \frac{0.98\, K_c}{0.062}$$

$$K_c = 0.0633$$

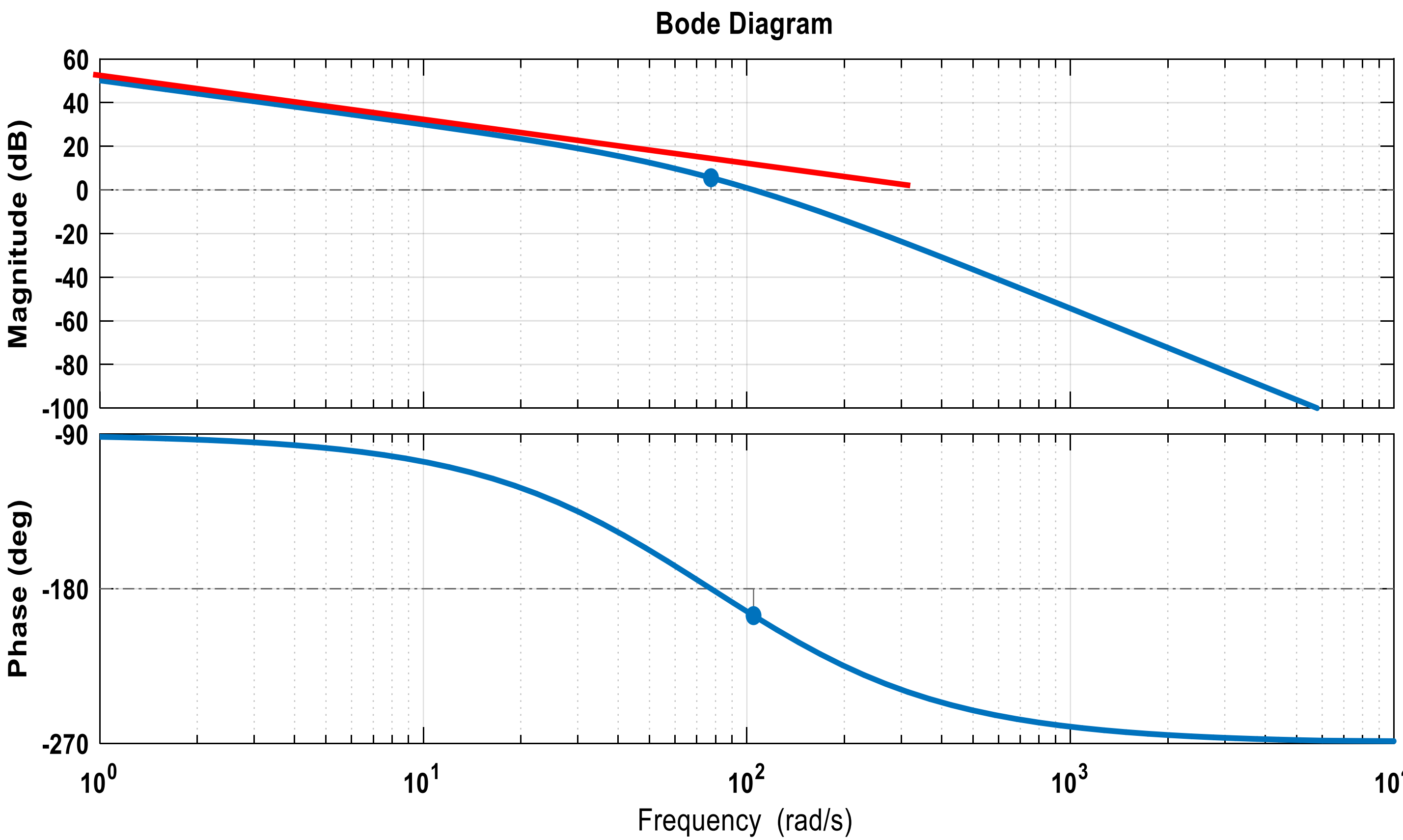
$$C_c(s) = \frac{0.0633\,(s + 0.98)}{(s + 0.062)}$$



$$G_{open}(s) = C_c(s)G(s) = \frac{0.0633*593900\,(s+0.98)}{s(s+0.062)(s+36)(s+100)}$$

PROBLEM: Design a lag compensator for the system in Skill-Assessment Exercise 11.1 that will improve the steady-state error tenfold, while still operating with 20% overshoot.

$$G(s) = \frac{1{,}942{,}000}{s(s+50)(s+120)}$$



$$K_v = \lim_{s \to 0} sG(s) = K\frac{\prod_{i=1}^{n} z_i}{\prod_{i=1}^{m} p_i} = \lim_{s \to 0} \frac{s\,1942000}{s(s+50)(s+120)} = 323.67$$

$$e_{ss}(\infty) = \frac{1}{K_v} = \frac{1}{323.67} = 0.0030896$$

This analysis for uncompensated system.

