Functions (1.1) exercises

Find the domain and range from (1 → 6)

1) $f(x) = 1 + x^2$
D: $\mathbb{R}$
R: $[1, \infty)$

2) $f(x) = 1 - \sqrt{x}$
D: $x \geq 0$
R: $(-\infty, 1]$

3) $f(x) = \sqrt{5x+10}$
D: $5x + 10 \geq 0 \Rightarrow x \geq -2$
R: $y \geq 0$

4) $g(x) = \sqrt{x^2 - 3x}$
D: $x^2 - 3x \geq 0 \Rightarrow x(x-3) = 0$, $x = 0, 3$
D: $(-\infty, 0] \cup [3, \infty)$
R: $[0, \infty)$

5) $f(t) = \frac{4}{3-t}$
D: $t \neq 3$
R: $y \neq 0$

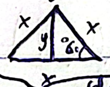
6) $G(t) = \frac{2}{t^2 - 16}$
D: $t \neq \{4, -4\}$
R: $y \neq 0$

7: a is a relation and b is a function because of VLT

8: a and b are relations because of VLT

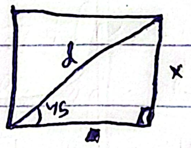
9) suppose $f(x)$ is a function that rules the perimeter of an triangle $\Rightarrow h(x) = 3x$, $x > 0$

suppose $g(x)$ is a function that rules the ~~perimeter~~ Area of a triangle $\Rightarrow g(x) = \frac{\sqrt{3}}{4} x^2$, $x > 0$



$\sin 60° = \frac{y}{x} \Rightarrow y = \frac{\sqrt{3}}{2} x$

10) suppose $f(d)$ is a function that rules the ~~diagonal~~ side length of a square $\Rightarrow f(d) = \frac{d}{\sqrt{2}}$

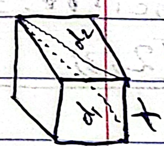


$\sin 45° = \frac{x}{d} \Rightarrow d = \sqrt{2} x$

suppose $g(d)$ is a function that rules the area of a square $\Rightarrow g(d) = \left(\frac{d}{\sqrt{2}}\right)^2 = \frac{d^2}{2}$

11) suppose $f(d)$ is a function that rules the ~~side~~ edge length of a cube $f(d) = \frac{d}{\sqrt{3}}$

$d_1^2 = d_2^2 + x^2 \Rightarrow d_1^2 = 3x^2 \Rightarrow x = \frac{d}{\sqrt{3}}$



suppose $g(d)$ is a function that rules the ~~area~~ surface area of a cube $g(d) = 2d_1^2$

suppose $h(d)$ is a function that rules the volume of a cube $h(d) = \left(\frac{d}{\sqrt{3}}\right)^3$

12) the coordinates of p is $(x, f(x))$

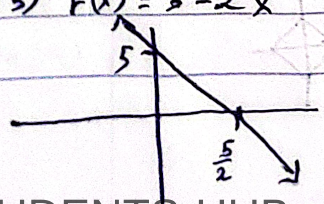
$s = \frac{\sqrt{x} - 0}{x - 0} = \frac{1}{\sqrt{x}} = \frac{1}{f(x)} \Rightarrow f(x) = \frac{1}{s}$

13) $L = f(x) = \sqrt{\left(\frac{5}{4} - \frac{1}{2}x\right)^2 + x^2}$

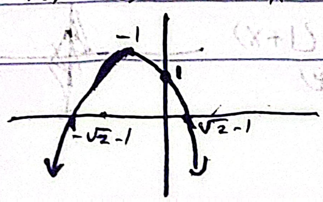
14) $l = f(y) = \sqrt{2y - 1}$

graph functions from 15 → 20

15) $f(x) = 5 - 2x$

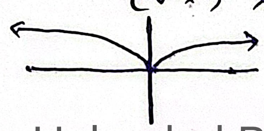


16) $f(x) = 1 - 2x - x^2$

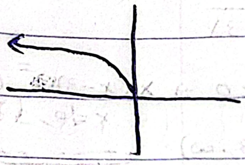


17) $g(x) = \sqrt{|x|}$

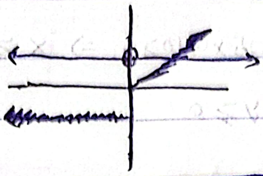
$g(x) = \begin{cases} \sqrt{x}, & x \geq 0 \\ \sqrt{-x}, & x < 0 \end{cases}$



