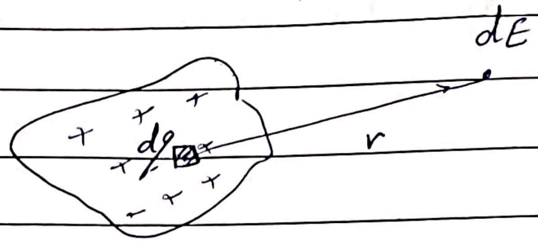
22-4: The electric field due to a line of charge: [Continous charge distribution]

$$dE = k\frac{dq}{r^2}$$



$$E = \int dE \quad ✓$$

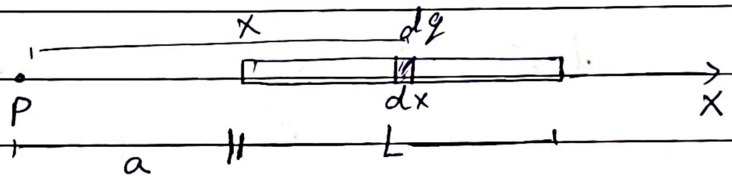
* we will consider several cases:

① line of charge

problem (22-13) linear charge distribution.

→ Find $\vec{E}$ due to a line of charge

* the rod has linear charge density λ.



$$\lambda = \frac{dq}{dx}$$

$$dE = k\frac{dq}{r^2} = k\frac{dq}{x^2} \quad , dq = \lambda\, dx$$

$$\int dE = \int \frac{k\lambda\, dx}{x^2}$$

$$E = k\lambda \int_a^{L+a} \frac{dx}{x^2} = k\lambda\left[\frac{-1}{x}\right]_a^{L+a}$$

$$= k\lambda\left[\frac{-1}{L+a} - \left(\frac{-1}{a}\right)\right]$$

$$= k\lambda\left[\frac{-a + L + a}{a(L+a)}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda L}{a(L+a)} = \frac{q}{4\pi\varepsilon_0 a(L+a)}$$

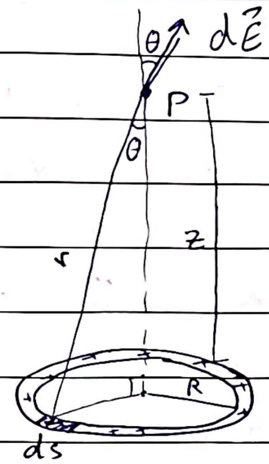
note that $\lambda = \frac{q}{L} \Rightarrow q = \lambda L$ (total charge)

d) if $a \gg L \Rightarrow a(L+a) \Rightarrow a(0+a) = a^2$

$E = \frac{kq}{a^2}$ as a point charge

example: a thin ring of radius R, with a uniform distribution of positive charge. (P'567)

Find $\vec{E}$ at point P (on the z-axis)



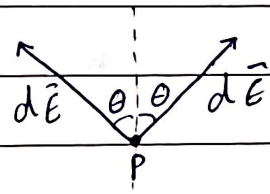
$dE = k\frac{dq}{r^2}$, but $\lambda = \frac{dq}{ds}$

$dq = \lambda\, ds$

$dE = k\frac{\lambda\, ds}{r^2}$

$r^2 = R^2 + z^2$

$dE = \frac{k\lambda\, ds}{(R^2+z^2)}$



dE_x cancel each other

$\Rightarrow$ All the remaining components are in the positive direction of the z-axis.