This is the analysis for the compensated system.

$$e_{ss-new}(\infty) = 0.1 * e_{ss}(\infty) = 0.00030896 = \frac{1}{K_{v-new}}$$

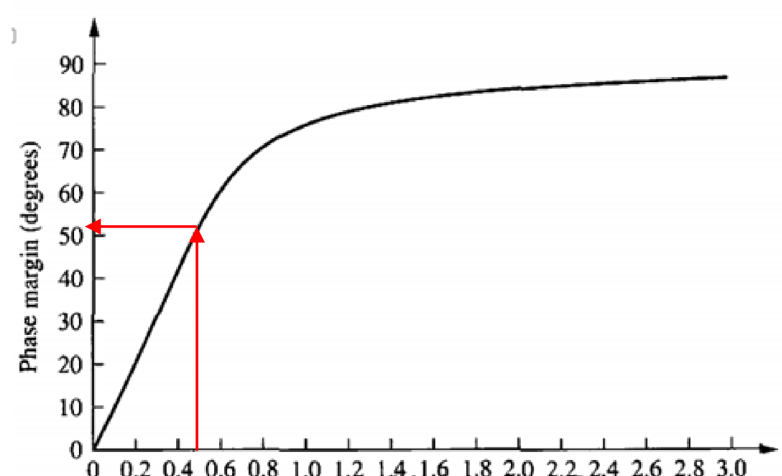
$$K_{v-new} = \frac{1}{K_{v-new}} = 3236.7$$

$$K_{v-new} = \lim_{s \to 0} \frac{s\, 1942000K}{s(s+50)(s+120)} = 3236.7$$

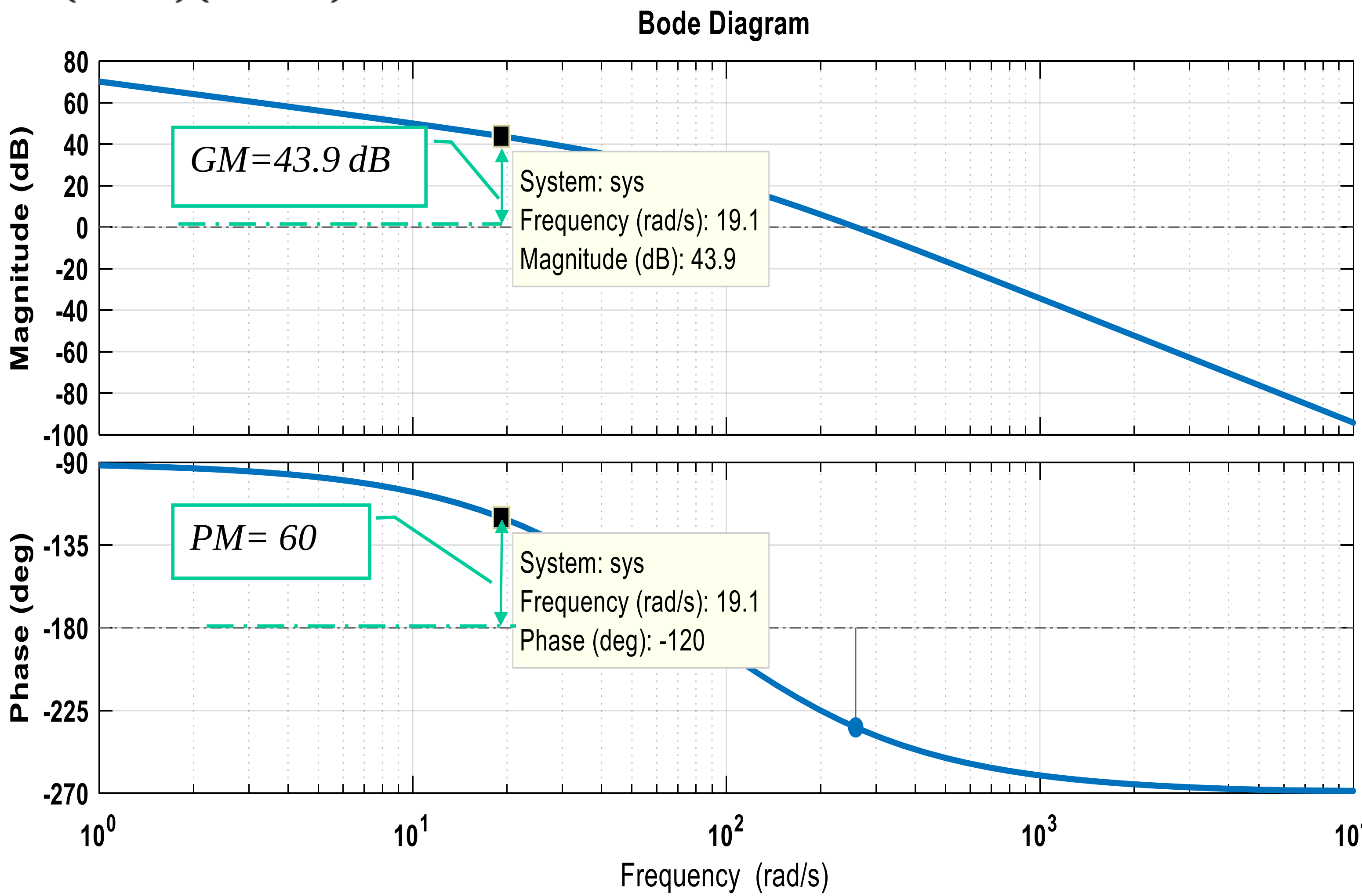
$$\frac{1942000K}{(50)(120)} = 3236.7 \longrightarrow K = 10$$

