

2) $(1+x)^{\frac{1}{3}} = 1 + \sum_{k=1}^{\infty} \left(\frac{1}{3}\right)_k x^k$

$$= 1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3 - \dots$$

110.101

25) $f(x) = \int_0^x \sin t^2 dt, [0,1] \quad E < 10^{-3}$

$$= \int_0^x \left(x^2 - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \right) dx$$

$$= \int_0^x \left(x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \dots \right) dx$$

$$= \frac{x^3}{3!} - \frac{x^7}{7 \cdot 3!} + \frac{x^{11}}{11 \cdot 5!} - \frac{x^{15}}{15 \cdot 7!} + \dots$$

$$\approx \frac{x^3}{3!} - \frac{x^7}{7 \cdot 3!} + \frac{x^{11}}{11 \cdot 5!}$$

$$|E| < \frac{1}{15 \cdot 7!} \approx 0.000013$$

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10) $\frac{x}{\sqrt[3]{1+x}}$

$$= x(1+x)^{-\frac{1}{3}}$$

$$= x \left[1 + \sum_{k=1}^{\infty} \left(\frac{-1}{3}\right)_k x^k \right]$$

$$= x \left[1 + \frac{-1}{3}x + \frac{2}{9}x^2 - \frac{14}{81}x^3 + \dots \right]$$

$$= x - \frac{x^2}{3} + \frac{2x^3}{9} - \frac{14x^4}{81} + \dots$$

$$15] \int_0^{0.2} \frac{e^{-x} - 1}{x} dx$$

$$16] \int_0^{0.2} \frac{1}{x} \left[1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \dots - 1 \right] dx$$

$$\int_0^{0.2} \left[-1 + \frac{x}{2} - \frac{x^2}{6} + \frac{x^3}{24} - \dots \right] dx$$

$$= -x + \frac{x^2}{4} - \frac{x^3}{18} + \frac{x^4}{96}$$

$$|E| \leq \frac{(0.2)^4}{96} \approx 0.00002$$

$$\approx -x + \frac{x^2}{4} - \frac{x^3}{18} + \dots \Big|_0^{0.2} \approx -0.19044$$

$$26] f(x) = \int_0^x t^2 e^{-t^2} dt, [0, 1] \quad E < 10^{-3}$$

$$e^t = \sum_{n=0}^{\infty} \frac{t^n}{n!} \rightarrow e^{-t^2} = \sum_{n=0}^{\infty} \frac{(-t^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n}}{n!}$$

$$t^2 e^{-t^2} = t^2 \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n}}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+2}}{n!} = t^2 - t^4 + \frac{t^6}{2!} - \frac{t^8}{3!} + \frac{t^{10}}{4!} - \frac{t^{12}}{5!} + \dots$$

$$\int_0^x t^2 e^{-t^2} dt$$

$$= \frac{x^3}{3} - \frac{x^5}{5} + \frac{x^7}{7 \cdot 2!} - \frac{x^9}{9 \cdot 3!} + \frac{x^{11}}{11 \cdot 4!} - \frac{x^{13}}{13 \cdot 5!}$$

$$E < \left| \frac{x^{13}}{13 \cdot 5!} \right| = \frac{x^{13}}{13 \cdot 5!} < \frac{1}{13 \cdot 5!} = 6.41 \times 10^{-4} < 10^{-3}$$

$$f(x) \approx \frac{x^3}{3} - \frac{x^5}{5} + \frac{x^7}{7 \cdot 2!} - \frac{x^9}{9 \cdot 3!} + \frac{x^{11}}{11 \cdot 4!}$$

$$30) \lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x}$$

$$\lim_{x \rightarrow 0} \frac{1}{x} \left[1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots - 1 + x - \frac{x^2}{2!} + \frac{x^3}{3!} - \dots \right]$$

$$\lim_{x \rightarrow 0} \frac{1}{x} \left[2x + \frac{2x^3}{3!} + \dots \right]$$

$$\lim_{x \rightarrow 0} \left[2 + \frac{2x^2}{3!} + \dots \right] = 2$$

$$33) \lim_{y \rightarrow 0} \frac{y - \tan^{-1} y}{y^3}$$

$$\lim_{y \rightarrow 0} \frac{1}{y^3} \left[y - \left[y - \frac{y^3}{3} + \frac{y^5}{5} - \frac{y^7}{7} + \dots \right] \right]$$

$$\lim_{y \rightarrow 0} \frac{1}{y^3} \left[y - y + \frac{y^3}{3} - \frac{y^5}{5} + \frac{y^7}{7} - \dots \right]$$

$$\lim_{y \rightarrow 0} \left[\frac{1}{3} - \frac{y^2}{5} + \frac{y^4}{7} - \dots \right] = \frac{1}{3}$$