1.6 Applications of functions in Business and Economics

Assume a firm produces x units and sells each unit with price p. Then:

- Total Revenue from sale x units = (price per unit)(number of units)

$$TR(x) = px$$

- Total cost of production and sale = $\underbrace{\text{variable cost}}_{VC}$ + $\underbrace{\text{fixed costs}}_{FC}$

$$TC(x) = mx + b$$

m: marginal cost $\overline{MC}$ ← is the cost of producing an additional unit at any level of production

→ slope of the line

b: y-intercept = C(0)

- Total Profit from sale of x units = Total Revenue - Total Cost

$$\pi(x) = TP(x) = px - (mx + b)$$

$$= px - mx - b$$

$$= (p - m)x - b$$

- The point where the total revenue line crosses the total cost line is called the Break-Even point. That is the Break-Even point x makes the total profit zero:

$$P(x) = 0 \iff (p-m)x - b = 0$$

$$(p-m)x = b$$

$$\boxed{x = \frac{b}{p-m}}$$

Exp Suppose that when a company produes its product, the fixed costs are \$ 12500 and the variable cost per item is \$ 75. Assume x represents the number of units. Find

1. total cost

$FC = 12500 = b$

$$C(x) = mx + b$$
$$= 75x + 12{,}500$$

$\overline{MC} = m = 75$

2. Are $FC = C(0)$?

Yes since $C(0) = 75(0) + 12{,}500 = 0 + 12500 = 12500 = b$

so $C(0) = b$ which is the y-intercept

3. If the company sells its product for \$ 175. Find

a) total revenue

$p = 175 = \overline{MR}$

$$R(x) = px = 175x$$

b) total revenue if 100 units are sold

$$R(100) = 175(100) = 17{,}500 \text{ dollars}$$

c) total profit

$$P(x) = R(x) - C(x)$$
$$= (p-m)x - b$$
$$= (175-75)x - 12{,}500$$
$$= 100x - 12{,}500$$