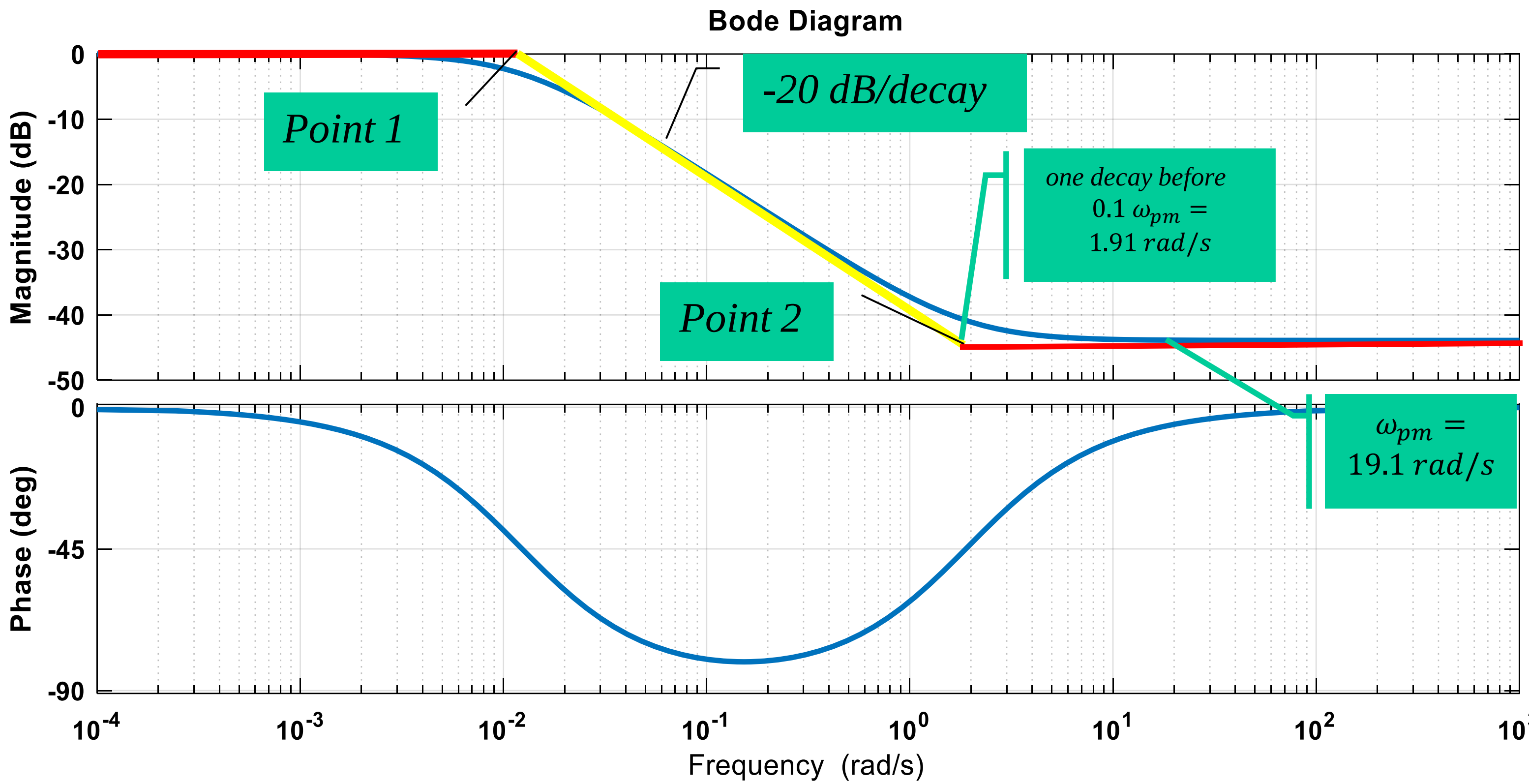
- $OS\% = 20\%$ $\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}$ =0.456

