Leen Sabri

Q6 $a_n = \frac{2^n - 1}{2^n}$

$a_1 = \frac{2^1-1}{2^1} = \frac{1}{2}$,, $a_2 = \frac{2^2-1}{2^2} = \frac{4-1}{4} = \frac{3}{4}$,, $a_3 = \frac{2^3-1}{2^3} = \frac{8-1}{8} = \frac{7}{8}$,,, $a_4 = \frac{2^4-1}{2^4} = \frac{15}{16}$

Q10 $a_1 = -2$, $a_{n+1} = \frac{n a_n}{n+1}$

$a_2 \Rightarrow a_{1+1} = \frac{(1) a_1}{1+1} = \frac{-2}{2} = -1$,,, $a_3 \Rightarrow a_{2+1} = \frac{(2) a_2}{2+1} = \frac{(2)(-1)}{3} = \frac{-2}{3}$,,, $a_4 \Rightarrow a_{3+1} = \frac{(3) a_3}{3+1} = \frac{(3)(\frac{-2}{3})}{4} = \frac{-2}{4} = \frac{-1}{2}$,, ...

Q22 $\underset{n=1}{2}, \underset{n=2}{6}, \underset{n=3}{10}, \underset{n=4}{14}, \underset{n=5}{18}, \dots$

$a_n = 4n - 2$, $n = 1, 2, 3 \dots$

Q26 $0, 1, 1, 2, 2, 3, 3, 4, \dots$

$a_n = [\frac{n}{2}]$, $n = 1, 2, 3, \dots$

ex : $a_1 = [\frac{1}{2}] = 0$, $a_2 = [\frac{2}{2}] = 1$, $a_3 = [\frac{3}{2}] = [1.5] = 1$

Q30 $a_n = \frac{2n+1}{1-3\sqrt{n}} \Rightarrow$ درجة البسط > درجة المقام $\therefore \lim_{n\to\infty} a_n = \infty$ $\therefore a_n$ diverge

Q35 $a_n = 1 + (-1)^n$

حسب القاعدة ④ في الدفتر :

$|x| < 1$, $-1 < x < 1$ $\xrightarrow{\text{Then}} \lim_{n\to\infty} x^n = 0$ $\xrightarrow{\text{لكن}}$ $x = -1 \rightarrow |-1| = 1$ $\therefore$ لم يتحقق الشرط $\therefore \lim_{n\to\infty} (-1)^n = \infty \Rightarrow 1 + \infty = \infty$ $\therefore a_n$ diverge

Q36 $a_n = (-1)^n(1 - \frac{1}{n}) \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} (-1)^n (1 - \frac{1}{n}) = \lim_{n\to\infty} (-1)^n \lim (1 - \frac{1}{n})$

أوجدناها في السؤال السابق Q35 وتساوي ∞

$\lim (1 - \frac{1}{n}) = \lim 1 - \lim \frac{1}{n} = 1 - 0 = 1$

$\infty \times 1 = \infty$ $\therefore a_n$ is a diverge

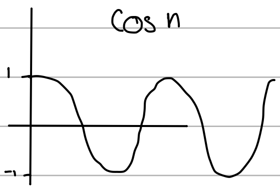
Q41 $a_n = \sqrt{\frac{2n}{n+1}} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty} \sqrt{\frac{2n}{n+1}} = \sqrt{\lim_{n\to\infty} \frac{2n}{n+1}} = \sqrt{\frac{\lim 2n}{\lim n+1}} \rightarrow \frac{\infty}{\infty} \xrightarrow{\text{لوبيتال}} \sqrt{\frac{2}{1}} = \sqrt{2} \Rightarrow$ converge

Q44 $a_n = n\pi \cos(n\pi) \Rightarrow \lim_{n\to\infty} n\pi \cos(n\pi) \Rightarrow$

قاعدة : $\lim_{n\to\infty} \cos n$ مترددة بين ال 1 و -1

$\Rightarrow \lim_{n\to\infty} \underset{\infty}{n\pi} \underset{\text{تساوي } -1 \text{ أو } 1}{(-1)^n}$ $\therefore = \infty$ or $-\infty$ $\therefore$ diverge

:: بنحولها $(-1)^n$



Q46 $a_n = \frac{\sin^2 n}{2^n} \Rightarrow \lim_{n\to\infty} \frac{\sin^2 n}{2^n} \Rightarrow$ by sandwich theorem

