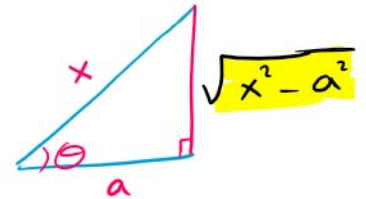
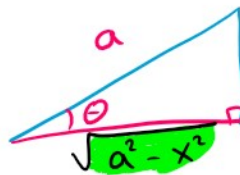
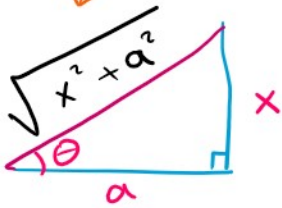


Exp How to evaluate $\int \sqrt{9+x^2} dx$?

$\int \sqrt{x^2 - 9} dx$?

$\int \sqrt{9-x^2} dx$?

$\int \frac{dx}{9+x^2}$?



$x = a \tan \theta$ ✓
 $dx = a \sec^2 \theta d\theta$
 $\tan \theta = \frac{x}{a} \Rightarrow \theta = \tan^{-1} \frac{x}{a}$

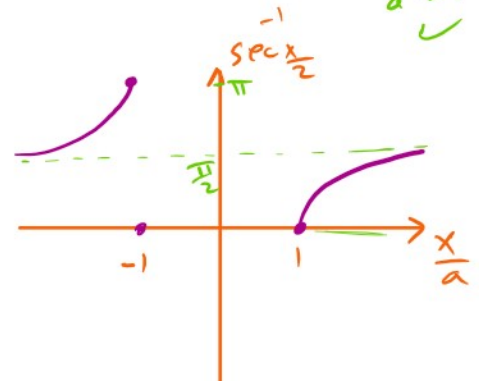
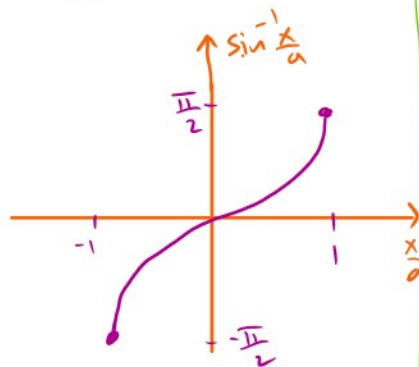
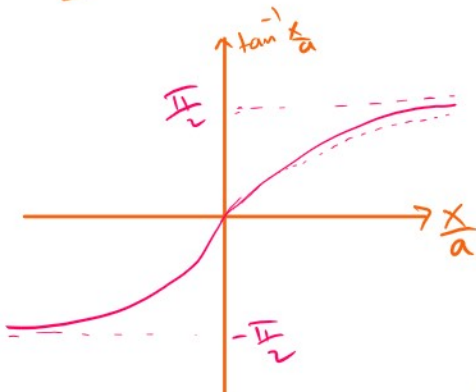
$x = a \sin \theta$ ✓
 $dx = a \cos \theta d\theta$
 $\sin \theta = \frac{x}{a} \Rightarrow \theta = \sin^{-1} \frac{x}{a}$

$x = a \sec \theta$
 $dx = a \sec \theta \tan \theta d\theta$
 $\sec \theta = \frac{x}{a} \Rightarrow \theta = \sec^{-1} \frac{x}{a}$

$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

$0 \leq \theta < \pi$ ✓



$\sqrt{x^2 + a^2} = \sqrt{a^2 \tan^2 \theta + a^2}$
 $= \sqrt{a^2 (\tan^2 \theta + 1)}$

$\sqrt{a^2 - x^2} = \sqrt{a^2 - a^2 \sin^2 \theta}$
 $= \sqrt{a^2 (1 - \sin^2 \theta)}$

$\sqrt{x^2 - a^2} = \sqrt{a^2 \sec^2 \theta - a^2}$

$$\begin{aligned}
 &= \sqrt{a^2 (\tan^2 \theta + 1)} \\
 &= a \sqrt{\sec^2 \theta} \\
 &= a |\sec \theta| \\
 &= a \sec \theta
 \end{aligned}$$