$dE_{tot} = dE\cos\theta = \frac{k\lambda\, ds\cos\theta}{(R^2+z^2)}$

$$\cos\theta = \frac{z}{r} = \frac{z}{(z^2+R^2)^{1/2}}$$

$$\Rightarrow \quad dE_{tot} = k \frac{z\lambda}{(z^2+R^2)^{3/2}} \, ds$$

$$E = \int dE_{tot} = \int_0^{2\pi R} \frac{k z \lambda}{(z^2+R^2)^{3/2}} \, ds$$

$$= \frac{k z \lambda}{(z^2+R^2)^{3/2}} \int_0^{2\pi R} ds$$

$$= \frac{k z \lambda (2\pi R)}{(z^2+R^2)^{3/2}}$$

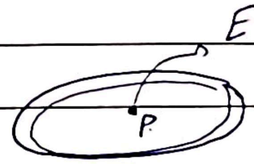
$$* \quad \lambda = \frac{q}{2\pi R} \left(\frac{\text{total charge}}{\text{total length}} \right)$$

$$\Rightarrow \quad q = \lambda (2\pi R)$$

$$\Rightarrow \quad \vec{E} = \frac{k z q}{(z^2+R^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0} \frac{z q}{(z^2+R^2)^{3/2}} \hat{k}$$

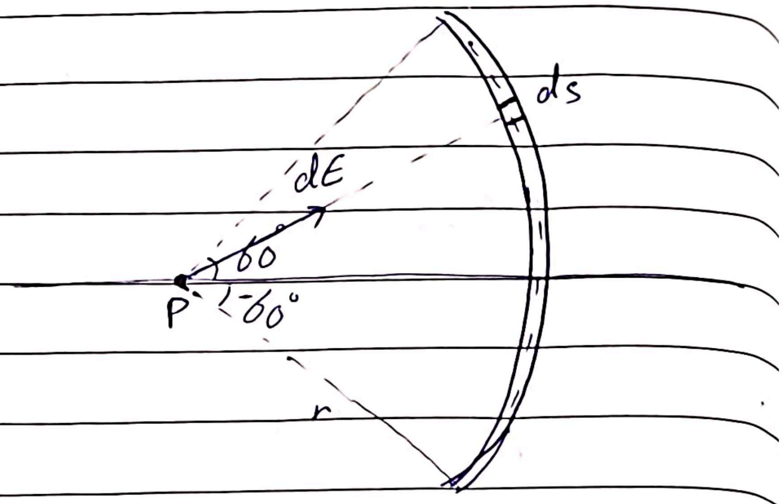
if $z \gg R \Rightarrow E = \frac{1}{4\pi\varepsilon_0} \frac{q}{z^2} \hat{k}$ (as a point cha

if $z = 0 \Rightarrow E = 0$



S.P (22.3)

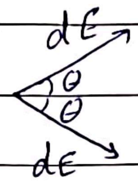
Find $\vec{E}$ at P ?



$$dE = \frac{k\,dq}{r^2}$$

from symmetry

$dE_y = 0$



$$\Rightarrow \quad dE_x = \frac{k\,dq}{r^2}\cos\theta$$

$$dq = \lambda\, ds \qquad , \quad ds = r\,d\theta$$

$$dE_x = \frac{k\,\lambda\,\cancel{r}\,d\theta}{r^{\cancel{2}}}\cos\theta$$

$$= \frac{k\,\lambda\cos\theta\,d\theta}{r}$$

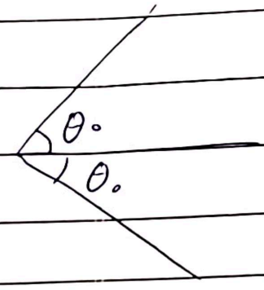
$$E = \int dE_x = \frac{k\lambda}{r}\int_{-60}^{60}\cos\theta\,d\theta$$

$$= \frac{k\lambda}{r}\Big[\sin\theta\Big]_{-60}^{60}$$

$$= \frac{k\lambda}{r}\Big[\sin 60 - \sin -60\Big]$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r}\left(2\sin 60\right) = \frac{1.73\,\lambda}{4\pi\varepsilon_0\, r}$$

$$\left[\text{In general} \quad E = \frac{\lambda}{2\pi\varepsilon_0 r}\sin\theta_0\right]$$



to find $\lambda = \dfrac{\text{total charge}}{\text{length}}$

$$= \frac{Q}{r\theta} = \frac{Q}{r(2\pi/3)} = \frac{0.477\,Q}{r}$$

$$\Rightarrow E = \frac{1.73\,(0.477\,Q)}{4\pi\varepsilon_0 r^2} = \frac{0.83\,Q}{4\pi\varepsilon_0 r^2}\,\hat{i}$$