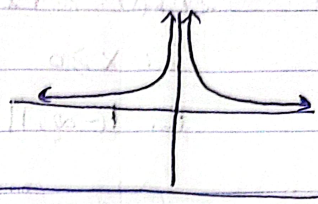
18) $g(x) = \sqrt{-x}$



19) $f(t) = \frac{t}{|t|} \Rightarrow |t| = \begin{cases} t, & t > 0 \\ -t, & t < 0 \end{cases}$ $t \neq 0$



20) $G(t) = \frac{1}{|t|}$



21) find the domain of $y = \frac{x+3}{4-\sqrt{x^2-9}}$

$y = \frac{x+3}{-5-|x|} \Rightarrow$ when $x \geq 0 \Rightarrow y = \frac{x+3}{-(5+x)}$; when $x < 0$ $y = \frac{x+3}{-(5-x)}$

D: $\mathbb{R}$

Donemirator $\neq 0$ if $x \geq 0$, Donemirator $\neq 0$ if $x < 0$

22) find the range of $y = 2 + \frac{x^2}{x^2+4}$

$y = 2 + \frac{x^2+4-4}{x^2+4} = 3 - \frac{4}{x^2+4}$

suppose $f = \frac{4}{x^2+4} \longrightarrow$

$x^2+4 \geq 4$

$\frac{1}{x^2+4} \leq \frac{1}{4}$ $\times 4$

$\frac{4}{x^2+4} \leq 1$

and $\frac{4}{x^2+4} > 0$

R: $(0,1]$

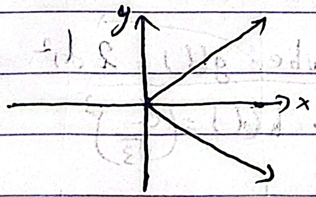
y Range is: $3-(0,1]$

R: $[2,3)$

23)

a) $|y| = x$

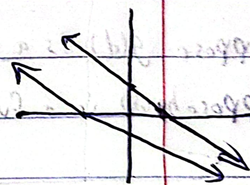
$x = \begin{cases} y, & y \geq 0 \\ -y, & y < 0 \end{cases}$



by VLT it is a relation

b) $|x+y| = 1$

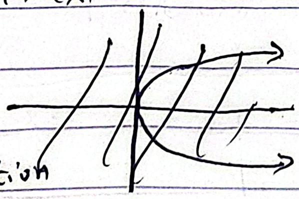
$|x+y| = \begin{cases} x+y, & x+y \geq 0 \\ -(x+y), & x+y < 0 \end{cases}$



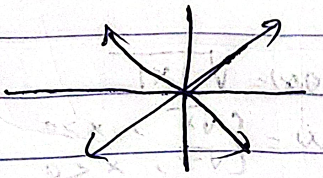
by VLT it is a relation

24)

