

4.4 Conditional Probabilities

(52)

Let A and B be two events:

- The conditional probability of A given B is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- The conditional probability of B given A is

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

- The events A and B are independent if

$$P(A \cap B) = P(A) P(B)$$

Otherwise, they are dependent.

- If A and B are independent, then

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) P(B)}{P(B)} = P(A)$$

$$P(B|A) = P(B)$$

- The events A and B are mutually exclusive

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$$\text{if } P(A \cap B) = 0$$

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Example: Suppose that we have two events A and B, ⁽⁵³⁾
 with $P(A) = 0.2$, $P(B) = 0.4$, $P(A \cap B) = 0.08$

(a) Find $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.08}{0.4} = 0.2$

(b) Find $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.08}{0.2} = 0.4$

(c) Are A and B independent? why or why not?

Yes Because $P(A|B) = P(A)$

$P(B|A) = P(B)$

Example (Q32 page 168) Sample data representative
 of the national health insurance coverage are shown here:

		Health Insurance		Total
		Yes	No	
Age	18-34	750	170	920
	35 and older	950	130	1080
Total		1700	300	2000

(a) Develop a joint probability table for these data
 ← joint probabilities

		Health insurance		Total
		Yes	No	
Age	18-34	$\frac{750}{2000} = 0.375$	0.085	0.46
	35 and older	0.475	0.065	0.54
Total		0.850	0.150	1.00

→ Marginal probabilities

b) what do the marginal probabilities tell you about the age of the US population?

46 % of the population are within age 18-34
54 % of the population are 35 and older.

c) what is the probability that a randomly selected individual does not have health insurance?

0.15

d) If the individual is between the ages 18-34, what is the probability that the individual does not have health insurance?

$$P(\text{No} | 18-34) = \frac{0.085}{0.46} = 0.1848$$

e) If the individual age is 35 or older, what is the probability that the individual does not have health insurance.

$$P(\text{No} | 35 \text{ or older}) = \frac{0.065}{0.54} = 0.1204$$

f) If the individual does not have health insurance, what is the prob. that the individual is in 18-34 age?

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$$P(18-34 | \text{No}) = \frac{0.085}{0.150} = 0.567$$

g) what does the prob. information tell you about the health insurance in United state?

From d and e, we see higher probability of No for 18-34