$-1 \leq \sin n \leq 1 \xrightarrow{\text{نربع}} 0 \leq \sin^2 n \leq 1 \rightarrow 0 \leq \frac{\sin^2 n}{2^n} \leq \frac{1}{2^n}$

$\Rightarrow \underbrace{\lim_{n\to\infty} 0}_{0} \leq \lim_{n\to\infty} \frac{\sin^2 n}{2^n} \leq \underbrace{\lim_{n\to\infty} \frac{1}{2^n}}_{0}$ $\therefore \lim_{n\to\infty} \frac{\sin^2 n}{2^n} = 0$

Q48 $a_n = \frac{3^n}{n^3} \Rightarrow \lim_{n\to\infty} \frac{3^n}{n^3} \rightarrow \frac{\infty}{\infty} \xrightarrow{\text{لوبيتال}} \lim_{n\to\infty} \frac{3^n \ln 3}{3n^2} \rightarrow \frac{\infty}{\infty} \xrightarrow{\text{لوبيتال}} \lim_{n\to\infty} \frac{3^n (\ln 3)^2}{6n} \rightarrow \frac{\infty}{\infty} \xrightarrow{\text{لوبيتال}} \lim_{n\to\infty} \frac{3^n (\ln 3)^3}{6} = \infty$ $\therefore$ diverge

Q50 $a_n = \frac{\ln n}{\ln 2n} \Rightarrow \lim_{n\to\infty} a_n = \frac{\infty}{\infty} \xrightarrow{\text{لوبيتال}} \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{2}{2n}} = \lim_{n\to\infty} 1 = 1$ $\therefore$ converge

Q54 $a_n = (1 - \frac{1}{n})^n \Rightarrow \lim_{n\to\infty} (1 - \frac{1}{n})^n \xrightarrow{\text{حسب القاعدة ⑤ في}} e^{-1}$ $\therefore$ converge

Q58 $a_n = (n+4)^{1/(n+4)} \Rightarrow$ let $(n+4) = x \Rightarrow a_n = x^{1/x} \xrightarrow{\text{حسب القاعدة ② في}} \lim_{n\to\infty} x^{1/x} = 1$ $\therefore$ converge

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Q60 $a_n = \ln n - \ln(n+1) = \ln\left(\frac{n}{n+1}\right) \Rightarrow \lim_{n\to\infty} \ln\left(\frac{n}{n+1}\right) = \ln \lim_{n\to\infty}\left(\frac{n}{n+1}\right) = \frac{\infty}{\infty}$ —لوبيتال→ $\ln \lim_{n\to\infty}(1) = \ln 1 = 0 \therefore$ converge

Q63 $a_n = \frac{n!}{n^n} \Rightarrow \lim_{n\to\infty} \frac{n!}{n^n} \leq \lim_{n\to\infty} \frac{1}{n} \Rightarrow \lim_{n\to\infty} \frac{1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n}{n \cdot n \cdot n \cdots n} < \underbrace{\lim_{n\to\infty} \frac{1}{n}}_{=\text{zero}}$,,, and $\frac{n!}{n^n} \geq 0$ → [illegible]

$\therefore \lim_{n\to\infty} \frac{n!}{n^n} = 0 \Rightarrow$ converge

Q70 $a_n = \left(\frac{n}{n+1}\right)^n \Rightarrow \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty} e^{\ln\left(\frac{n}{n+1}\right)^n} = \lim_{n\to\infty} e^{n\ln\left(\frac{n}{n+1}\right)} = \lim_{n\to\infty} e^{\frac{\ln\left(\frac{n}{n+1}\right)}{\frac{1}{n}}} = \lim_{n\to\infty} e^{\frac{\ln(n)-\ln(n+1)}{\frac{1}{n}}} = e^{\lim_{n\to\infty}\frac{\ln(n)-\ln(n+1)}{\frac{1}{n}}} \Rightarrow \frac{\infty}{\infty}$ —لوبيتال→ $e^{\lim_{n\to\infty}\frac{\frac{1}{n}-\frac{1}{n+1}}{-\frac{1}{n^2}}} = e^{\lim_{n\to\infty}\left(\frac{-n^2}{n(n+1)}\right)} = e^{-\lim_{n\to\infty}\left(\frac{n}{n+1}\right)} \Rightarrow \frac{\infty}{\infty}$ —لوبيتال→