- $PM = 50$ *from the following figure.*
- *we increase 5-12 to the phase margin in order to compensate for the phase angle contribution of the lag compensator.*
- *So PM=50+10=60.*
- *Now find where the phase margin is 60. You can find it from the bode diagrams*
- *This frequency occurs at phase angle of -180+60=-120. Therefore* $\omega_{PM} = 19.1\ rad/s$.
- $M(\omega_{PM})$*=+43.9 dB.*
- *Thus the lag compensator must provide -43.9 dB attenuation at* $\omega_{PM} = 19.1\ rad/s$



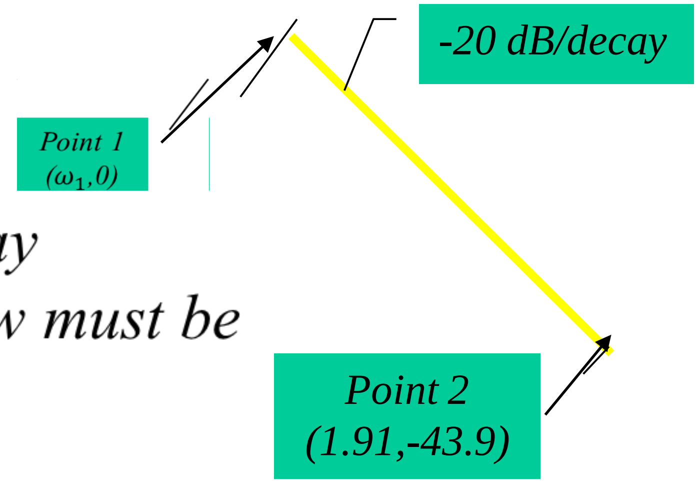
$$G(s) = \frac{19420000}{s(s+50)(s+120)}$$





- First draw the high frequency asymptote at -43.9 dB.
- Arbitrarily select the higher break frequency to be about one decade below the phase-margin frequency, or 1.91 rad/s.
- Starting at the intersection of this frequency with the lag compensator's high-frequency asymptote, draw a -20 dB/decade line until 0 dB is reached.

- at point 1: $(\omega_1, 0\ dB)$
- at point 2: (1.91,-43.9 dB)
- the slope of the yellow line is -20 dB/decay
- To compute ω_1 by using the slope the draw must be 20log (G(jw)) vs log(ω). So:



$$-20 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-20 = \frac{-43.9 - 0}{\log(1.91) - \log(\omega_1)}$$

$$-5.6207 + 20\log(\omega_1) = -43.9 \longrightarrow \frac{-38.279}{20} = \log(\omega_1)$$

