Series with some $\boxed{-ve}$ terms.

$$\sum_{n=1}^{\infty} (-1)^{n+1} U_n \text{ (alternating)}$$

1. $\lim_{n \to \infty} U_n$ ✓

$\neq 0$ $\Downarrow$ div by nth term test

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$= 0$ $\Downarrow$

2. $\sum |a_n|$

conv. ✓ $\Downarrow$ $\boxed{\text{abs. conv.}}$

#

$\boxed{\text{div.}}$ $\Downarrow$

3. 1. $U_n \geq 0$
   2. U_n nonincreasing
   3. $\lim U_n = 0$

$\Downarrow$

conv. by A.S.T

$\Downarrow$

Cond. conv.

6. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2+5}{n^2+4}$

$$\lim_{n\to\infty}\frac{n^2+5}{n^2+4}=1\neq 0\Rightarrow \text{div.}$$

by nth term test.

$n^n > n! > x^n$

8. $\sum_{n=1}^{\infty}(-1)^n\frac{10^n}{(n+1)!}$

$$\lim_{n\to\infty}\frac{10^n}{(n+1)!}=0$$

$$\sum_{n=1}^{\infty}\left|(-1)^n\frac{10^n}{(n+1)!}\right|=\sum_{n=1}^{\infty}\frac{10^n}{(n+1)!}$$

$$\Rightarrow \lim_{n\to\infty}\frac{10^{n+1}}{(n+2)!}\cdot\frac{(n+1)!}{10^n}=$$

$$=\lim_{n\to\infty}\frac{\cancel{10^n}\,10\,\cancel{(n+1)!}}{(n+2)\cancel{(n+1)!}\,\cancel{10^n}}=0<1$$

$\Rightarrow$ conv. by Ratio test.

$\Rightarrow$ the series abs. conv. $\Rightarrow$ conv.

18. $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\sqrt{n}}$

① $\lim_{n\to\infty} \frac{1}{1+\sqrt{n}} = 0$

② $\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{1+\sqrt{n}} \right| = \sum_{n=1}^{\infty} \frac{1}{1+\sqrt{n}}$ a_n

Let $b_n = \frac{1}{\sqrt{n}}$

$\Rightarrow \lim_{n\to\infty} \frac{1}{1+\sqrt{n}} \cdot \sqrt{n} = \lim_{n\to\infty} \frac{\sqrt{n}}{1+\sqrt{n}} = 1$

both $\sum b_n$ and $\sum a_n$ con. or both div.

but $\sum b_n = \sum \frac{1}{n^{1/2}}$ div

p-series $p = \frac{1}{2} < 1$

$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{1+\sqrt{n}}$ div. by LCT

③ 1. $U_n = \dfrac{1}{1+\sqrt{n}} \geq 0$ for $n \geq 1$

2. U_n nonincreasing (decreasing)

$f(x) = \dfrac{1}{1+\sqrt{x}} \longrightarrow f' = \ldots\ldots$

f' — — — —

f ↘

3. $\lim_{n \to \infty} U_n = 0$

$\Rightarrow$ the series conv. by A.S.T

$\Rightarrow$ $\boxed{\text{cond. conv.}}$

Let : $\sum_{n=1}^{\infty} (-1)^{n+1} U_n$ be an alternating series.

or

and $\boxed{\text{conv. by A.S.T}}$

exact sum = L

approx. sum = S_n

remainder = $L - S_n$

$\Rightarrow \boxed{|E| < U_{n+1}}$

the sign of E = the same of sign U_{n+1}

50. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{10^n}$

con. by A.S.T

$n=4$

$|E| < u_{n+1}$

$|E| < u_5$

$|E| < \frac{1}{10^5} \Rightarrow \boxed{|E| < 10^{-5}}$

the sign of the error is +ve

$-10^{-5} < E < 10^{5}$

$\Rightarrow \boxed{0 < E < 10^{-5}}$ *

$-\left[\sum_{n=1}^{\infty}\left(\frac{-1}{10}\right)^n\right] \Rightarrow \frac{\frac{-1}{10}}{1+\frac{1}{10}} = \frac{\frac{-1}{\cancel{10}}}{\frac{11}{\cancel{10}}}$

$L = \frac{1}{11}$

$= 0.09$

$S_4 = \frac{1}{10} - \frac{1}{10^2} + \frac{1}{10^3} - \frac{1}{10^4}$

$= 0.1 - 0.01 + 0.001 - 0.0001 = 0.0909$

$n = ?$

In Exercises 53–56, determine how many terms should be used to estimate the sum of the entire series with an error of less than 0.001.

54. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\underbrace{\frac{n}{n^2+1}}_{U_n}$ $\Rightarrow$ conv. by A.S.T

$$|E| < 0.001$$

$$\Rightarrow \boxed{U_{n+1} < 0.001}$$

$$\frac{n+1}{(n+1)^2+1} < \frac{1}{1000}$$

$$\frac{n+1}{n^2+2n+2} < \frac{1}{1000}$$

$$\frac{n^2+2n+2}{n+1} > 1000$$

$$n^2+2n+2 > 1000n + 1000$$

$$+n^2 - 998n - 998 \boxed{> 0}$$

$+ \quad - \quad - \quad +$

$-n_1 \qquad +n_2$

$$\boxed{n > n_2}$$

$$n \geq 999$$