$\overline{MP} = p - m$

d) the break-even point

Find x such that $P(x^*) = 0 \iff x = \dfrac{b}{p-m}$

$$= \frac{12{,}500}{175-75}$$

$$x = \frac{12{,}500}{100} = 125$$

note that we can find the break-even point using the idea of g

e) the marginal revenue $\overline{MR}$

" " cost $\overline{MC}$

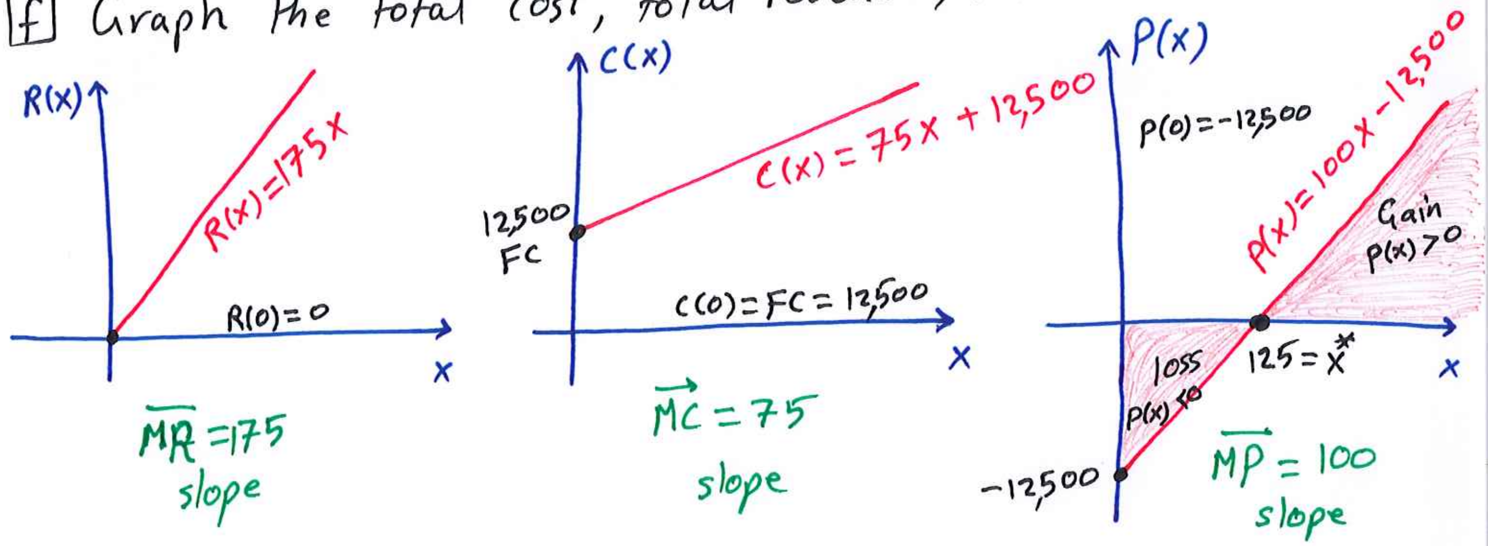
" " profit $\overline{MP}$

$\overline{MR} = p = 175$

$\overline{MC} = m = 75$

$\overline{MP} = p - m = 175 - 75 = 100$

f) Graph the total cost, total revenue, total profit



g) the intersection of the two lines R(X) and C(X)

$R(X) = 175X$

$C(X) = 75X + 12{,}500$

The intersection occurs when

$R(X) = C(X)$

$75X + 12{,}500 = 175X$

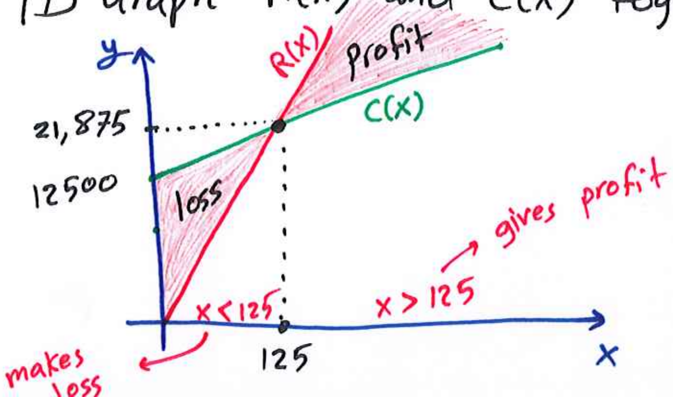
$12{,}500 = 100X$

$X = \frac{12{,}500}{100} = 125$

h) what does the intersection point in g) means?

The company will make zero profit if it produces X=125 units

i) Graph R(X) and C(X) together



- The company makes profit only if $R(X) > C(X)$
- This happens when the company produces more than the break-even level
- If $R(X) < C(X) \Rightarrow$ there is loss

j) Find the revenue and cost at the break-even

$R(x) = px = 175x \quad \Rightarrow R(x^*) = R(125) = 175(125)$

$= 21,875$

$C(x) = 75x + 12,500 \quad \Rightarrow C(x^*) = C(125)$

$= 75(125) + 12,500$

$= 9,375 + 12,500$

$= 21,875$

At the break-even point $\Rightarrow$ revenue = cost

$\Rightarrow$ profit = 0

The law of demand: the quantity demanded increases as price decreases

and " " " decreases " " increases

The law of supply: the quantity supplied for sale increases as the price of product increases