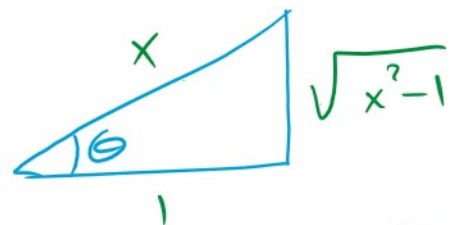
✓

$$\begin{aligned}
 &= \sqrt{a^2 (1 - \sin^2 \theta)} \\
 &= a \sqrt{\cos^2 \theta} \\
 &= a |\cos \theta| \\
 &= a \cos \theta
 \end{aligned}$$

✓

$$\begin{aligned}
 \sqrt{x^2 - a^2} &= \sqrt{a^2 \sec^2 \theta - a^2} \\
 &= \sqrt{a^2 (\sec^2 \theta - 1)} \\
 &= a \sqrt{\tan^2 \theta} \\
 &= a |\tan \theta| \\
 &= a \tan \theta
 \end{aligned}$$

Exp 14 $\int \frac{2 dx}{\sqrt{x^2 - 1}}, \quad x > 1$



$a = 1$

$$\begin{aligned}
 x &= a \sec \theta \\
 &= \sec \theta
 \end{aligned}$$

$$dx = \sec \theta \tan \theta d\theta$$

$$\sqrt{x^2 - 1} = a \tan \theta = \tan \theta$$

$$\int \frac{2 \cancel{\sec \theta} \cancel{\tan \theta} d\theta}{\cancel{\sec^2 \theta} \cancel{\tan \theta}} = 2 \int \frac{d\theta}{\sec^2 \theta} = 2 \int \cos^2 \theta d\theta$$

$$= 2 \int \left(\frac{1 + \cos 2\theta}{2} \right) d\theta = \int (1 + \cos 2\theta) d\theta$$

$$= \theta + \frac{\sin 2\theta}{2} + C$$

$$= \sec^{-1} x + \frac{2 \sin \theta \cos \theta}{2} + C$$

$$= \sec^{-1} x + \frac{\sqrt{x^2-1}}{x} \cdot \frac{1}{x} + C$$

(24) $\int_0^1 \frac{dx}{(4-x^2)^{\frac{3}{2}}} = \int_0^1 \frac{dx}{(\sqrt{4-x^2})^3}$

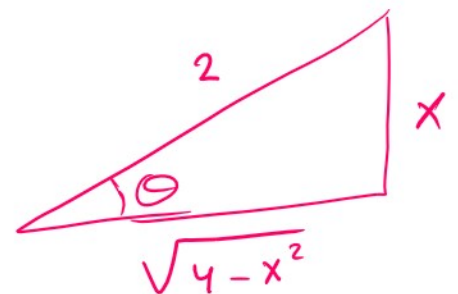
$$x = a \sin \theta$$

$$= 2 \sin \theta$$

$$dx = 2 \cos \theta d\theta$$

$$\sqrt{4-x^2} = a \cos \theta = 2 \cos \theta$$

$$\int_0^{\frac{\pi}{6}} \frac{2 \cos \theta d\theta}{(2 \cos \theta)^3}$$



$$x = 2 \sin \theta$$

$$\theta = \sin^{-1} \frac{x}{2}$$

$$x=0 \Rightarrow \theta = \sin^{-1} 0 = 0$$

$$x=1 \Rightarrow \theta = \sin^{-1} \frac{1}{2} = \frac{\pi}{6}$$

$$\int_0^{\frac{\pi}{6}} \frac{2 \cos \theta}{4 \cos^3 \theta} d\theta$$

$$= \frac{1}{4} \int_0^{\frac{\pi}{6}} \sec^2 \theta d\theta$$

$$= \frac{1}{4} \tan \theta \Big|_0^{\frac{\pi}{6}}$$

$$= \frac{1}{4} \left[\tan \frac{\pi}{6} - \tan 0 \right]$$

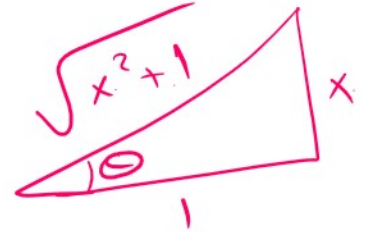
$$= \frac{1}{4} \cdot \frac{1}{\sqrt{3}} = \frac{1}{4\sqrt{3}}$$

$$= \frac{1}{4} - \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{4\sqrt{3}}$$