a) $y^2 = x^2 \Rightarrow$ $|y| = |x|$

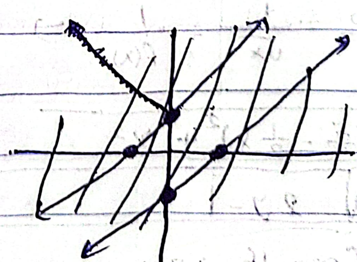


by VLT it is a relation



b) $|x| + |y| = 1$

by VLT it is a relation

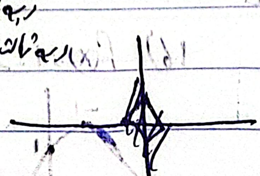


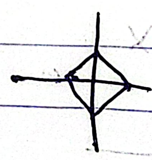
$y = 1-x$

$y = 1+x$

$y = -(1-x)$

$y = -(1+x)$

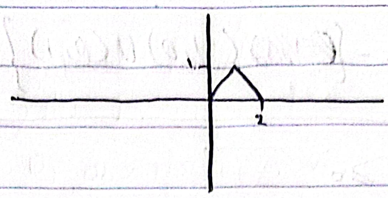




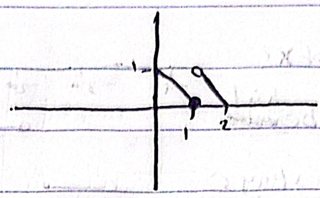
Function(1.1) exercises

graph these functions

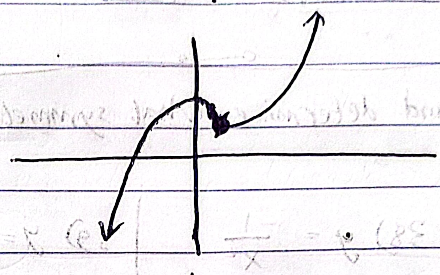
25) $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2-x, & 1 < x \le 2 \end{cases}$



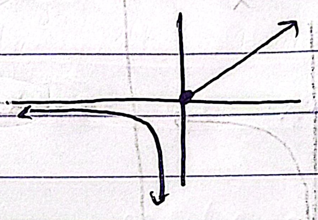
26) $g(x) = \begin{cases} 1-x, & 0 \le x \le 1 \\ 2-x, & 1 < x \le 2 \end{cases}$



27) $f(x) = \begin{cases} 4-x^2, & x \le 1 \\ x^2+2x, & x > 1 \end{cases}$



$G(x) = \begin{cases} \frac{1}{x}, & x < 0 \\ x, & x \ge 0 \end{cases}$



Find the formula for 29 → 32

29) a) $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ -x+2, & 1 < x \le 2 \end{cases}$

b) $f(x) = \begin{cases} 2, & x \in [0,1) \cup [2,3) \\ 0, & x \in [1,2) \cup [3,4] \end{cases}$

30)

a) $f(x) = \begin{cases} x-2, & 0 < x \le 2 \\ -\frac{1}{3}x + \frac{5}{3}, & 2 < x \le 5 \end{cases}$

b) $f(x) = \begin{cases} -3x-3, & -1 < x \le 0 \\ -2x+3, & 0 < x \le 2 \end{cases}$

31) a) $f(x) = \begin{cases} -x, & -1 \le x < 0 \\ 1, & 0 < x \le 1 \\ -\frac{1}{2}x + \frac{3}{2}, & 1 < x < 3 \end{cases}$

b) $f(x) = \begin{cases} \frac{1}{2}x, & -2 \le x \le 0 \\ -2x+2, & 0 < x \le 1 \\ -1, & 1 < x \le 3 \end{cases}$

32) a) $f(x) = \frac{2}{T}x - 1$

b) $f(x) = \begin{cases} A, & x \in (0, \frac{T}{2}) \cup [T, \frac{3T}{2}) \\ -A, & x \in [\frac{T}{2}, T) \cup [\frac{3T}{2}, 2T] \end{cases}$

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33)