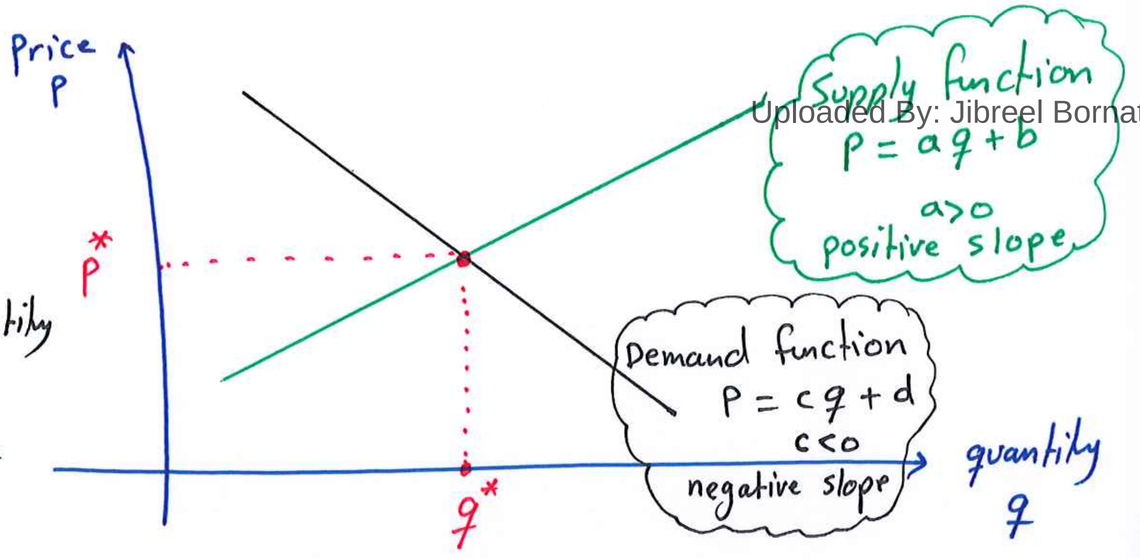
and " " " " " decreases " " " decreases

Market Equilibrium occurs when the quantity demanded for a commodity equals to the quantity supplied

(q^*, p^*)

q^*: Equilibrium quantity

p^*: Equilibrium price



Exp Find the market equilibrium point for the following demand and supply functions and graph both functions

Demand: $p = -3q + 36$

Supply: $p = 4q + 1$

Market equilibrium occurs when demand = supply

$-3q + 36 = 4q + 1$

$36 = 7q + 1$

$35 = 7q$

$\boxed{q^* = 5}$

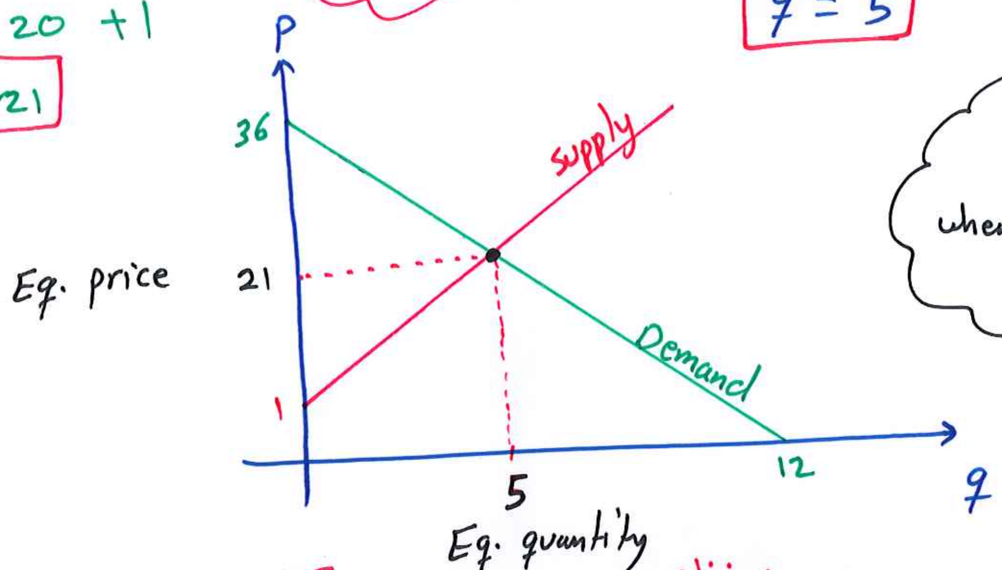
$p^* = 4q^* + 1$

$= 4(5) + 1$

$= 20 + 1$

$\boxed{p^* = 21}$

Equilibrium Point is (5, 21)



Demand
$p = -3q + 36$
when $q = 0 \Rightarrow p = 36$
$p = 0 \Rightarrow q = 12$

supply
$p = 4q + 1$
when $q = 0 \Rightarrow p = 1$
$q^* = 5 \Rightarrow p^* = 21$

Exp* A group of wholesalers (تجار جملة) will buy 50 dryers (مجففات) per month if the price is \$200 and they " " 30 " " " " " " " \$300.