18
Exp

$$\int \frac{dx}{x^2 \sqrt{x^2+1}}$$

$a = \sqrt{1} = 1$



$$x = a \tan \theta = \tan \theta$$

$$dx = \sec^2 \theta d\theta$$

$$\sqrt{x^2+1} = a \sec \theta = \sec \theta$$

$$\int \frac{\sec^2 \theta d\theta}{\tan^2 \theta \sec \theta} = \int \frac{\frac{1}{\cos \theta} d\theta}{\frac{\sin^2 \theta}{\cos \theta}}$$

$$= \int \frac{\cos \theta d\theta}{\sin^2 \theta}$$

$$u = \sin \theta$$

$$du = \cos \theta d\theta$$

$$= \int \frac{du}{u^2} = \int u^{-2} du = \frac{u^{-1}}{-1} + C$$

$$= -\frac{1}{\sin \theta} + C$$

$$= -\csc \theta + C$$

$$= -\frac{\sqrt{x^2+1}}{x} + C$$

⑩ Exp $\int \frac{5 dx}{\sqrt{25x^2 - 9}} = \int \frac{\cancel{5} dx}{\sqrt{\cancel{25}(x^2 - \frac{9}{25})}} = \int \frac{\cancel{dx}}{\sqrt{x^2 - \frac{9}{25}}}$

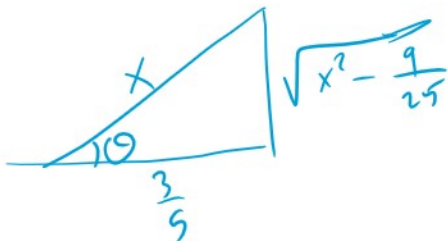
$a = \sqrt{\frac{9}{25}} = \frac{3}{5}$

$x = a \sec \theta$
 $= \frac{3}{5} \sec \theta$

$dx = \frac{3}{5} \sec \theta \tan \theta d\theta$
 $\sqrt{x^2 - \frac{9}{25}} = a \tan \theta = \frac{3}{5} \tan \theta$

$\int \frac{\cancel{\frac{3}{5}} \sec \theta \tan \theta d\theta}{\cancel{\frac{3}{5}} \tan \theta} = \int \sec \theta d\theta = \ln |\sec \theta + \tan \theta| + C$

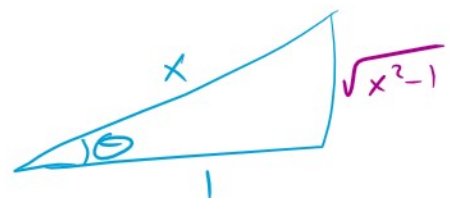
$= \ln \left| \frac{x}{\frac{3}{5}} + \frac{\sqrt{x^2 - \frac{9}{25}}}{\frac{3}{5}} \right| + C$



Exp 26 $\int \frac{x^2 dx}{(x^2 - 1)^{5/2}}, x > 1$

$\int \frac{x^2 dx}{(\sqrt{x^2 - 1})^5}$

$a = \sqrt{1} = 1$



$x = a \sec \theta = \sec \theta$
 $dx = \sec \theta \tan \theta d\theta$
 $\sqrt{x^2 - 1} = a \tan \theta = \tan \theta$

$\int \frac{\sec^2 \theta \sec \theta \tan \theta d\theta}{(\tan \theta)^4}$

$\int \frac{1}{\tan^3 \theta} \cdot \frac{\cos \theta}{\sin \theta} d\theta = \int \frac{\cos \theta d\theta}{\sin^4 \theta}$

$$\int \frac{\sec^3 \theta}{\tan^4 \theta} d\theta = \int \frac{1}{\cos^3 \theta} \frac{\cos^4 \theta}{\sin^4 \theta} d\theta = \int \frac{\cos \theta d\theta}{\sin^4 \theta}$$

$u = \sin \theta$
 $du = \cos \theta d\theta$

$$\int \frac{du}{u^4} = \int u^{-4} du = \frac{u^{-3}}{-3} + C$$

$$= -\frac{1}{3} \frac{1}{u^3} + C = -\frac{1}{3} \frac{1}{\sin^3 \theta} + C$$

$$= -\frac{1}{3} \csc^3 \theta + C$$

$$= -\frac{1}{3} \left(\frac{x}{\sqrt{x^2-1}} \right)^3 + C$$