22-5: The electric field due to a charged Disk:

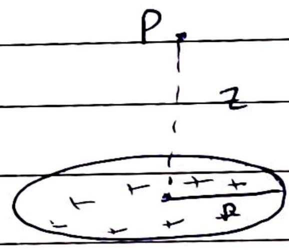
Disk: → radius $= R$

→ charge $= q$

→ area $= \pi R^2$

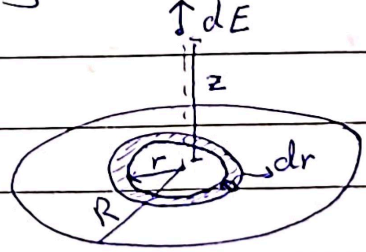
→ surface charge density

$$\sigma = \frac{q}{A} = \frac{q}{\pi R^2} \left(\frac{C}{m^2}\right)$$



Find E at P? (at a distance (z) above the center)

① Divide the disk to rings, each ring with the following properties.



Ring: radius $= r$

width $= dr$

area of the ring $dA = 2\pi r\,dr$

charge on the ring $dq = \sigma\,dA$

$dq = \sigma\,(2\pi r\,dr)$

Electric field of the ring $\quad dE_{ring} = \dfrac{z\,dq}{4\pi\varepsilon_0 [z^2 + r^2]^{3/2}}$

$$E_{disk} = \int dE_{ring} = \int \frac{z(\sigma(2\pi r\,dr))}{4\pi\varepsilon_0 [z^2+r^2]^{3/2}}$$

$$= \frac{\sigma z}{4\varepsilon_0} \int_0^R (z^2+r^2)^{-3/2} (2r\,dr)$$

let $u = z^2 + r^2$

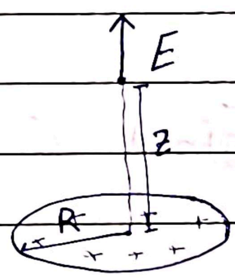
$du = 2r\,dr$

$$E_{disk} = \frac{\sigma z}{4\varepsilon_0} \int u^{-3/2}\,du$$

$$= \frac{\sigma z}{4\varepsilon_0}\left[\frac{u^{-1/2}}{-1/2}\right] = \frac{-\sigma z}{2\varepsilon_0}\left[\frac{1}{\sqrt{z^2+r^2}}\right]_0^R$$

$$= \frac{-\sigma z}{2\varepsilon_0}\left[\frac{1}{\sqrt{z^2+R^2}} - \frac{1}{z}\right]$$

$$E_{disk} = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$



* for very large disk $R \gg z \Rightarrow \dfrac{z}{\sqrt{z^2+R^2}} \to 0$

$$E_{disk} = \frac{\sigma}{2\varepsilon_0}$$

* if the point (P) very close to the disk $(z \to 0)$

 $\Rightarrow E_{disk} = \dfrac{\sigma}{2\varepsilon_0}$

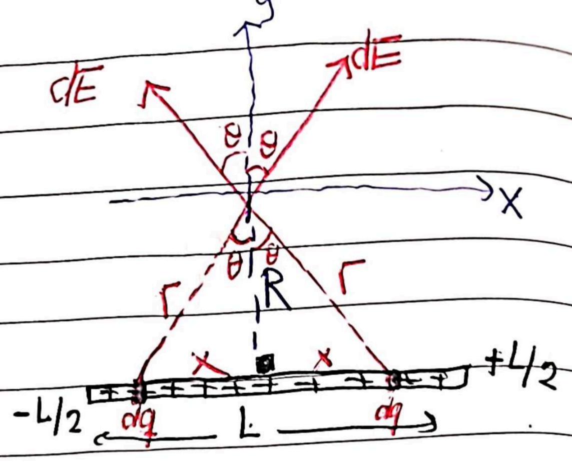
Problem (22-10)