$$\omega_1 = 10^{-1.914} = 0.01219$$

$$C_c = \frac{K_c(s + 1.91)}{(s + 0.01219)}$$

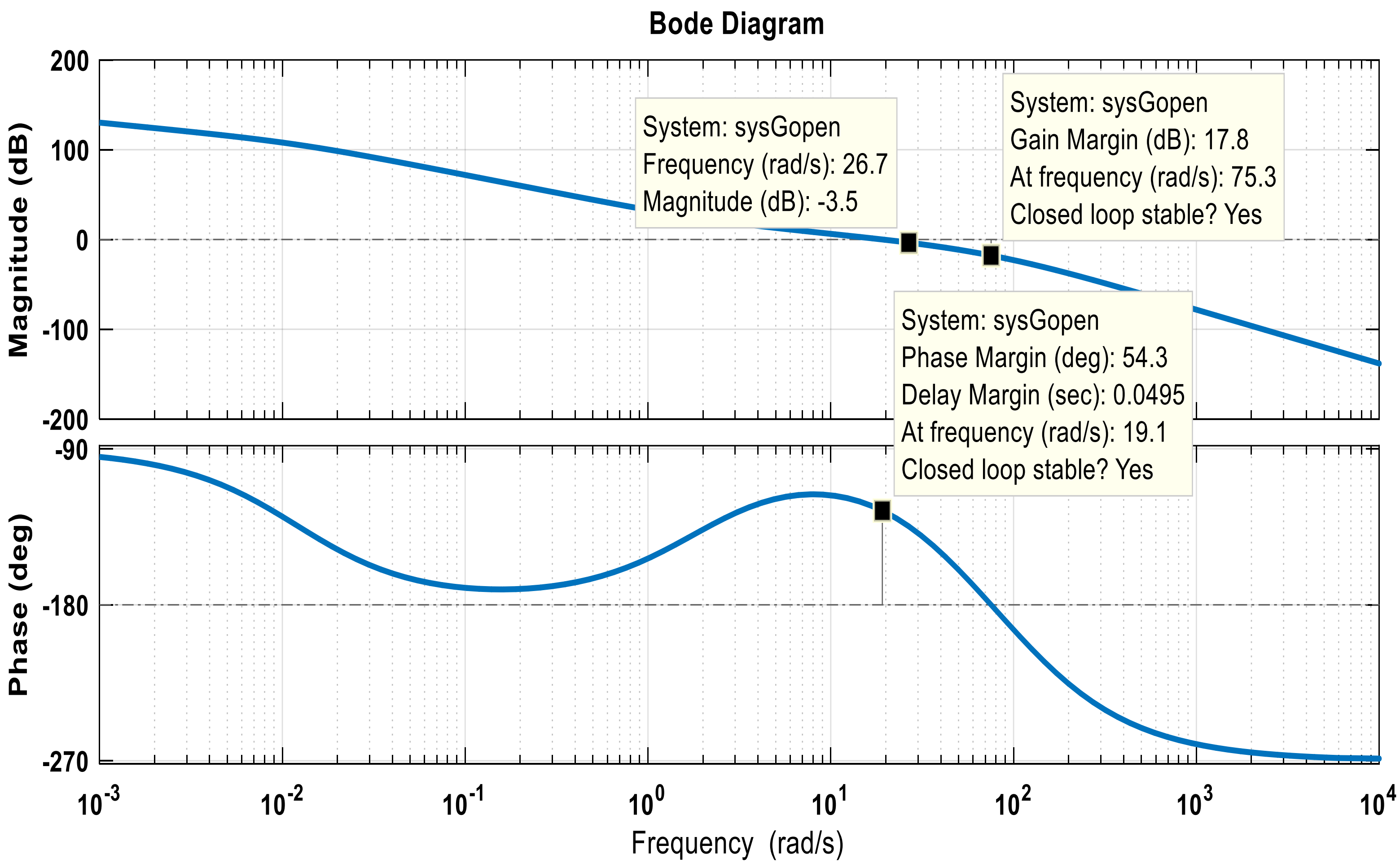
$$C_c(s) = \frac{K_c(s + 1.91)}{(s + 0.01219)}$$ ← *The dc gain for $C_c(s)$ must be unity*