$= e^{-\lim_{n\to\infty}(1)} = e^{-1} \Rightarrow$ converge

Q72 $a_n = \left(1-\frac{1}{n^2}\right)^n \Rightarrow \lim_{n\to\infty}\left(1-\frac{1}{n^2}\right)^n = \lim_{n\to\infty} e^{\ln\left(1-\frac{1}{n^2}\right)^n} = \lim_{n\to\infty} e^{n\ln\left(1-\frac{1}{n^2}\right)} = \lim_{n\to\infty} e^{\frac{\boxed{\ln\left(1-\frac{1}{n^2}\right)}}{\frac{1}{n}}} = e^{\lim_{n\to\infty}\frac{\boxed{\ln\left(1-\frac{1}{n^2}\right)}}{\frac{1}{n}}} \Rightarrow \ln\left(1-\frac{1}{\infty}\right) = \ln(1-0) = \ln 1 = 0$ $\boxed{0 = \infty \text{ [illegible]}}$ $\Rightarrow \frac{0}{0}$ —لوبيتال→ $e^{\lim_{n\to\infty}\frac{\frac{2}{n^3}/\left(1-\frac{1}{n^2}\right)}{\left(-\frac{1}{n^2}\right)}} = e^{\lim_{n\to\infty}\frac{\frac{2}{n^3}}{\left(1-\frac{1}{n^2}\right)\left(-\frac{1}{n^2}\right)}} = e^{\lim_{n\to\infty}\frac{\frac{1}{n^2}\left(\frac{2}{n}\right)}{\left(1-\frac{1}{n^2}\right)\left(-\frac{1}{n^2}\right)}} = e^{\lim_{n\to\infty}\frac{2}{-n\left(1-\frac{1}{n^2}\right)}}$ توحيد مقامات

$= e^{\lim_{n\to\infty}\frac{2}{-n\left(\frac{n^2-1}{n^2}\right)}} = e^{\lim_{n\to\infty}\frac{-2}{\frac{n^2-1}{n}}} = e^{\lim_{n\to\infty}\frac{-2n}{n^2-1}} \Rightarrow \frac{\infty}{\infty}$ —لوبيتال→ $= e^{\lim_{n\to\infty}\frac{-2}{2n}} = e^{\lim_{n\to\infty}\frac{-1}{n}} = e^0 = 1 \therefore$ converge

Q76 $a_n = \sinh(\ln n) = \frac{e^{\ln n} - e^{-\ln n}}{2} = \frac{e^{\ln n} - e^{\ln\left(\frac{1}{n}\right)}}{2} = \frac{n-\frac{1}{n}}{2} \Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{n-\frac{1}{n}}{2} = \frac{1}{2}\lim_{n\to\infty} \underset{\infty}{n} - \underset{0}{\frac{1}{n}} = \infty \therefore$ diverge

Q82 $a_n = \frac{1}{\sqrt{n}}\tan^{-1} n \Rightarrow \lim_{n\to\infty}\frac{1}{\sqrt{n}}\tan^{-1} n = 0 \cdot \frac{\pi}{2} = 0 \therefore$ converge

Q111 $a_n = \frac{3n+1}{n+1} \Rightarrow$

$a_1 = \frac{3+1}{2} = 2$,,, $a_2 = \frac{6+1}{3} = \frac{7}{3}$,,, $a_3 = \frac{9+1}{4} = \frac{10}{4} = \frac{5}{2}$,,, $a_4 = \frac{12+1}{5} = \frac{13}{15}$

$\frac{4}{2}, \frac{7}{3}, \frac{10}{4}, \frac{13}{5}$

$M = \frac{4}{2}$
$m = \frac{4}{2}, 0, -1, -2 \ldots$ $\Big\}$ $\therefore a_n$ is bounded

The sequence is nondecreasing (يعني تتزايد) $\therefore$ The sequence is monotonic

Q92 Since a_n converges $\Rightarrow \lim_{n\to\infty} = L \Rightarrow \lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \frac{a_n+6}{a_n+2} \Rightarrow L = \frac{L+6}{L+2} \Rightarrow L(L+2) = L+6 \Rightarrow L^2+2L = L+6 \Rightarrow L^2+L-6=0 \Rightarrow (L+3)(L-2)=0$ $L=-3,2$

إذن $0 < s_n$ لما $2 \leq n$ $\therefore$ $\therefore 0 < s_n$ $2 = L$