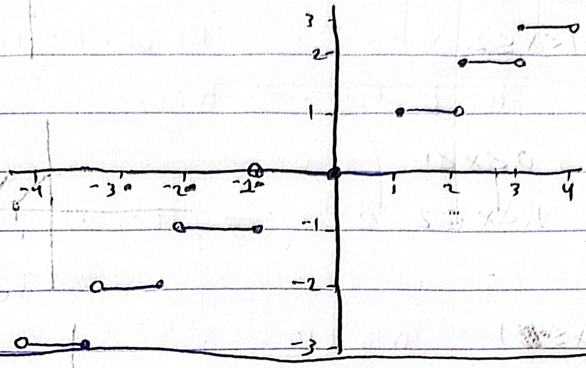
a) $\lfloor x \rfloor = 0$ when $0 \le x < 1$, b) $\lceil x \rceil = 0$ when $-1 < x \le 0$

34) $\lfloor x \rfloor = \lceil x \rceil$ when $x \in \mathbb{Z}$

Function (1.1) exercises

35) no, it is equal to it in $\mathbb{R} - \{(-1,0) \cup (0,1)\}$ because none of them will equal 0

36) Graph $f(x) = \begin{cases} \lfloor x \rfloor, & x \geq 0 \\ \lceil x \rceil, & x < 0 \end{cases}$

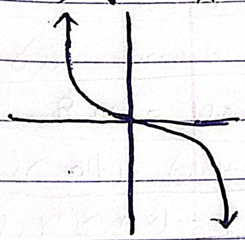


why is f(x) called the integer part of x?

because it removes every number behind the decimal point and the integer stays

Graph the functions (37→46) and determine what symmetries they have, and specify where they increase and decrease?

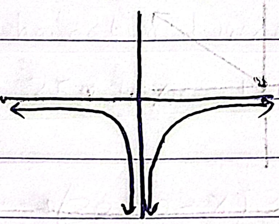
37) $y = -x^3$



Symmetric about the origin.