- The manufacturer (مصنع) is willing to supply 20 dryers if the price is \$210 and 30 " " " " " \$230.
- Assume the demand and supply are linear. Find the Equilibrium point for this market and sketch.

Demand function passes through the points $(\underset{q_0}{50}, \underset{p_0}{200})$ and $(\underset{q_1}{30}, \underset{p_1}{300})$

$p - p_0 = m(q - q_0)$ where the slope $m = \dfrac{p_1 - p_0}{q_1 - q_0} = \dfrac{300 - 200}{30 - 50}$

$= \dfrac{100}{-20}$

$= -5$

$p - 200 = -5(q - 50)$

$p - 200 = -5q + 250$

$\boxed{p = -5q + 450}$

Supply function passes through the points $(\overset{q_0}{20}, \overset{p_0}{210})$ and $(\overset{q_1}{30}, \overset{p_1}{230})$

$p - p_0 = m(q - q_0)$ where the slope $m = \dfrac{p_1 - p_0}{q_1 - q_0} = \dfrac{230 - 210}{30 - 20}$

$p - 210 = 2(q - 20)$

$= \dfrac{20}{10}$

$p - 210 = 2q - 40$

$= 2$

$\boxed{p = 2q + 170}$

$q = 0 \Rightarrow p = 170$

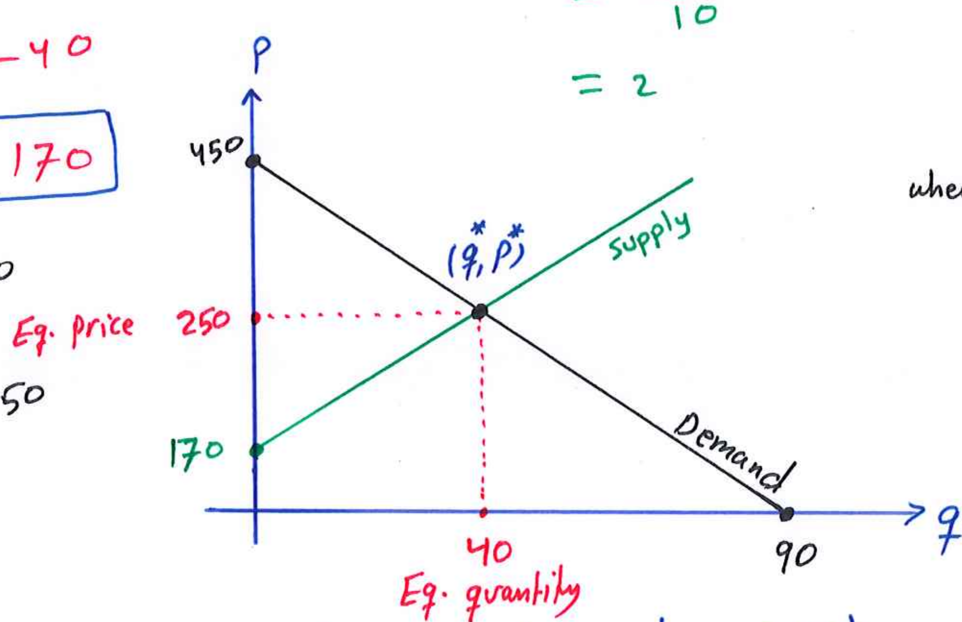
$q^* = 40 \Rightarrow p^* = 250$

demand function

$p = -5q + 450$

when $q = 0 \Rightarrow p = 450$

$p = 0 \Rightarrow q = 90$



To find Equilibrium point $\Rightarrow$ demand = supply

$-5q + 450 = 2q + 170$

$450 = 7q + 170$

$280 = 7q$

$\boxed{q^* = 40}$

$p^* = 2q^* + 170$

