

Series with some -ve terms.

$$\sum_{n=1}^{\infty} (-1)^{n+1} u_n \text{ (alternating)}$$

① $\lim_{n \rightarrow \infty} u_n$ ✓

$\neq 0$

↓
div by
nth term test

② $\sum |a_n|$

conv. ✓

div.

#

abs. conv.

#

③

- 1. $u_n \geq 0$
- 2. u_n non-increasing
- 3. $\lim u_n = 0$

conv. by A.S.T

Conv. Conv.

$$6. \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2 + 5}{n^2 + 4}$$

$$\lim_{n \rightarrow \infty} \frac{n^2 + 5}{n^2 + 4} = 1 \neq 0 \Rightarrow \text{div.}$$

by n^{th} term test.

$$8. \sum_{n=1}^{\infty} (-1)^n \frac{10^n}{(n+1)!}$$

$$n^n > n! > x^n$$

$$\lim_{n \rightarrow \infty} \frac{10^n}{(n+1)!} = 0$$

$$\sum_{n=1}^{\infty} \left| (-1)^n \frac{10^n}{(n+1)!} \right| = \sum_{n=1}^{\infty} \frac{10^n}{(n+1)!}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{10^{n+1}}{(n+2)!} \cdot \frac{(n+1)!}{10^n} =$$

$$= \lim_{n \rightarrow \infty} \frac{10^n 10 (n+1)!}{(n+2)(n+1)! 10^n} = 0 < 1$$

\Rightarrow conv. by Ratio test.

\Rightarrow the series abs. conv. \Rightarrow conv.

$$18. \sum_{n=1}^{\infty} \frac{(-1)^n}{1 + \sqrt{n}}$$

① $\lim_{n \rightarrow \infty} \frac{1}{1 + \sqrt{n}} = 0$

② $\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{1 + \sqrt{n}} \right| = \sum_{n=1}^{\infty} \frac{1}{1 + \sqrt{n}}$

Let $b_n = \frac{1}{\sqrt{n}}$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{1}{1 + \sqrt{n}} \cdot \sqrt{n} = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{1 + \sqrt{n}} = 1$$

both $\sum b_n$ and $\sum \alpha_n$ conv. or both
div.

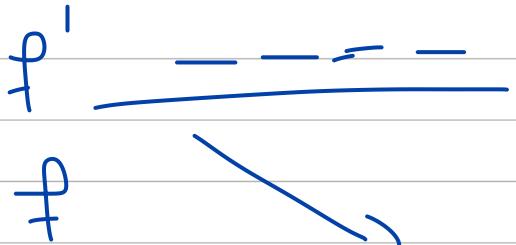
but $\sum b_n = \sum \frac{1}{n^{1/2}}$ div.
p-series
 $p = \frac{1}{2} < 1$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{1 + \sqrt{n}} \text{ div. by LCT}$$

$$\textcircled{3} \quad 1. U_n = \frac{1}{1+\sqrt{n}} \geq 0 \quad \text{for } n \geq 1$$

2. U_n non increasing (decreasing)

$$f(x) = \frac{1}{1+\sqrt{x}} \rightarrow f' = \dots$$



$$3. \lim_{n \rightarrow \infty} U_n = 0$$

\Rightarrow the series converges by A.S.T

\Rightarrow cond. conv.

Let : $\sum_{n=1}^{\infty} (-1)^{n+1} u_n$ be an alternating series.

and conv. by A.S.T

$$\text{exact sum} = L$$

$$\text{approx. sum} = S_n$$

$$\text{remainder} = L - S_n$$

$$|E| < \underline{u_{n+1}}$$

the sign of E = the same of sign u_{n+1}

$$50. \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{10^n}$$

con. by A.S.T ✓

$$|E| < u_{n+1}$$

$$|E| < u_5$$

$$|E| < \frac{1}{10^5} \Rightarrow |E| < 10^{-5}$$

the sign of the error. is +ve

$$\begin{aligned} -10^{-5} &< E < 10^{-5} \\ \Rightarrow [0 &\leq E < 10^{-5}] * \end{aligned}$$

$$\begin{aligned} -\left[\sum_{n=1}^{\infty} \left(\frac{-1}{10} \right)^n \right] &\Rightarrow \frac{-1}{1 + \frac{1}{10}} = \frac{-1}{\frac{11}{10}} \\ L &= \frac{1}{11} = 0.0909 \end{aligned}$$

$$S_4 = \frac{1}{10} - \frac{1}{10^2} + \frac{1}{10^3} - \frac{1}{10^4}$$

$$= 0.1 - 0.01 + 0.001 - 0.0001 = 0.0909$$

$n = ?$

In Exercises 53–56, determine how many terms should be used to estimate the sum of the entire series with an error of less than 0.001.

$$54. \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2 + 1}$$

U_n

→ conv. by A.S.T

$$|E| < 0.001$$

$\Rightarrow U_{n+1} < 0.001$

$$\frac{n+1}{(n+1)^2 + 1} < \frac{1}{1000}$$

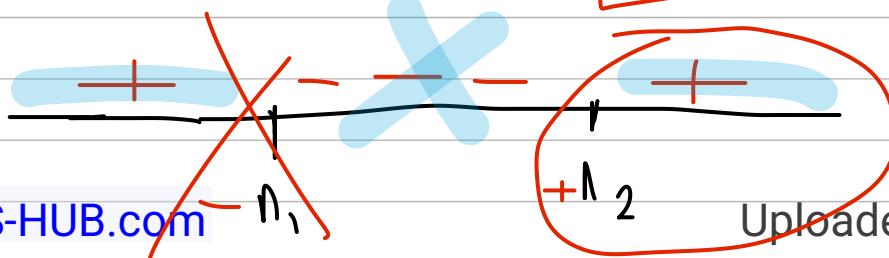
$$\frac{n+1}{n^2 + 2n + 2} < \frac{1}{1000}$$

$$\frac{n^2 + 2n + 2}{n+1} > 1000$$

$$n^2 + 2n + 2 > 1000n + 1000$$

$$+ n^2 - 998n - 998 > 0$$

$$n > n_2$$



$$n > 999$$