A line of charge

rod → Length $= L = 16cm$

→ $q = 9.25\ pC = 9.25 \times 10^{-12} C$

→ $\lambda = \frac{q}{L} = 5.78 \times 10^{-11}\ C/m$



Find $\vec{E}$ at a point a distanc $R = 6cm$ from the rod along its perpendicular bisector

$\cos\theta = \frac{R}{r}$

$r = \frac{R}{\cos\theta} = R\sec\theta = R\sec\theta$

take dq ~~to~~ be a distance (x) from the origin

$dE = k\frac{dq}{r^2}$, $dq = \lambda dx$,

$\tan\theta = \frac{x}{R}$

$x = R\tan\theta$

$dx = R\sec^2\theta\, d\theta$

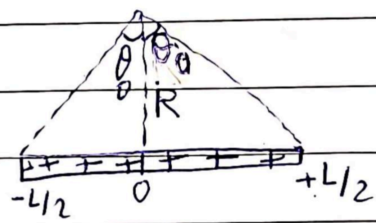
$dE_x = 0$ from symmetry

$E_x = 0$

$dx = R\sec^2\theta\, d\theta$

$(dE)_y = dE\cos\theta$

$$E_y = \int dE_{\cos\theta} = k\int \frac{dq_{\cos\theta}}{r^2} = k\int_{-L/2}^{+L/2} \frac{\lambda dx}{r^2}\cos\theta$$



$$E_y = k\lambda\int \frac{(R\sec^2\theta\, d\theta)}{R^2\sec^2\theta}\cdot\cos\theta = \frac{k\lambda}{R}\int_{-\theta_0}^{+\theta_0}\cos\theta\, d\theta$$

$\theta_0 = \tan^{-1}\left(\frac{L/2}{R}\right)$

$$= \frac{\lambda}{4\pi\epsilon_0 R}\left[\sin\theta\right]_{-\theta_0}^{\theta_0} = \frac{\lambda}{4\pi\epsilon_0 R}\left[\sin\theta_0 - \sin{-\theta_0}\right]$$

$$= \frac{\lambda}{4\pi\epsilon_0 R}\left[2\sin\theta_0\right]$$

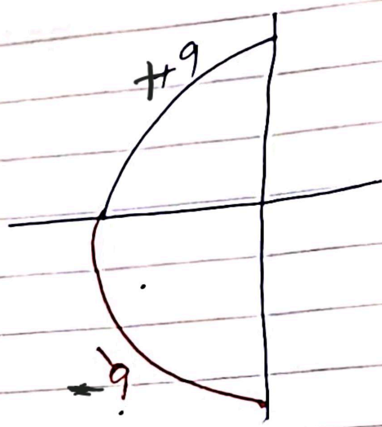
$\theta_0 = \tan^{-1}\left(\frac{8}{6}\right) = 53°$

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 R}\sin\theta_0\ \hat{j}$$

$$E = \frac{(5.78\times10^{-11})(2)\,9\times10^{9}}{6\times10^{-2}}\sin 53\ \hat{j}$$

$$\vec{E} = 13.87\hat{j}\ N/C$$

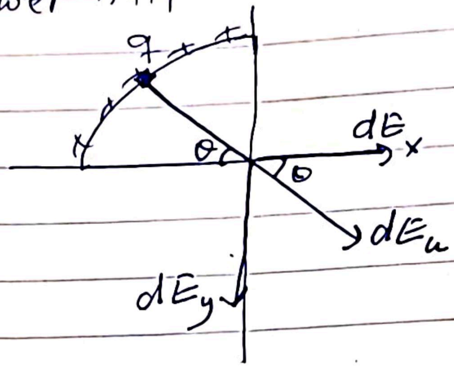
(22-14)



