Chapter 7.1, Problem 38E

Problem

Let $X = \{a, b, c\}$ and $Y = \{r, s, t, u, v, w\}$. Define $f: X \to Y$ as follows: $f(a) = v$, $f(b) = v$, and $f(c) = t$.

a. Draw an arrow diagram for f.

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b. Let $A = \{a, b\}$, $C = \{t\}$, $D = \{u, v\}$, and $E = \{r, s\}$. Find $f(A)$, $f(X)$, $f^{-1}(C)$, $f^{-1}(D)$, $f^{-1}(E)$, and $f^{-1}(Y)$.

Step-by-step solution

Step 1 of 2

Consider the sets,

$$X = \{a, b, c\} \text{ and } Y = \{r, s, t, u, v, w\}$$

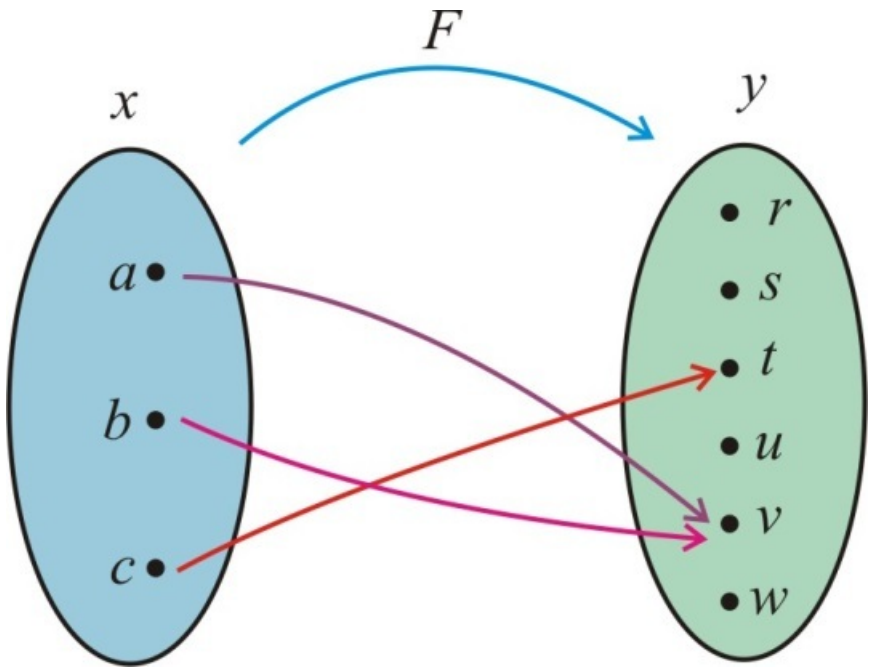
The mapping $f : x \to y$ is defined as follows:

$$f(a) = v,$$
$$f(b) = v \text{ and}$$
$$f(c) = t$$

(a)

List the elements in X, Y and draw an arrow from each element in X to the corresponding element in Y.

Draw the arrow diagram for f as follows:



Note that this arrow diagram represents a function.

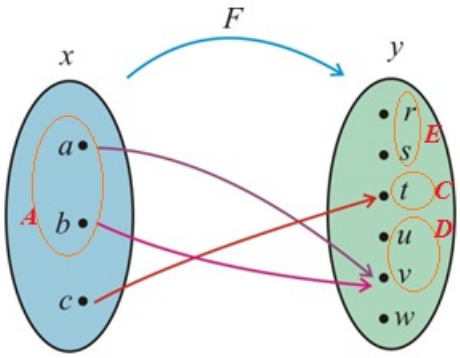
Step 2 of 2

(b)

Write the sets,

$A=\{a,b\}$,$C=\{t\}$, $D=\{u,v\}$, and $\{r,s\}$

Show these in the below diagram:



As the image for a,b is *v,* write $f(A)=\{v\}$

As the range of the map is $\{t,v\}$, write $f(X)=\{t,v\}$

The inverse map of the set $D=\{u,v\}$ is,

$$f^{-1}(D)=f^{1}\{u,v\}$$
$$=\{a,b\}$$

The inverse map of the set $C=\{t\}$ is,

$$f^{-1}(C)=\{c\}$$

As the elements r,s are not mapped, the inverse map of the set $E=\{r,s\}$ is,

$$f^{-1}(E)=f^{-1}\{r,s\}$$
$$=\phi$$

The inverse map of the image set Y is,

$$f^{-1}(Y)=\{a,b,c\}$$
$$=X$$