$$C_c(s) = \frac{1.91K_c(\frac{s}{1.91}s + 1)}{0.01219(\frac{s}{0.01219} + 1)}$$

$$\textit{Dc gain} = \lim_{s\to 0}(C_c(s)) = 1 = \frac{1.91K_c}{0.01219}$$

$$K_c = 0.006382$$

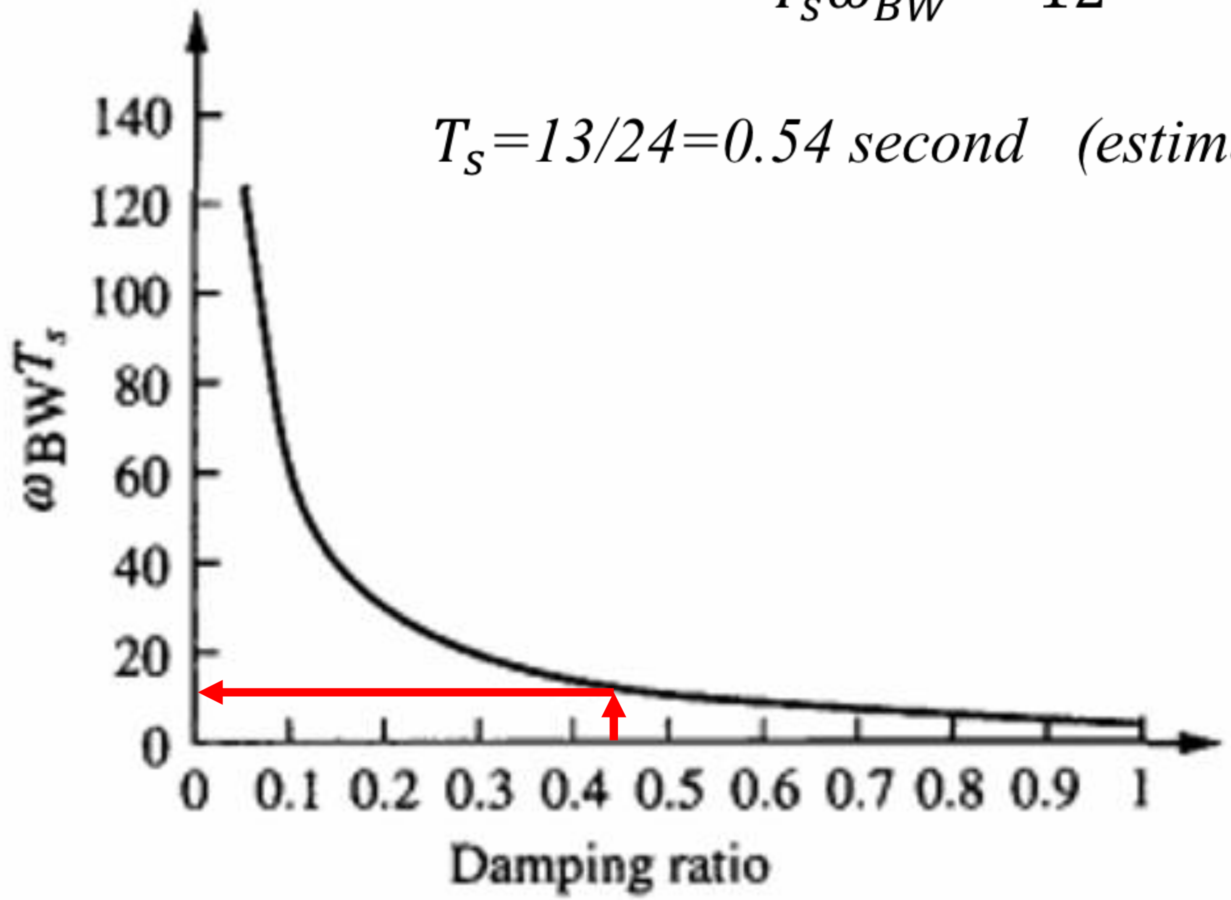
$$C_c(s) = \frac{0.006382\,(s + 1.91)}{(s + 0.01219)}$$



$$G_{open}(s) = C_c(s)G(s) = \frac{0.006382*19420000(s+1.91)}{s(s+0.01219)(s+50)(s+120)}$$

$$\omega_{BW} = 24\, rad/s$$
$$T_s\omega_{BW} = 12$$

T_s=13/24=0.54 second (estimated)