$= 2(40) + 170$

$= 80 + 170$

$\boxed{p^* = 250}$

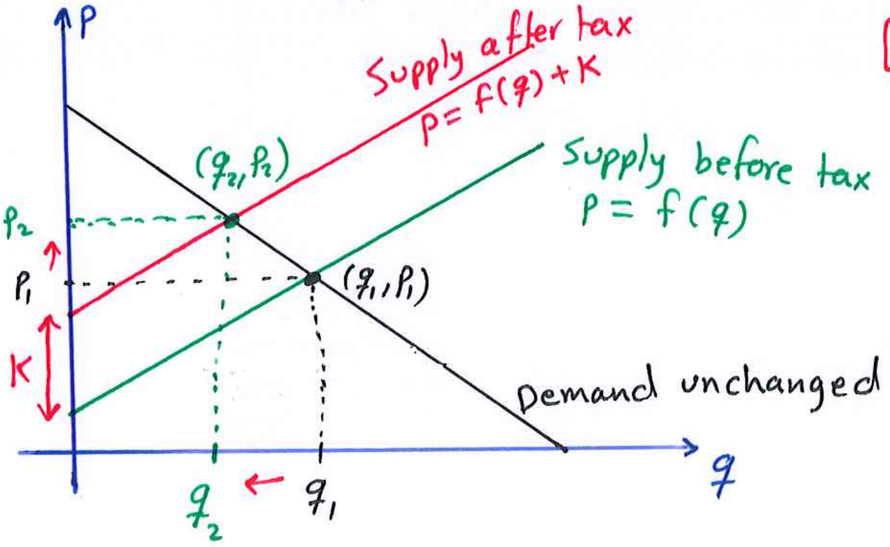
Equilibrium point is $(40, 250)$

Exp what is the effect of proposing tax K in dollars by the supplier on each unit sold?

- If the supplier imposes \$K tax on each unit sold, then this tax is passed to the consumer by adding \$K to the selling price of product
- That is, if the original supply is $p = f(q)$, then the new supply function after adding tax is $p = f(q) + K$
- Since the value of product is not changed $\Rightarrow$ demand function is unchanged
- The Equilibrium quantity will decrease while the Equilibrium price will increase increase

(q_1, p_1) is the market Equilibrium before tax

(q_2, p_2) is the market Equilibrium after tax



Exp Recall that from Exp* page 5 : Supply: $p = 2q + 170$

Demand: $p = -5q + 450$

and the Equilibrium point was $(q^*, p^*) = (40, 250)$.

Now suppose that the wholesaler is taxed \$14 per unit sold. What is the new Equilibrium point? Graph.

- New Supply is $p = 2q + 170 + 14$, $\boxed{p = 2q + 184}$ where the demand function is unchanged $\boxed{p = -5q + 450}$

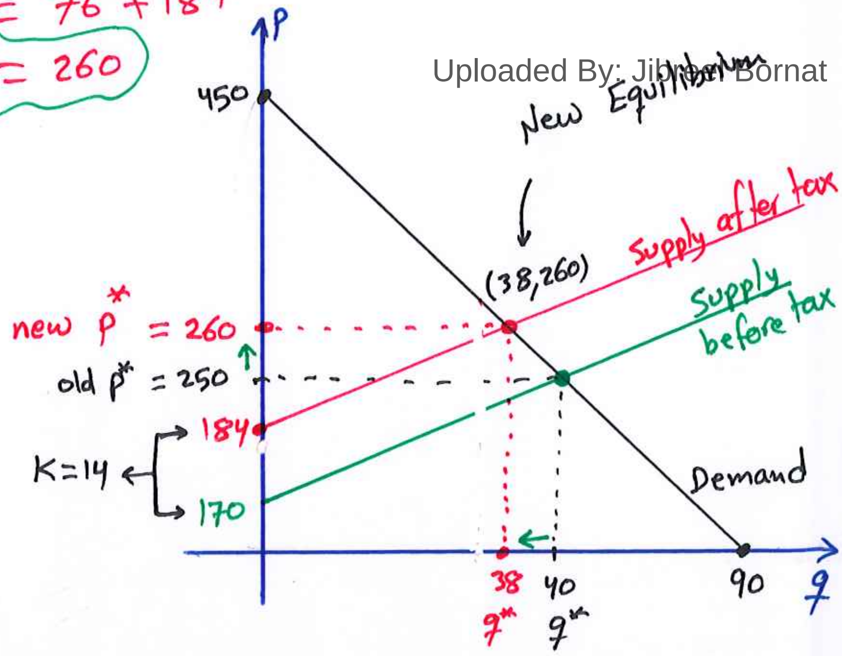
- To find the Eq. point we solve the linear system: $p = 2q + 184$, $p = -5q + 450$