$r = 3cm$

$+q = 4.5pC$ upper half

$-q = -4.5pC$ lower half



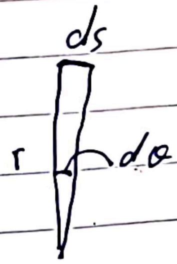
$\lambda_+ = \frac{q}{r(\pi/2)}$

$\lambda_u = \frac{4.5PC}{(3\times10^{-2})\frac{\pi}{2}} =$

$\lambda_u = \frac{4.5PC}{4.7\times10^{-2}}$

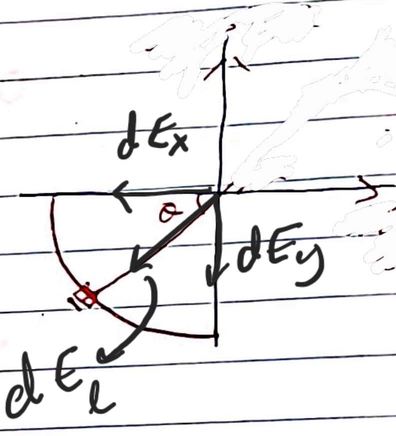
$\lambda_u = 9.57\times10^{-11} C/m$

$= 95.7 \; pC/m$



$dE_u = \frac{k\,dq}{r^2}$, $dq = \lambda ds = \lambda r d\theta$

$dE_u = \frac{k\lambda r d\theta}{r^2}$ → $dE_x = dE\cos\theta$; → $dE_y = dE\sin\theta$

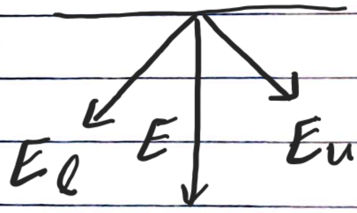


$(E_u)_x = \int_0^{\pi/2} \frac{k\lambda r d\theta}{r^2} \cdot \cos\theta$

$(E_u)_x = \frac{k\lambda}{r}\int_0^{\pi/2} \cos\theta\, d\theta = \frac{k\lambda}{r}(\sin\frac{\pi}{2} - \sin 0) = \frac{k\lambda}{r}\hat{i}$

$(E_{uy}) = \int_0^{\pi/2} \frac{k\lambda r d\theta}{r^2}\sin\theta = \frac{k\lambda}{r}\int_0^{\pi/2}\sin\theta\, d\theta = \frac{k\lambda}{r}{}^{-}[\cos\frac{\pi}{2} - \cos 0]$

$(\vec{E}_u)_y = \frac{k\lambda}{r}(-\hat{j})$



$(\vec{E}_l)_x = -\frac{k\lambda}{r}\hat{i}$, $(\vec{E}_l)_y = -\frac{k\lambda}{r}\hat{j}$

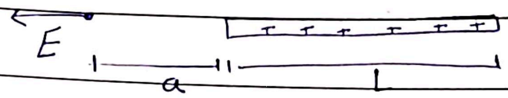
$\vec{E} = \vec{E}_u + \vec{E}_l = \left(\frac{k\lambda}{r}\hat{i} - \frac{k\lambda}{r}\hat{j}\right) + \left[\left(-\frac{k\lambda}{r}\hat{i}\right) - \left(\frac{k\lambda}{r}\right)\hat{j}\right] = 2\frac{k\lambda}{r}\hat{i}$

* Summary:

$\vec{E}$ due to a continous charge distribution:

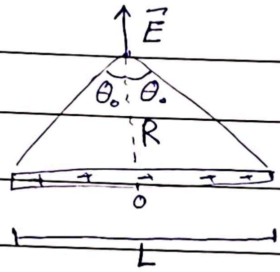
1) $\vec{E}$ due to a uniformly charged rod

$$E = \frac{q}{4\pi\varepsilon_0 a(L+a)}$$

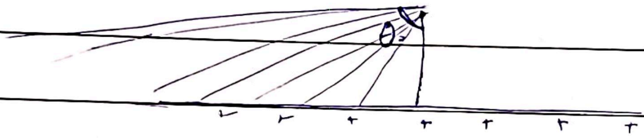


2) $\vec{E}$ due to a uniformly charged rod: (finite)

$$E = \frac{\lambda}{2\pi\varepsilon_0 R}\sin\theta_0$$



* if L is very long $\Rightarrow L \to \infty \Rightarrow \theta_0 \to \frac{\pi}{2}$