Decreasing on $\mathbb{R}$

38) $y = \frac{-1}{x^2}$

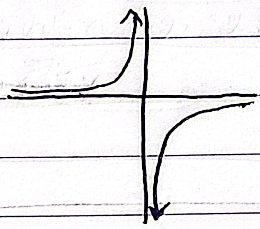


symmetric on y-axis

Decreasing on $(-\infty, 0)$

increasing on $(0, \infty)$

39) $y = \frac{-1}{x}$

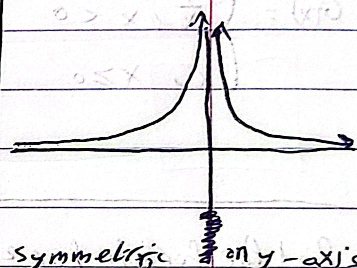


symmetric about the origin

increasing on $(-\infty, 0)$

Decreasing on $(0, \infty)$

40) $y = \frac{1}{|x|}$

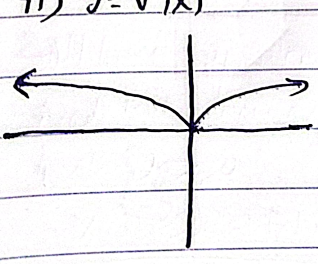


symmetric on y-axis

increasing on $(-\infty, 0)$

Decreasing on $(0, \infty)$

41) $y = \sqrt{|x|}$

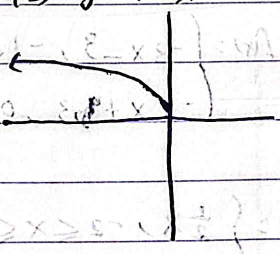


symmetric on y-Axis

Decreasing on $(-\infty, 0]$

increasing on $[0, \infty)$

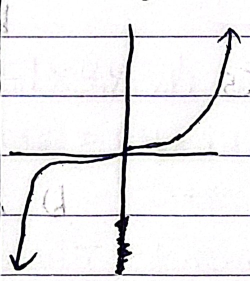
42) $y = \sqrt{-x}$



not symmetric

Decreasing on $(-\infty, 0]$

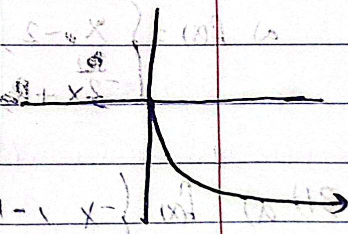
43) $y = \frac{x^3}{8}$



symmetric about the origin

Increasing on $\mathbb{R}$

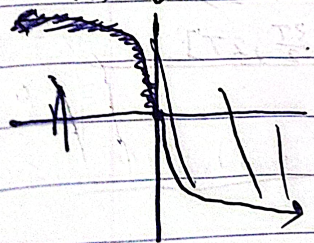
44) $y = -4\sqrt{x}$

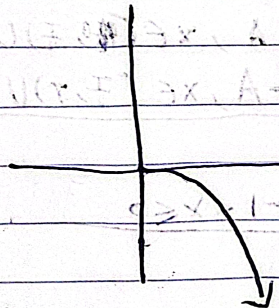


not symmetric

Decreasing on $[0, \infty)$

45) $y = -x^{\frac{3}{2}}$

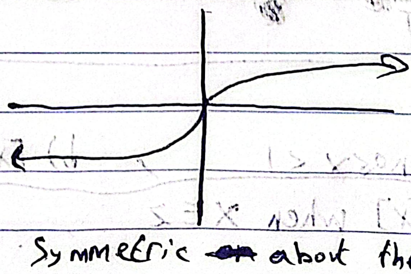




not symmetric

Decreasing on $[0, \infty)$

$y = (-x)^{\frac{2}{3}}$



Symmetric about the origin point

Increasing on $\mathbb{R}$

function (1.1) exercises

• determine which of these functions are odd or even or neither?

47) $f(x)=3$
$f(x)=f(-x)=3$
even

48) $f(x)=x^2+1$
$f(x)=f(-x)=x^2+1$
even

49) $f(x)=x^{-5}=\frac{1}{x^5}$
$f(x)=-f(-x)$
odd

50) $f(x)=x^2+x$
$f(x)\neq f(-x)\neq -f(-x)$
neither

51) $g(x)=\frac{1}{x^2-1}$
$g(x)=g(-x)=\frac{1}{x^2-1}$
even

52) $g(x)=x^4+3x^2-1$
$g(x)=g(-x)=x^4+3x^2-1$
even

53) $g(x)=x^3+x$
$g(x)=-g(-x)=-(-x^3-x)$
odd

54) $g(x)=\frac{x}{x^2-1}$
$g(x)\neq g(-x)\neq -g(-x)$
neither

55) $h(t)=\frac{1}{t-1}$
$h(t)\neq h(-t)\neq -h(-t)$
neither

56) $h(t)=|t^3|$
$h(t)=h(-t)$
even

57) $h(t)=2t+1$
$h(t)\neq h(-t)\neq -h(-t)$
neither

58) $h(t)=2|t|+1$
$h(t)=h(-t)$
even

59) $\frac{s}{t}=c \Rightarrow \frac{25}{75}=c \Rightarrow c=\frac{1}{3}$
$s=\frac{1}{3}t$
$t=3s \Rightarrow t=3\times 60=180$

60) $\frac{u}{m}=c \Rightarrow \frac{12960}{18}=720 \Rightarrow 720=\frac{u}{10} \Rightarrow u=7200J$

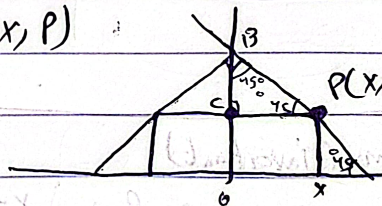
61) $r=\frac{c}{s} \Rightarrow c=6\times 4=24 \Rightarrow s=\frac{24}{10}=2.4$

62) $VP=c \Rightarrow c=850\times 16390=13931500 \Rightarrow V\times 1350=13931500 \Rightarrow V=1031.63\,cm^3$