$$2q + 184 = -5q + 450$$
$$7q + 184 = 450$$
$$7q = 266$$
$$q^* = 38$$

$$p^* = 2q^* + 184$$
$$= 2(38) + 184$$
$$= 76 + 184$$
$$p^* = 260$$

Equilibrium before tax is (40, 250)

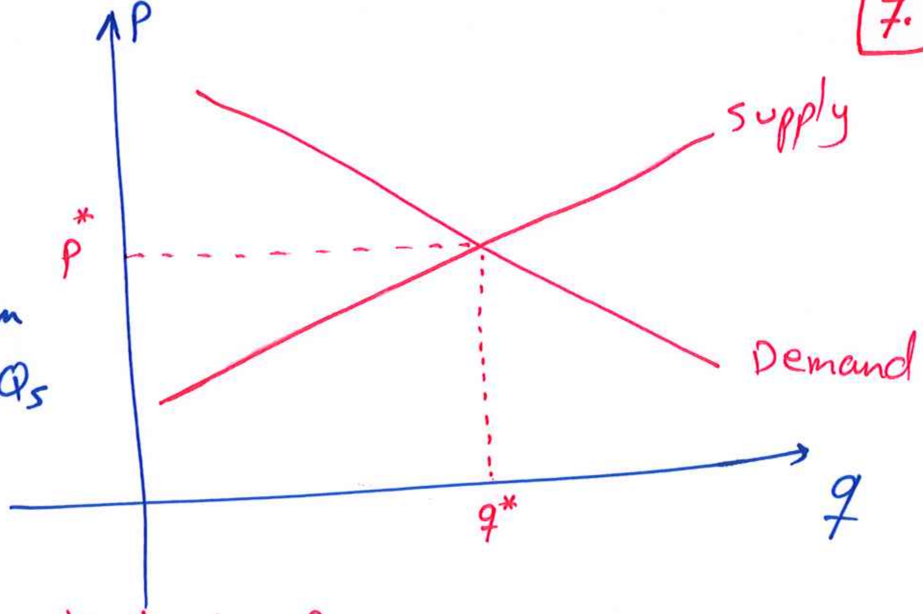
Equilibrium after tax is (38, 260)



p^* : Equilibrium Price

q^* : Equilibrium Quantity

(q^*, p^*) : Market Equilibrium occurs when $Q_D = Q_S$



Q_S : Quantity supplied by firms

Q_D : Quantity demanded by consumers

- If $Q_S = Q_D$ the Market is at Equilibrium
- If $Q_S > Q_D \Rightarrow$ There is surplus (firms should produce less)
- If $Q_S < Q_D \Rightarrow$ There is shortage (firms should produce more)

<u>Exp</u> Assume a market with Equilibrium price \$20.

- At any price less than \$20 $\Rightarrow$ there is shortage
  - $\Rightarrow$ so the firms should supply more the product in the market
  - $\Rightarrow$ the price should be increased until we reach the Eq. Price \$20

- At any price more than \$20 $\Rightarrow$ there is surplus
  - $\Rightarrow$ so the firms should supply less the product in the market
  - $\Rightarrow$ the price should be decreased until we reach the Eq. Price \$20.