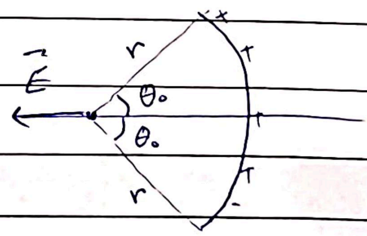
$$\Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 R}\sin \pi/2$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 R}$$ (Very long line of charge)

$L \gg R$

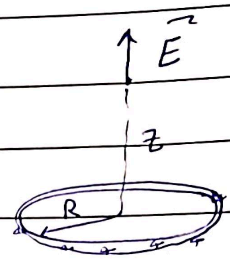
3) $\vec{E}$ due to a uniformly charged circular arc:

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}\sin\theta$$



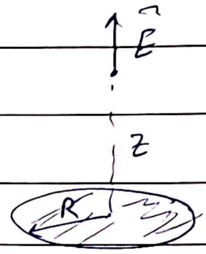
4- $\vec{E}$ due to a uniformly charged ring:

$$E = \frac{q z}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}}$$



5- $\vec{E}$ due to a uniformly charged disk:

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

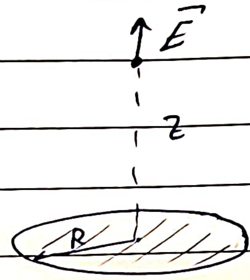


p17:

Case (1)

disk : Radius = R

$z = 2R$



$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

$z = 2R$

$$\Rightarrow E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{2R}{\sqrt{(2R)^2+R^2}}\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{2\cancel{R}}{\sqrt{5}\,\cancel{R}}\right]$$

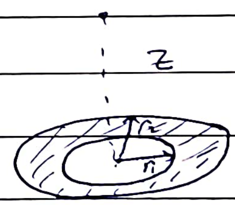
$$= 0.0528\,\frac{\sigma}{\varepsilon_0}$$

Case (2)

disk : $r_1 = \frac{R}{4}$

$r_2 = R$

$z = 2R$



$$E'_{disk} = \int_{R/4}^{R} dE_{ring}$$

, Back to E_{disk} & change the integral limits ($R/4 \rightarrow R$)

$$E'_{disk} = \frac{-\sigma z}{2\varepsilon_0}\left[\frac{1}{\sqrt{z^2+r^2}}\right]_{R/4}^{R}$$

$$E'_{disk} = \frac{-\sigma z}{2\varepsilon_0}\left[\frac{1}{\sqrt{z^2+R^2}} - \frac{1}{\sqrt{z^2+\left(\frac{R}{4}\right)^2}}\right]$$

$$z = 2R$$

$$E' = \frac{-\sigma 2R}{2\varepsilon_0}\left[\frac{1}{\sqrt{(2R)^2+R^2}} - \frac{1}{\sqrt{(2R)^2+\frac{R^2}{16}}}\right]$$

$$= \frac{-\sigma R}{\varepsilon_0}\left[\frac{1}{\sqrt{5R^2}} - \frac{4}{\sqrt{65R^2}}\right]$$

$$= \frac{-\sigma R}{\varepsilon_0}\left[\frac{1}{R}\left(\frac{1}{\sqrt{5}} - \frac{4}{\sqrt{65}}\right)\right]$$

$$= 0.0489\,\frac{\sigma}{\varepsilon_0}$$

At what percentage will the electric field decrease?

$$\frac{E - E'}{E}(100\%) = \frac{0.0528\left(\frac{\sigma}{\varepsilon_0}\right) - 0.0489\left(\frac{\sigma}{\varepsilon_0}\right)}{0.0528\left(\frac{\sigma}{\varepsilon_0}\right)}$$

$$= 7.3\ \%$$