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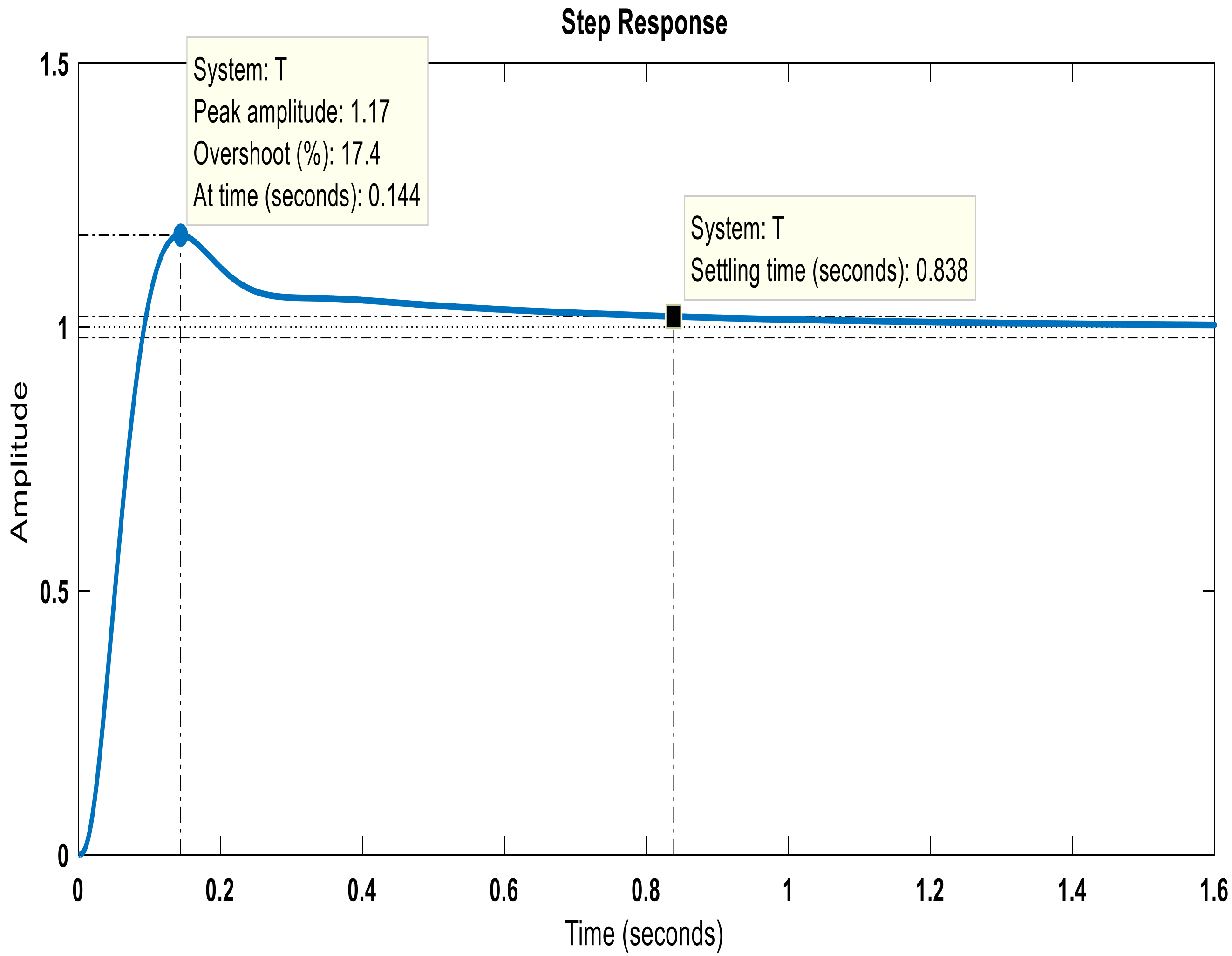
$$T = \frac{G_{open}(s)}{1+G_{open}(s)H(s)} = \frac{1.2394*10^5(s+1.91)}{(s+131.4)(s+2.112)(s^2+36.4s+852.9)}$$



Matlab Commands:

Writing a transfer functions: $G(s)= \frac{5(s+1)}{s(s+10)(s+10)}$

- *G=tf([num],[den])*

G=tf([5 5],[1 20 100 0])

- *G=zpk([zeros],[poles],gain)*

G=zpk([-1],[0 ,-10, -10],5)

Bode plot for the open loop system:

- *bode(G)*

Calculate the closed transfer function $T(s)=\frac{G(s)}{1+G(s)H(s)}$

- *T=feedback(G,H)*

Bode plot for the closed loop system:

bode(T)