63) $L=55-2x$, $W=35-2x$, $H=x$
$V=(55-2x)(35-2x)x,\ x>0$

64) with triangles similarity: (B,c,P), (A,x,P)

a) $\tan 45^\circ=\frac{y}{1-x} \Rightarrow y=1-x$



b) $f(x)=2x(1-x)=2x-2x^2$

65) $y=x^4$ is h, $y=x^7$ is f, $y=x^{10}$ is g ($y=x^7$ is symmetric about the origin, $y=x^{10}$ is more flatten than $y=x^4$)

66) $y=5x$ is f, $y=5^x$ is g, $y=x^5$ is h ($y=5x$ is a straight line, $y=5^x$ is a power function, $y=x^5$ is an odd function)

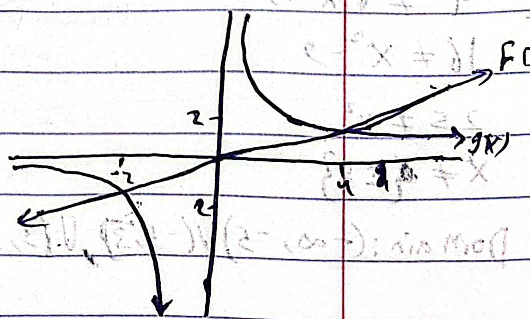
67) determine where $f(x)=\frac{x}{2}>g(x)=1+\frac{4}{x}$?

$\frac{x}{2}>1+\frac{4}{x}$ multiple by x

$\frac{x^2}{2}-x>4$
$x^2-2x-8>0$
$x^2-2x-8=0$
$(x-4)(x+2)=0$
$x=\{-2,4\}$

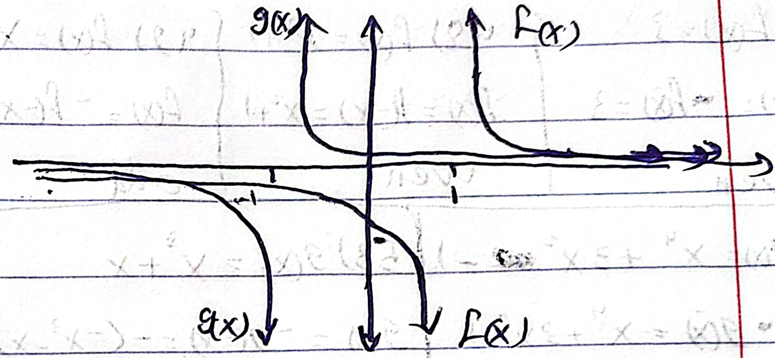
$\frac{x}{2}>1+\frac{4}{x}$

where $x\in(-\infty,-2)\cup(4,\infty)$



68) Graph $f(x) = \frac{3}{x-1}$ and $g(x) = \frac{2}{x+1}$ and indetify the values of x where

$$\frac{3}{x-1} < \frac{2}{x+1}$$



$$\frac{3}{x-1} < \frac{2}{x+1}$$

$$\frac{3}{x-1} - \frac{2}{x+1} < 0$$

$$\frac{3(x+1) - 2(x-1)}{x^2-1} < 0 \Rightarrow \frac{x+5}{x^2-1} < 0$$

$$- - - - \;\; + + + + + \;\; - - - - - - - \;\; + + +$$
$$-5 \qquad -1 \qquad 1$$

where $x \in (-\infty, -5) \cup (-1, 1)$

69) because a x value will have two different y value $\{y, -y\}$

70) $R(x) = 12000 + 300.5x - 25.45x = 12000 + 375x$

71) $C(h) = (20 + 15\sqrt{2})x$

72) $C(x) = 5040\sqrt{x^2+240^2} + (3200 - x)300$

$x^2 + 240^2 = y^2$

$\cancel{x = \sqrt{y^2 - 240^2}}$

$y = \sqrt{x^2 + 240^2}$